

ILLUSTRATING THE MATHEMATICS STANDARDS



The following examples of student work illustrate achievement at the mathematics standards for years 6, 7, and 8.

Stained Glass Windows

The task used in this illustration was part of a visual arts and technology unit in which students were asked to design a stained glass window. The students explored geometric shapes used in stained glass patterns and the technology associated with joining pieces of glass.

For this task, the students were asked to use a “maths lens” and to document aspects of number, geometry, and measurement evident in their designs.

The task relates to achievement objectives for Number and Algebra and for Geometry and Measurement from the mathematics and statistics learning area in *The New Zealand Curriculum*.

Stained Glass Windows

1. *Design a stained glass window to fit a 1-square-metre frame.*
2. *After you have designed your window, divide a piece of paper into three sections, Number, Geometry, and Measurement, and describe your design, using numbers, symbols, and mathematical language. For example:*
 - *under Number, calculate the fraction that each shape is of the whole*
 - *under Geometry, comment on transformations in your design*
 - *under Measurement, calculate the areas of coloured glass needed for your design and the perimeter of the shapes (you can use this later to work out how much lead you will need).*

Some features of students' work used to make judgments in relation to the mathematics standards are described below.

Stained Glass Windows

New Zealand Curriculum: Level 3

In solving problems and modelling situations, students will:

Number and Algebra

- use a range of additive and simple multiplicative strategies with whole numbers, fractions ... (number strategies)

Geometry and Measurement

- use ... whole numbers of metric units for length, area ...
- find areas of rectangles ... by applying multiplication (measurement)
- describe the transformations (reflection, rotation, translation, or enlargement) that have mapped one object onto another (transformation)

Mathematics Standard: By the end of year 6

Number and Algebra

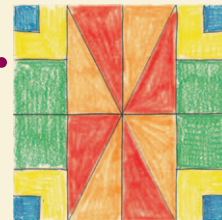
- apply additive and simple multiplicative strategies flexibly to:
 - combine or partition whole numbers
 - find fractions of ... shapes ...

Geometry and Measurement

- measure ... the attributes of objects, choosing appropriate standard units
- represent and describe the results of reflection [and] rotation ... on shapes ...

Riley stated what fraction of the whole window the green squares and the triangles were. He used repeated division (halving) to work out what fraction 1 green square was of the window, given that 4 green squares were one-quarter.

Riley and Tino created this design. Riley described it.



Riley used a partitioning strategy to multiply 25×25 in order to find the area of 1 green square. He expressed this in an appropriate standard unit (square centimetres).

Number -
8 squares
8 triangles
Squares are $\frac{1}{4}$ of this space
Triangles are $\frac{1}{2}$ of the same
2 green squares = $\frac{1}{8}$ 1 green square = $\frac{1}{16}$

geometry -
2 of the same size make a rectangle just flip over
Triangles
Squares
rotation
Symmetry To cut in half and reflect the same squares for the squares are in reflection patterns
4 small make 1 large
you can go a half turn

Measurement
1 meter \times 1 meter = 1 m² - this is the area of whole window
area of 1 green square 25cm \times 25cm = 625cm²
25 \times 10 = 250
25 \times 10 = 250
25 \times 5 = 125
so 625 altogether

He identified examples of reflective and rotational symmetry.

The teacher noted as a teaching point that Riley needed help to expand his repertoire of strategies for multiplying two 2-digit numbers.

Discussion

This task provides some of the evidence needed to show that Riley is achieving at curriculum level 3 and the year 6 standard in Number, Geometry, and Measurement. He is able to find areas of squares, use appropriate measurement units, and describe symmetry in terms of reflection and rotation. He has demonstrated that he is able to use a multiplicative strategy with whole numbers, which suggests that he is working at the Advanced Additive stage of the Number Framework.

Stained Glass Windows

New Zealand Curriculum: Level 4

In solving problems and modelling situations, students will:

Number and Algebra

- find fractions, decimals ... of amounts expressed as whole numbers, simple fractions ... (number strategies and knowledge)

Geometry and Measurement

- use appropriate ... metric units for length, area ...
- use side or edge lengths to find the perimeters and areas of rectangles [and] triangles (measurement)

Mathematics Standard: By the end of year 7

Number and Algebra

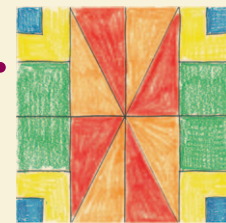
- apply additive and multiplicative strategies flexibly to whole numbers ... and equivalent fractions

Geometry and Measurement

- measure ... the attributes of objects, using metric ... measures
- use side or edge lengths to find the perimeters and areas of rectangles ..., given whole-number dimensions
- identify and describe the transformations that have produced given shapes or patterns

Tino used multiplicative strategies to calculate how many blue squares would make up the whole. She realised that the 4 blue squares together were $\frac{1}{4}$ of a $\frac{1}{4}$ of the window or $\frac{1}{16}$ of the whole.

Tino and Riley created this design. Tino described it.



Tino converted between fractions and decimals to work out the area of the window that was red. She could see that the green squares had the same area as the red triangles.

She could see that there was 3 times as much yellow as blue. She simplified $\frac{12}{64}$ to get the equivalent fraction $\frac{3}{16}$.

She used additive strategies with equivalent fractions ($\frac{1}{2} = \frac{8}{16}$, $1 = \frac{16}{16}$) to calculate the relative proportions of warm and cool colours.

Tino also identified that the angles meeting in the middle of the window combined to equal 360° and that two right-angled triangles formed a rectangle.

NUMBER

4 blue squares
 $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$
 64 bluesquares make the whole
 blue squares fill up $\frac{1}{4}$ of $\frac{1}{4} = \frac{1}{16}$
 yellow squares make $3 \times \frac{1}{64} = \frac{3}{64}$ $\times 4 = \frac{12}{64} = \frac{3}{16}$
 Warm colours Yellow + red + orange
 $\frac{1}{2} + \frac{3}{16} = \frac{8}{16} + \frac{3}{16} = \frac{11}{16}$
 Cool colours must be $1 - \frac{11}{16} = \frac{5}{16}$ bit bigger than $\frac{1}{4}$
 almost $\frac{3}{4}$

MEASUREMENT

window is $1m \times 1m$
 area = $1m^2$ perimeter = $4m$
 area for 4 red triangles
 $0.5 = \frac{1}{2}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25m^2$
 4 green squares is the same area as the red triangles
 perimeter of blue square $20 + 1m = 21m$
 $12.5 \times 4 = 50cm$
 $12.5 + 12.5 = 25$
 $25 + 25 = 50$

GEOMETRY

uses squares
 uses right-angled triangles
 put shapes together to make rectangles
 all angles of triangles that touch centre make 360° full turn
 Cool colours are reflected
 warm colours show rotation 180°

The teacher noticed that Tino calculated the perimeter of a blue square aloud, using an additive strategy flexibly, and asked her to record her thinking.

In describing her window, Tino identified reflective and rotational symmetries.

Discussion

This task provides some of the evidence needed to show that Tino is achieving at early curriculum level 4 and the year 7 standard in Number, Geometry, and Measurement. She has demonstrated that she is able to calculate areas and perimeters of rectangles and triangles and to identify and describe transformations. She is able to use additive strategies flexibly with equivalent fractions and to use multiplicative strategies to solve problems involving fractions, which suggests that she is working at the Advanced Multiplicative stage of the Number Framework.

Stained Glass Windows

New Zealand Curriculum: Level 4

In solving problems and modelling situations, students will:

Number and Algebra

- find fractions, decimals, and percentages of amounts expressed as whole numbers, simple fractions, and decimals (number strategies and knowledge)

Geometry and Measurement

- convert between metric units, using whole numbers and commonly used decimals
- identify classes of two- ... dimensional shapes by their geometric properties (shape)

Mathematics Standard: By the end of year 8

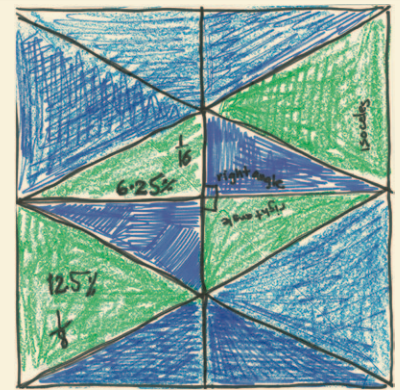
Number and Algebra

- apply multiplicative strategies flexibly to whole numbers ... and equivalent fractions (including decimals and percentages)

Geometry and Measurement

- make simple conversions between units, using decimals
- sort two- ... dimensional shapes into classes, considering the relationships between the classes and justifying the decisions made

Keegan created this design and then described it.



Keegan classified the triangles in his stained glass window based on angles and lengths of sides.

1. geometry
 8 right angle triangles
 4 isosceles ones
 parallellograms put Δ together
 rotate top $\frac{1}{2}$ and set bottom $\frac{1}{2}$ go 180°

2. measurement
 area 1 \square $1m^2$
 $\frac{1}{2}$ $0.5m^2$
 $\frac{1}{4}$ $0.25m^2$
 $\frac{1}{8}$ $0.125m^2$
 blue is $6.250cm^2$
 $0.625m^2$
 $3 \times 0.125 = 0.375m^2$ or $3750cm^2$
 3 is green

He renamed common fractions as decimals.

He converted square metres to square centimetres, using numbers involving decimals.

3. number
 fractions
 isosceles Δ are $\frac{1}{2}$ of each $\frac{1}{4} = \frac{1}{8}$ 12.5%
 right angle Δ are $\frac{1}{16}$ because 2 make 1 of those
 green $\frac{2}{8} + \frac{2}{16} = \frac{3}{8}$
 $\frac{1}{4}$ 25% + 12.5% = 37.5% is green!

He worked flexibly with fractions, decimals, and percentages when calculating what fraction each triangle was of the window and the percentage of green in his window.

He used the multiplicative relationship between $\frac{1}{8}$ and $\frac{3}{8}$ when calculating the area of green in the window.

Discussion

This task provides some of the evidence needed to show that Keegan is achieving at curriculum level 4 and the year 8 standard in Number, Geometry, and Measurement. He has demonstrated that he is able to group shapes by property and justify his classifications. He is also able to use fractions, decimals, and percentages flexibly to solve problems and can make conversions between standard units with numbers involving decimals, which suggests that he is working at the Advanced Multiplicative stage of the Number Framework.