# Going Public: Students' Views about the Importance of Communicating Their Mathematical Thinking and Solution Strategies

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This paper reports on data from 183 nine- to eleven-year old students attending six Waikato schools, four of which had participated in the Numeracy Development Project (NDP) and two that had not. Students' responses to questions about communicating their mathematical thinking and strategies to peers were analysed and patterns identified. There was considerable variation from school to school in students' ideas about the value of communicating mathematically with others. Students at the two schools that had not yet been involved in the NDP did not differ markedly from those at participating schools in the interviews about communicating mathematically with others. Overall, the students were more positive about the value of explaining their own thinking to other people than they were about knowing what strategies their peers used to solve mathematics problems.

For more than a decade, mathematics reform documents have been calling for a shift in approach to teaching and learning mathematics (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). A key aspect of the reforms is the idea that mathematics learning should no longer be a private, individual activity but instead involve groups of learners who challenge and support one another as they reason their way through problems (Lampert & Cobb, 2003). Students benefit not just by learning to communicate mathematically; by communicating, they also learn mathematics (NCTM, 2000). According to the current curriculum document for New Zealand schools, communication is essential if students are "to express ideas, and listen and respond to the ideas of others" (Ministry of Education, 1992, p. 9).

As part of the mathematics reforms, much work has been done on the ways that teachers and students approach mathematics teaching and learning in classrooms, the so-called "sociomathematical norms" identified by Yackel and Cobb (1996). Such studies provide exemplars of what is possible when conditions are favourable and teachers are exceptionally capable and committed. Less is known about implementing mathematics reforms in "ordinary" classrooms and schools.

In a separate but parallel development, there has been substantial work on the value of listening to and talking with students themselves (Carr, 2000; Davies, 1982; Devereux, 2001; Paley, 1986; Roberts, 2000; Rudduck & Flutter, 2000; Smith, 1995; Smith, Taylor, & Gollop, 2001). Recent research has focused on the issue of student "voice" and, in particular, on the importance of finding out how students see themselves as learners (Fielding, Fuller, & Loose, 1999; Freeman, McPhail, & Berndt, 2002; Phelan, Davidson, & Cao, 1992; Pollard, Thiessen, & Filer, 1997; Young-Loveridge & Taylor, 2003). According to McCallum, Hargreaves, and Gipps (2000), pupils' voice is important in understanding schools and schooling. The UN Declaration on Human Rights states explicitly that children should be given a voice on matters that have an impact on them (New Zealand Ministry of Foreign Affairs & Trade, 1997).

New Zealand has put in place a major initiative in numeracy that is designed to raise mathematics achievement by improving the professional capability of teachers in teaching mathematics (see *Curriculum Update no. 45 and no. 51.* Wellington: Ministry of Education,

2001). Key components of this initiative include a Number Framework that outlines progressions in students' learning about number, an assessment tool to pinpoint students' learning needs, professional development programmes for teachers to help them become familiar with these tools, and additional resources to support students' learning. As part of the evaluation process, the perspectives of teachers, principals, and facilitators have been researched, but little has been done to find out from students themselves how they see the project and its impact on their mathematics learning.

This paper presents data from part of a larger project that set out to explore students' perceptions and dispositions towards learning mathematics. A range of different issues were explored with students, including their views about the nature of mathematics, mental computation processes, communication of solution strategies with others, and teachers' and family/whānau roles in supporting mathematics learning. This paper focuses on one aspect of the larger study: namely, students' ideas about communicating mathematical thinking and strategies with their peers.

## Method

### *Participants*

The participants in this study consisted of 183 year 5 and 6 students (nine- to eleven-year-olds) at six schools. Table 1 shows the composition of the sample. More than half of the students were Māori, over a third were European, a tenth were Pasifika, and the remainder were Asian or another ethnic group. Four of the schools (marked with an asterisk) had participated in the NDP and were from a large urban centre. The two schools that had not yet participated in the numeracy projects (non-NDP) were from a small neighbouring town.

Table 1
*Composition of the Sample in Terms of Gender, Ethnicity, and Decile (Eur = European, Ma = Māori, Pas = Pasifika, As = Asian)*

| School | Decile | Boys | Girls | | Eur | Ma | Pas | As | Other | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Arch* | low | 21 | 17 | | 9 | 20 | 5 | 2 | 2 | 38 |
| Bank* | low | 8 | 15 | | 1 | 17 | 5 | | | 23 |
| City* | middle | 16 | 14 | | 21 | 3 | 1 | 5 | | 27 |
| Dale* | low | 15 | 15 | | 11 | 11 | 3 | 4 | 1 | 30 |
| Edge | low | 19 | 11 | | 2 | 28 | | | | 30 |
| Farm | low | 17 | 15 | | 9 | 22 | 1 | | | 32 |
| | | | | | | | | | | |
| Total | | 96 | 87 | | 53 | 101 | 15 | 11 | 3 | 183 |

### *Procedure*

Schools were asked to nominate about 30 year 5 and 6 students from across a range of mathematics levels. Students were interviewed individually in a quiet place away from the classroom. Students were told that the interviewer was interested in finding out more about "how kids learn maths and how their teachers can help them" and "what kids themselves think about learning maths". Interviews were transcribed for later analysis. Once the interviews were complete, schools were asked to identify each student's current stage on the Number Framework, or, if at a non-NDP school, whether the student was "average", "above average" or "below average" in mathematics, using their knowledge of the students' achievement that year.

This information was used to categorise students as *high* (above average or at stage 6: Advanced Additive Part–Whole or above), *middle* (average or at stage 5: Early Additive Part–Whole), or *low* (below average or at stage 4: Advanced Counting or below).

During the interview, students were asked to comment on a range of topics, including the importance of working out problems mentally, of getting answers correct, and whether they thought there was only one way or several different ways of working out an answer. They were then asked the following questions and the reasons for their responses:

> Do you think it is important for you to know how other people get their answers? Is it important for you to be able to explain to other people how you worked out your answer?

## Results

Table 2 presents a summary of students' responses to the questions explored in the study. This table shows considerable variation from school to school in students' responses to the questions.

Table 2

*Percentages of Students Who Responded to the Questions about Knowing and Sharing Solution Strategies with Peers (numbers are shown in brackets)*

*(DK = Don't know or an ambiguous response)*

| | Importance of knowing others' strategies | | | | Importance of explaining one's own strategies to others | | | |
|---|---|---|---|---|---|---|---|---|
| School | Yes | No | DK | Total | Yes | No | DK | Total |
| Arch* | 27.8 (10) | 52.8 (19) | 19.4 (7) | 36 | 48.6 (18) | 27.0 (10) | 24.3 (9) | 37 |
| Bank* | 39.1 (9) | 43.5 (10) | 17.4 (4) | 23 | 56.5 (13) | 39.1 (9) | 4.3 (1) | 23 |
| City* | 77.8 (21) | 11.1 (3) | 11.1 (3) | 27 | 74.1 (20) | 7.4 (2) | 18.5 (5) | 27 |
| Dale* | 46.7 (14) | 46.7 (14) | 6.7 (2) | 30 | 60.0 (18) | 23.3 (7) | 16.7 (5) | 30 |
| Edge | 17.9 (5) | 71.4 (20) | 10.7 (3) | 28 | 48.3 (14) | 17.2 (5) | 34.5 (10) | 29 |
| Farm | 31.3 (10) | 56.3 (18) | 12.5 (4) | 32 | 50.0 (16) | 46.9 (15) | 3.1 (1) | 32 |
| | | | | | | | | |
| *NDP* | *46.6 (54)* | *39.7 (46)* | *13.8 (16)* | *116* | *59.0 (69)* | *22.2 (26)* | *18.8 (22)* | *117* |
| *Non-NDP* | *25.0 (15)* | *63.3 (38)* | *11.7 (7)* | *60* | *49.2 (30)* | *32.8 (20)* | *18.0 (11)* | *61* |

Of the six schools, City had the highest levels of agreement from students about the importance of knowing others' strategies (77.8%) and also about the importance of explaining one's own strategies to others (74.1%). The majority of students at Edge School, on the other hand, felt strongly that it was *not* important to know how other people work out their answers (71.4%). Overall, there were higher levels of agreement about explaining one's thinking to other people than to knowing about other people's strategies and thinking (55.4% vs 38.8%).

As Table 2 shows, there was considerable variation from one school to another in students' views about the importance of communicating about mathematical thinking with their peers. When the responses of students from the four NDP schools were put together and compared with those of the non-NDP students, an interesting pattern emerged. There was a statistically significant difference between the two groups, but only with respect to knowing about how other people had worked out the answers to their problems [$\chi 2$ (2) = 9.47, $p < 0.01$]. Almost half of the students at NDP schools thought that knowing about how other students solved

problems was important, compared to only a quarter of the non-NDP group (see Figure 1). The reverse pattern was evident in the responses of students who thought that knowing others' solution strategies was *not* important; almost twice as many non-NDP as NDP students thought that knowing others' solution strategies was *not* important (63.3% vs 39.7%). However, the responses of students at non-NDP schools differed little from those of students at NDP schools on the question about the importance of explaining one's strategies to others (see Figure 1).

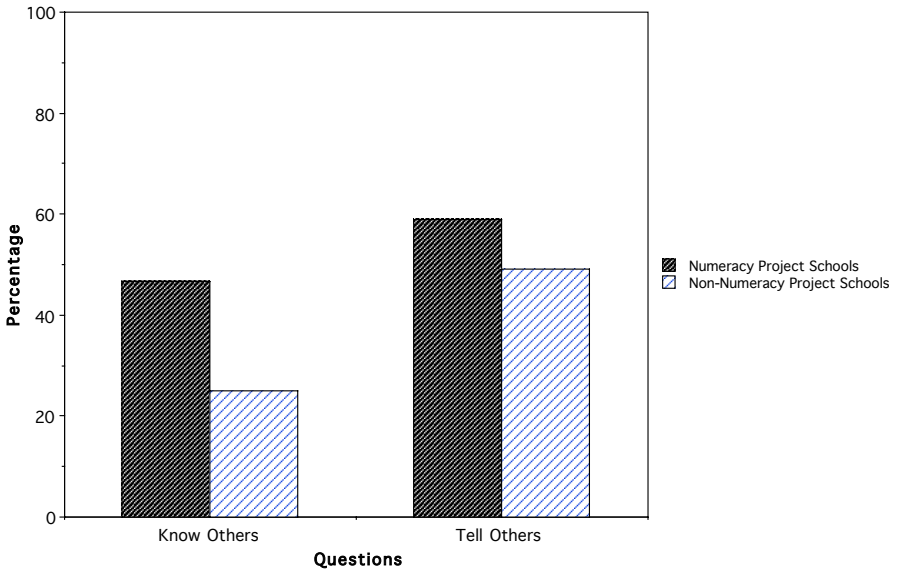


*Figure 1*. Percentages of year 5–6 students who thought that particular ideas were important as a function of participation in the NDP

Students' responses to the interview questions reflect the extent to which they thought communication was important for their mathematics learning. In the interviews, we endeavoured to find out *why* students held particular views. The following sections present students' responses to the two questions that are the focus of this paper, about *knowing* about other people's solution strategies and about *explaining* one's own solution strategies to others. The responses of those who thought that it was important are presented first, followed by the ideas of those who thought that it was *not* important.

*1. Knowing others' strategies is important*

Several students referred to the usefulness of having *alternative* ways of solving problems as their reason for saying that knowing others' strategies was important:

> They might have another way. (A11, boy, high)
>
> I could try and do it their way. (A24, girl, middle)
>
> Because other people may have different ways and you can learn off them and you maybe get better and better. (C18, boy, low)

Some students referred explicitly to the value for their own *learning*:

> Because then I can learn from it and other people can too. (B19, boy, high)
>
> So you can learn from them. (F12, girl, high)
>
> Because they're helping you understand. (F8, girl, high)
>
> Yes because then I can learn from it and other people can too. (B8, boy, middle)
>
> It could help your learning by getting better if they can show you how they got to work it out. Sometimes you can get very confused about what other people say, but it's good to hear their learning because sometimes it's most likely to help your learning as well. (C19, girl, low)

It was interesting to note that some of these responses (F12 & F8) came from students attending Farm School, a non-NDP school.

*Helping* other students learn was given as the reason by some students:

> Because you want to see people succeed with stuff because they do have a bit of trouble, but I just sometimes help them a bit. (F2, girl, high)

A group of students seemed preoccupied with the *correctness* of their answers, even though the NDP had tried to shift the focus away from correct answers onto examining the variety of strategies:

> To get the answer right. (B5, girl, middle)
>
> Because if they get what the answer is, if they get it right … it can be in your head too. (F32, boy, low)

Some students seemed more concerned about classroom *etiquette* and showing respect for their classmates:

> Because when you listen to other people, those other people might listen to you and learn your answer. (A29, girl, middle)

*Relationships* were an important consideration for some students. One student thought that knowing how other people solve problems was only important among close friends:

> Only if they're really close to you and they're really good friends. (A37, boy middle)

*2. Knowing others' strategies is <u>not</u> important*

Students who disagreed with the idea that knowing others' strategies is important had a range of different reasons for their view. The most frequent reason given referred to *dishonesty*:

> Because it's just like cheating. (F3, girl, middle)
>
> Because you're using their brains not yours. (A31, girl, low)
>
> Because you don't do that, because that's cheating. (A8, boy, middle)
>
> No, because then you're just like cheating off them and you don't really learn for yourself, you're learning off other people and when you go to do it by yourself you don't actually know because you've cheated off someone else. (A13, girl, high)

*Privacy* was referred to by some of the students who thought that knowing about other students' strategies was not important:

> Because it's their business and not mine. (F4, girl, middle)
>
> Because it's their work and not yours. (B7, boy, middle)
>
> Because you don't want to know about it, it's only important to them. (F31, boy, high)
>
> Because it's their own business how they do it … It's not important for me because I know my own way to work things out. (C15, boy, low)

> Because it's their way they do it, you don't have to, 'cause it's really none of your business. (C26, girl, low)

Some students thought that *individuality* was important:

> Because everyone's different and everyone has their own way (F5, girl, middle)
>
> Because you should know yourself. (F22, girl, middle)
>
> Because I like doing it by myself. (A2, boy, middle)

The idea of *reciprocity* came through in the responses of some students. They expected to alternate between being the "helper" and the person being helped.

> Because if they get the answer right then they can tell you and help you and then if they don't know a question, if they don't know the answer then we can help them. (A16, boy, high)

*3. Explaining One's Strategies to Others is Important*

Students tended to be in greater agreement about the importance of explaining their strategies to other students than they had been about knowing others' strategies. Many of these students referred to helping others with their *learning*:

> So they can learn how to do it next time they go to do it. (B15, girl, high)
>
> Because they can learn and they can go up another level. (B6, girl, middle)
>
> It's helping my learning as well so it's good to share what your side of the story is. (C19, girl, low)
>
> Because others may not understand it and you might. (A31, girl, low)
>
> So other people can learn from my answer. (B8, boy, middle)

Sometimes the helping involved withholding some information because of *concern* that too much information might prevent other students from working something out for themselves:

> Just how you solved it up, only a little bit, not too much, because people get out too much information. [Asked: "What would be the problem if you gave them too much information?"] They'll know heaps about it so they won't figure it out for themselves (D1, boy, middle)
>
> Sometimes if they're really stuck, but I won't actually give them my answer … I'll give them clues on how to work it out and that. (A13, girl, high)

Sometimes the reason for helping others with their learning was that the teacher was very busy and it facilitated *classroom organisation* if some of the more proficient mathematicians explained their ways of solving problems to less proficient classmates:

> Because if they wanted to learn something and they asked the teacher and the teacher's busy then they can go to a person that will know the answer and they can explain to them how you can add up to that. (B11, girl, high)

The value of *alternative* strategies was mentioned by some students:

> Because there's lots of different ways, and maybe I have a different way to them. (B17, boy, middle)
>
> Because it's good that I know how I worked out the answer first of all, and it's good 'cause I like sharing my ideas with other people and my point of view of how I can work it out, and so if I say my way and another person tells their way and their way's a bit easier, I can just try it their way, and then you get lots of different ways by telling your one, then other people go "I've got a simpler way of doing that", so it helps you to learn. (C2, boy, high)

The responses of some students hinted at the need to "*prove*" that they had solved the problem themselves before finding out how other students had come up with a solution:

> Because sometimes people ask you how you did it, so you've got to know how you did it, so before you take their answer, you say the answer, you think about how you did it. (C20, boy, high)
>
> So they know that you don't copy off other people. (B1, boy, middle).

Building and maintaining *relationships* was important for many students and this came through in their responses to the questions:

> So you can help your mates out. (A17, boy, middle).

*4. Explaining one's strategies to others is <u>not</u> important*

Relatively few students thought that explaining their thinking to others was *not* important. The reasons these students gave for their responses were similar. *Dishonesty* was the most frequent reason given:

> It's just cheating. (A19, girl, low).
>
> Because that's cheating. (F21, boy, middle)
>
> Because that's just like cheating. (F28, boy high)

*Individuality* was given as a reason for not explaining one's strategies to others:

> You should always worry about your work before you go to other people about their work. You never know when they could get a wrong answer and you could as well. (B1, boy, middle)
>
> Because they could have their own way to work it out. (C15, boy, low)
>
> Because different people have different ways of doing their answers. (F4, girl, middle)

*Privacy* was again referred to as a reason for not explaining one's own strategies to others:

> Because it's my work and not theirs. (F5, girl, middle)
>
> Because if we tell them how you worked out the answer, then they'll go round telling everybody else, then everybody else would get the same answers. (A38, girl, low)

Some students thought that explaining one's strategies to other students might *hinder learning r*ather than help it:

> If you told them the answer they won't be able to learn. (A18, girl, middle).
>
> Because they've got to learn, they've got to learn … (F10, boy, high)

## Discussion

Although the mathematics reforms have called for mathematics to be a more public activity with learners communicating openly about their ways of solving problems, the findings reported in this paper show that it is not easy to achieve this goal. There was considerable variation from school to school in the proportion of students who thought that the communication of mathematical thinking and strategies was important. On average, students at NDP schools were twice as likely as those from non-NDP schools to think that knowing about others' strategies was important. Both groups were similar in their view that explaining their strategies to others was important. However, there was still quite a number of students in NDP schools who, despite having experienced the sharing of mathematics strategies as part of the numeracy projects (in particular, in-class modelling by numeracy facilitators), still seemed to believe that mathematics should be private and was "no one else's business". This finding highlights the difficulties involved in bringing about changes in students' ideas about learning mathematics.

Similar comments have been made about the challenges of mathematics reform for teachers and students in the US (Lampert & Cobb, 2003).

The students' responses seem to reflect confusion about the difference between dishonesty/cheating and being helpful/co-operative. There appears to be a lack of clarity for students about what constitutes "cheating". It would seem that for them, the line between being dishonest and being helpful/co-operative is a very fine one. The fact that substantially more students thought it important to be able to explain their strategies to others (because they saw it as helpful for their peers) than to know about others' strategies (often seen as cheating) is consistent with this idea. Students may be receiving contrary messages in other contexts (for example, assessment situations) in which talking to others is seen as not appropriate.

Some of the students' responses reflected a concern about developing and maintaining relationships with others. This is consistent with the claims of some writers that having friends in a class is so important that sometimes students put being with their friends ahead of academic considerations (Duffield, Allan, Turner, & Morris, 2000; Phelan et al., 1992). Maintaining relationships may also have been an underlying issue for those students in our study who were worried about "cheating" or the risk of seeming too interested in "someone else's business". Relationships between students and their classroom teachers has been identified as a major issue that teachers need to pay more attention to (for example, Bishop et al., 2003; Hawk et al., 2003).

Related to this is the issue of power relations, not just between teachers and their students, but also among students themselves. According to some writers, students identify the feeling of emotional safety as an important feature of classroom climate (Phelan et al., 1992). Duffield and colleagues (2000) discussed the anxiety about social norms that many students experience at school as they become increasingly aware and concerned about what is, and is not, a "cool" topic for discussion with peers at school.

The extent to which students are active participants in helping to determine learning goals seems to be important. In another paper, we have analysed how students perceive the role of their teachers (Taylor, Hawera, & Young-Loveridge, in press). We identified four major roles that teachers seem to adopt, including the roles of mentor, manager, transmitter (of information), and arbiter. In City School, where students seem to value communication with peers very highly and there is a strong emphasis on formative assessment, students are encouraged to collaborate with their teacher in setting their own learning goals and record their learning in individual "logs". We found more students at City tended to see their teacher in the role of mentor. Further, the management team at City had previously been involved in a Ministry contract that focused on assessment and learning.

The findings of this study have some important implications for teachers and other educators. Although there is much rhetoric about the value of communication in mathematics (for example, Ministry of Education, 1992), there is clearly some way to go before students feel comfortable about and appreciate the benefits to their learning of communicating with peers about their thinking. To help students appreciate and value communication processes in mathematics, attention needs to be given to the messages that are conveyed about situations in which communication is to be encouraged and situations in which it is not. One way of supporting/encouraging students that communication in mathematics is valued is by making a point of reporting to parents/caregivers on how well students are doing in terms of their mathematical communication. In our experience, it is more usual to receive information about a child's proficiency on particular strands of the curriculum, such as number, measurement, geometry, algebra, or statistics (often after a test of some sort), than to hear about how well he or she is able to think, reason, and communicate mathematically (part of the mathematical processes strand).

Less obvious, but potentially just as powerful, is the need to keep families/whānau and the community abreast of changes in the approach to mathematics learning. Views of mathematics have shifted considerably over recent decades, and it is important for the community to be kept up-to-date about changes in the way that mathematical processes and thinking are being emphasised. We noticed that most students had family members willing and able to support their mathematics learning at home, but much of this help tended to reflect ways of thinking about mathematics more typical of past generations. The leaflets produced by the Ministry of Education in 2004 and 2005 go some way towards informing parents about the numeracy projects. A key message in the leaflets is that:

> There is usually more than one way to solve a problem. If your child has a strategy that works, praise them. If yours is different, that's quite OK.

The leaflets also list the kinds of things that children are learning on the NDP, including calculating "in their head where possible, rather than using a calculator or pen and paper" (Ministry of Education, 2004a, 2004b). It would be helpful if teachers were to make a point of highlighting the importance of communication in mathematics in their interactions with families/whānau.

The findings of this study, in particular the verbatim quotes from individual students, provide valuable insights into the students' unique perspectives on their mathematics learning and underline the importance of taking children's views into account (Civil & Planas, 2004; Cook-Sather, 2002; Young-Loveridge & Taylor, in press). We believe that teachers should be encouraged to make reciprocal communication in mathematics a major goal in their teaching. Furthermore, students should be encouraged by their teachers to collaborate in setting goals for their learning in mathematics. The findings of this study indicate that much more support is needed for teachers in order to sustain their learning from the NDP professional development over the long term.

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