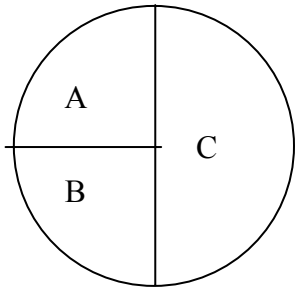
Mathematics in the New Zealand Curriculum
Second Tier

Strand: Statistics Thread: Probability Level: 2

Achievement Objective:
Investigate simple situations that involve elements of chance, recognizing equal and different likelihoods and acknowledging uncertainty.

Exemplars of student performance:
Exemplar One:
In an experiment with this spinner Larry predicts that C is more likely than A or B, which have about the same likelihood.
[See: http://nlvm.usu.edu/en/nav/frames_asid_186_g_1_t_5.html?open=activities&from=category_g_1_t_5.html]
He accepts that in one spin any of the 3 letters might come up (accepts uncertainty).
Larry tries twenty spins and records his results.
He compares his predictions with the recorded results but is unsure when the natural variation of these results contrasts with his predictions,
e.g. "I got eleven spins for C. I thought ten. I'm not sure why that happened."



A	B	C
𝍸	𝍸	𝍸 𝍸 I

Where the results vary markedly from his prediction Larry questions his original thinking or the validity of the experiment.
e.g. "I should get more C's than B's. Maybe the spinner is not working properly."

A	B	C
𝍸	𝍸 III	𝍸 III

Exemplar Two: In a trial of drawing red, yellow, and blue "lollies" (cubes) from a mystery bag of 100 cubes Rongopai records her results systematically. She organises the cubes she collects in rows to form a data display:

Blue 

Yellow 

Red 

Rongopai notes that there appears to be more yellow lollies than blue lollies" in the bag so getting a yellow lolly is more likely than getting a blue or red "lolly", and that getting a blue lolly is more likely than getting a red.
When asked if there could still be the same number of red and blue lollies in the bag she refutes the idea. Rongopai does <u>not</u> accept that this result may be due to variation rather than the actual numbers of each colour in the bag.

Exemplar Three: Mali tosses a coin ten times and records her results. She gets five heads and five tails. Mali is happy with that result as she expected the same number of heads (accepted equal chance).
Her teacher refers Mali to the pattern of heads and tails:

H H T H H T H T T T

Asked what will happen on the next throw, Mali says that either heads or tails might happen (uncertainty). However she suggests that heads should "come along shortly."
Asked what the next ten tosses might be, Mali says that she will probably get five heads and five tails again (does not accept variation).

Important teaching ideas

Students at level two need to accept uncertainty, that is, there is no definitive predictability to what will occur with any next event. Link the concepts of equal and unequal probability to events in student's lives. Pose questions such as "Is it more likely to be sunny or cloudy tomorrow?" or "Will you have takeaways this week?" or "Which television programme, X or Y, will the most people watch this week?" or "When you go to cross the road, will a vehicle be coming?" Continue to develop students' probability vocabulary. Use words such as "equal", "greater" and "less" to compare the chance of

different outcomes.
Use data displays such as pictographs to show the outcomes from situations involving elements of chance, e.g. weather each day over three weeks, number of birds sitting on the telephone wire at 9:00am, or children who bring fruit for lunch. The data displays can become a source of prediction, e.g. How many children do you expect walked to school today?

Everyday life events often have an element of repeating pattern that help with prediction, e.g. It has rained the last two days and it is still cloudy, the bus is usually three minutes late, or our family usually has fish and chips on Friday. Such pattern helps with the degree of confidence we feel about our predictions. However, in situations where we have no contextual knowledge then we need to either experiment or sample to find out or, in rare occasions, we can reason the possible outcomes.
Use simple one-stage experiments such as tossing coins and dice and spinning spinners. This makes it easy for students to make predictions using a list or diagram of all the outcomes. Get the students to keep record of their experimental results, and to use this record to predict the next outcome. Contrast what happens as many students find the next result. The structural idea here is independence, that is, given several possibilities, the next outcome can never be certain despite the history of previous events.
For example, the pattern from five coin tosses is H, T, H, T, T. Ask the students to predict the outcome of the next toss. A common misconception is that the previous pattern of outcomes will affect the outcome of the next toss, e.g. "There have been two tails in a row so it's got to be a head," or, "So it's got to be another tail." Compare students' predictions with actual occurrence.
Students often have difficulty regarding successive trials as replications of the same event, for example, they see each coin toss as an unrelated event. Students often find the pooled class results of a single trial, e.g. each student tosses a coin once, more easily related than a history of themselves tossing a coin many times. They need experiences with both types of experiment, individual and pooled.
Approach the concepts of equality and inequality of likelihood in two ways. Firstly, expect students to predict what outcomes are more likely than others from discussions of all the possibilities. For example, if a dice has four "yes" faces and two "no" faces ask the students predict if they will get more "yes" outcomes than "no" outcomes, the same number, or less "yes" outcomes. Use lists and diagrams to model all the possible outcomes of an experiment. This will develop student's ability to create theoretical models of all the outcomes. With level two students, equality or inequality of likelihood is established by count, e.g. three sides of the cube are blue, three sides are yellow.
Secondly, support or contradict their predictions by trialing an increasing number of times. Students' comparisons at this level will tend to be relative, e.g. "There were four more "yeses" than "noes"", rather than proportional, e.g. "Well over two-thirds of the tosses were "yeses"". Provide experiences that get students to question their use of the history of

outcomes as a basis for predicting the next outcome. Use simple chance events like drawing one card from a box containing ace, two, three, replacing the chosen card before each turn.
Carry out several trials and record the outcomes, e.g. 2, 3, 3, 2, 1, 2, 3… Ask the students to predict what the next card will be. Discuss their beliefs and resulting predictions. Be aware that work in algebra promotes pattern-seeking behaviour, e.g. "It will be 3 because the pattern goes 2, 3, 3," and this can interfere with the development of ideas about uncertainty. Contrast the success of different methods for choosing the next number. Apply the different methods of prediction across many experiments. The aim is to help students realise that each trial is an independent event. A theoretical probability found by working out all the possible outcomes is the best, yet uncertain, predictor of the next outcome.
Broaden students' methods for recording all the possible outcomes by involving them in simple two stage events, like tossing a coin twice, or drawing out two cards from the box containing ace, two, and three. Develop the use of organised lists, tables and tree diagrams as recording methods to find all of the outcomes (see level 3).

Possible resources
Figure It Out, Statistics Levels 2-3, Pages 17 – 24.
nzmaths.co.nz units:
http://www.nzmaths.co.nz/node/129 (Probability: That's Not Fair)
Learning Objects:
http://www.tki.org.nz/r/digistore/protected/objects/?id=118&vers=1.0
http://www.tki.org.nz/r/digistore/protected/objects/?id=168&vers=1.0
http://www.tki.org.nz/r/digistore/protected/objects/?id=3653&vers=2.0
http://www.tki.org.nz/r/digistore/protected/objects/?id=2392&vers=1.0
http://www.tki.org.nz/r/digistore/protected/objects/?id=3546&vers=1.0
http://www.tki.org.nz/r/digistore/links/1619e68b9ff93ab312da61163c5ea884/ec_002_utah_014/applet.html
Topic Based Mathematics One- Probability supplement (If available in your school)