Equipment-in-use in the Numeracy Development Project: Its Importance to the Introduction of Mathematical Ideas

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Teachers' orientation to equipment use in the classroom is an important factor in the opportunities afforded to students to discuss mathematical ideas. The opportunities arise through the extent to which the initial focus of student activity is on using equipment when introducing a new mathematical idea. This paper examines the ways in which teachers use equipment in the Numeracy Development Project (NDP) for new concepts. Particular attention is paid to the extent to which they orient students' use of equipment to promote student thinking and discussion. These teaching strategies are tools for equipment-in-use.[4]

The Teaching Model

The NDP is structured around the use of equipment to teach students mathematical concepts.[5] The teaching model describes the progression from the features and functions of the equipment, to imaging, and then to number properties in students' representations of number ideas. The uses are in contrast to an experiential "hands-on" orientation common in so-called child-centred approaches where the use of equipment is to keep students actively engaged (Higgins, 2001). Experiential use does not necessarily lead to students developing mathematical understanding, nor does it necessarily imply that students talk about what they are doing.

Previously, the use of equipment in New Zealand schools has been associated with teaching mathematics to younger students, with the expectation that older students progress to book-based studies. The NDP emphasises the use of equipment as the starting point for any new idea. The levels of abstraction described in the NDP teaching model are referred to as a progression from materials, to imaging, and on to number properties, and are followed each time students are introduced to new learning, irrespective of their stage on the Number Framework. This progression can also be thought of as a shift for students from an externalised representation to a visualised idea and then to an internalised representation.

The teacher's role is one of leading the focus in these initial stages of learning new ideas. However, that role lessens as students' knowledge and strategies develop so that they are ultimately able to independently solve problems abstractly, using mathematical properties rather than equipment. This progressive cycle is repeated when a new mathematical idea is introduced. The teacher's focusing strategies can be described as shifting from an emphasis on demonstrating using materials to mediating using number properties.

As shown in Figure 1, the initial teaching–learning sequence presents the teaching model described in the NDP materials. It depicts the integration of student activities and the associated teacher strategies. The traditional, experiential use of equipment is not depicted.

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[4] The term *equipment-in-use* has been coined to emphasise the purpose of equipment use from a socio-cultural perspective. This paper describes equipment use in terms of four orientations.

[5] One of the key Ministry of Education goals is to reduce disparity. To help address this goal through the NDP, supplementary funding was provided to lower decile schools to purchase equipment and provide ancillary support.

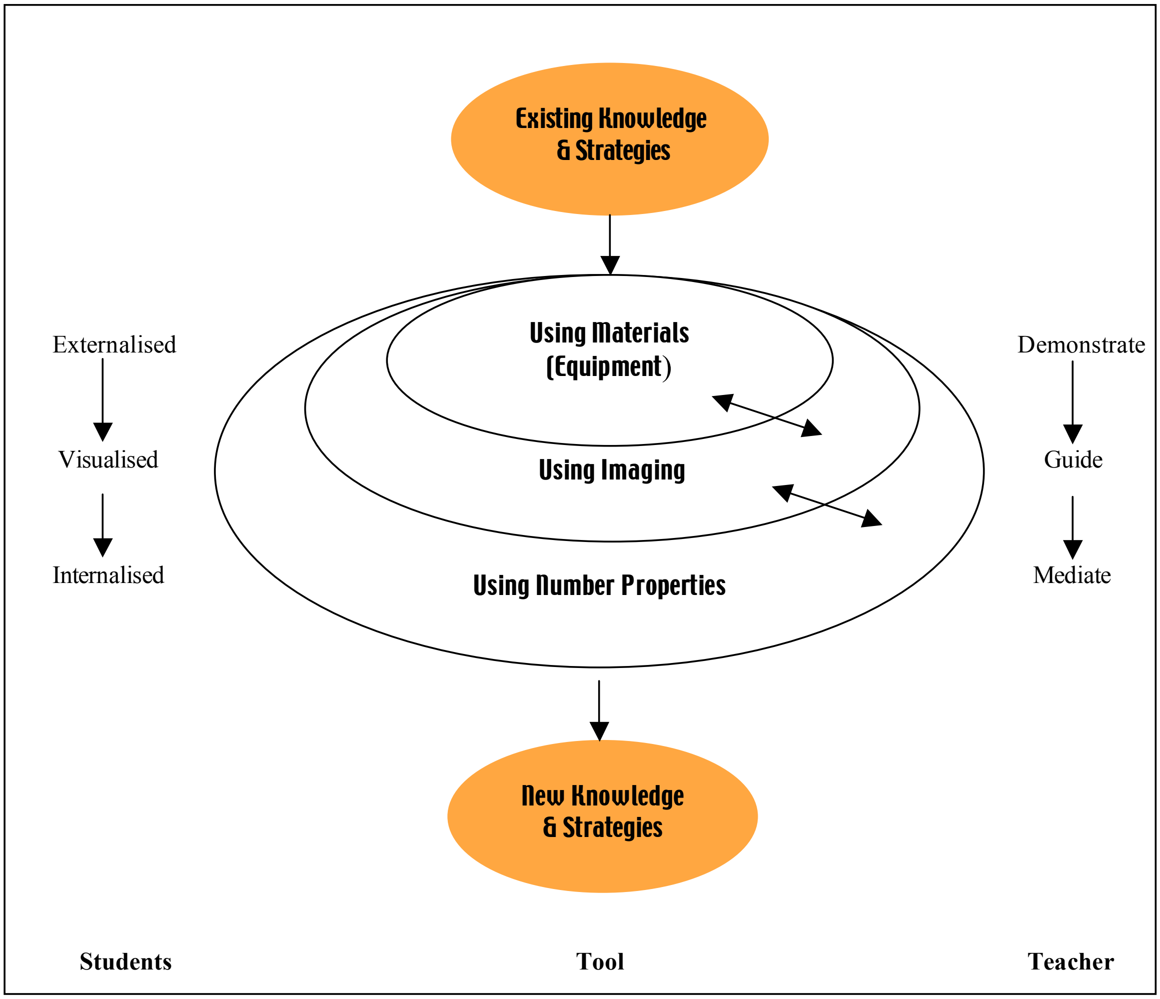

Figure 1. The initial teaching–learning sequence when introducing new mathematical ideas[6]

A Comparative Analysis: Four Orientations to the Use of Equipment

The use of equipment in early mathematics is a well-established practice for New Zealand primary school teachers. Its use can vary, according to the age of the students, from an experiential focus on mathematics operations with younger students to a more formalised, structured approach with older students.

The schematic, Figure 1, identifies the three orientations of equipment use: externalised, visual/conceptual, and internalised. These contrast in significant ways with the traditional experiential approach focused on algorithms. The differences between all four orientations are summarised in Table 1. The table sets the distinction across the orientations according to three elements: the equipment-in-use, the teacher's focusing strategies, and the student's progression to intellectual independence and peer discussion.

[6] This diagram is a modified version of The Teaching Model (Ministry of Education, 2004).

Table 1
Comparison of orientations of actions towards equipment use

Element	Algorithmically oriented activity	Externally oriented activity	Conceptually oriented activity	Dialogically oriented activity
Tool (equipment-in-use)	The physical action on the equipment demonstrates the “working form” of the algorithm.	Equipment provides a concrete manipulative reference point.	Equipment represents the thinking in solving a mathematics problem.	Equipment mediates discussion.
Teacher	The teacher gives directions to the students about the actions to take with the equipment to complete the algorithm.	The teacher demonstrates the elements of the problem using equipment.	The teacher guides the students to use the equipment to show their thinking.	The teacher mediates student dialogue in justifying their mathematical thinking using equipment.
Students	Students imitate the teacher’s demonstration of words and actions of the algorithm with the equipment.	Students follow the teacher’s demonstration or model.	Students represent their mathematical thinking through equipment.	Students use equipment as a dialogical support for participating in a mathematical discussion.

In the first orientation (algorithmically oriented activity), equipment is frequently used in middle and senior primary classes in a more structured way to support the teaching of addition, subtraction, multiplication, and division algorithms. Used in this way, equipment is portrayed as an external representation of the procedures of the computational algorithm.

In contrast, a second orientation (externally oriented activity), arises from students’ experiences. From this perspective, equipment is used as a tool for the support and guidance of students’ thinking sequences in solving a mathematical problem. Equipment can be used to provide a concrete manipulative reference point for students in the introduction of new ideas. This strategy uses the equipment as an external representation of the thinking process, not just for the operations of calculations. Such use of equipment highlights the elements of mathematical concepts rather than the procedural stages of algorithmic operations as in the first orientation.

The third orientation (conceptually oriented activity), supports students conceptually to understand the structural elements of a mathematical idea.

The fourth orientation to equipment use (dialogically oriented activity) centres on the structural elements of an idea by encouraging discussion and explanation. The equipment becomes a reference point in the justification and negotiation of meaning of mathematical ideas as students work towards a shared understanding.

Across the three orientations of the NDP (excluding the algorithmic), the focus shifts from individual students (with the external) to collective students (with the dialogical). Similarly, the responsibility for the ways in which the equipment is used progresses from wholly the teacher (with the external) across to wholly the students (with the dialogical).

The comparison of the orientations draws on excerpts from interviews informed by classroom observations in three mainstream classrooms across two regions in New Zealand. These interviews were part of the evaluation of the NDP in 2004 aimed at creating an image of each orientation to equipment-in-use.

In Table 1, each element is discussed in terms of its orientation to use. The excerpts illustrate the element for a particular orientation.[7] This is followed below by a discussion of the impact of the NDP on equipment use.

Comparison of Orientations of Teachers' Actions Towards the Use of Equipment

Equipment: Algorithmic Orientation

Use of equipment from this orientation is limited to representing the working procedures of a mathematical algorithm. This algorithmic approach uses equipment as a tool for procedures. It mirrors the steps in the working form of an algorithm. This orientation is associated with a transmission model of teaching where emphasis is given to knowledge of the procedures to be followed. The teacher's quote below highlights the restriction of the use of equipment to being steps to follow.

> The procedural way of doing it doesn't ask them to think about what they know. (Vicki,[8] interview)

The choice and use of equipment is therefore based on how well the piece of equipment represents the procedures of the algorithm,[9] rather than how well aspects of a mathematical idea are represented.

Equipment: External Orientation

The equipment provides a concrete manipulative reference point when introducing new learning to students. The quote below is from a NDP teacher who is trying to help her son, whose school has not yet participated in the project.

> He had not seen the equipment to understand the idea … so you know the equipment is essential for the kids … it's not to keep their hands busy. (Vicki, interview)

At this stage, equipment is useful as a concrete representation introducing elementary aspects of a mathematical idea and, as such, might be seen as a tool for "getting started".

Equipment: Conceptual Orientation

In a conceptual orientation, equipment-in-use becomes a tool to show aspects of mathematical ideas rather than a tool for showing the procedures to be followed in solving a mathematics problem. Gravemeijer (1994) compares the shift in equipment use from a *working* to a *thinking* model. The distinction he draws is not about the equipment per se, but about its use as a tool for thinking mathematically. He suggests that actions with equipment should provide a "frame of reference" or *thinking model* when doing recording. The degree and ways in which equipment is structured will shape but not necessarily limit its use. In the NDP, much

[7] Excerpts to illustrate the algorithmically-oriented activity are comments by teachers that are in contrast to their current use of equipment guided by the NDP.

[8] Teachers' names have been changed.

[9] The renaming method for subtraction replaced the equal additions algorithm because place value blocks widely available in New Zealand schools in the 1980s were useful for demonstrating the steps in renaming, but not for equal additions.

of the equipment is structured around groups of five or ten because the emphasis is on part–whole schema.

> So you need all the equipment to show the different parts of a number, the different properties of a number. (Vicki, interview)

Teachers' knowledge of the design of equipment gives a basis for its usefulness in developing an understanding of a particular mathematical idea. That knowledge guides teachers' choice and use of equipment.

Equipment: Dialogical Orientation

The use of equipment to support mathematical discussion emphasises the use to which the equipment is put. This focus on activity "shifts away from the analysis of symbols [including equipment] as external supports for reasoning and moves towards students' participation in practices that involve symbolising" (Cobb, 2002, p. 187). McClain's (2002) study, investigating the role of tools in both supporting and constraining communication in the classroom, underscored the importance of a teacher's understanding of the student's activity of explaining and justifying their thinking.

> I just feel that the activity-based [approach] works really well with the language type [approach]. … If they don't have that oral language and that ability to talk and explain things … That's the crux of it … It's the talking, it's the explaining, it's the using the words, the language … They'll talk to their partner, even if it's just working out who's putting up how many fingers ... They're still talking with somebody and they're still using that language. (Nancy, interview)

An item of equipment does not have any inherent meaning apart from that arising from the context of use (Roth & McGinn, 1998). In the quote above, the students use their fingers in the context of their discussion of mathematical ideas.

Teacher: Algorithmic Orientation

When there is a focus on procedures, the teacher's role is to give directions to the students about the actions to take with the equipment to complete an algorithm. Emphasis is given to students' "proper" use of the materials. Concern is given to physical dependency on the equipment and so this use of equipment is frequently followed by individual bookwork to practise the procedures learned. In this orientation, bookwork is seen as a more "advanced stage" than using equipment and is one possible reason for minimal use of equipment with older classes.

> That's the thing because in their previous classrooms they were not allowed to share, they had to do it in their books, so they reduced down to an individual. (Vicki, interview)

The emphasis is on following procedures rather than asking students to think about what they are doing and talk about it with others.

Teacher: External Orientation

The teacher demonstrates the elements of the problem with the equipment. In the quote below, the teacher clearly sees her use of equipment as a part of a progression in the teaching of mathematical ideas.

> I like using equipment for the simple fact that I find it easier for children to understand, particularly at the beginning stages … even if they are on … you know … early part–whole … I just find it easier to use it to start with and then they can wean off it again … just the teaching of the initial concept. (Nancy, interview)

The teacher here, in her use of equipment, is focused on giving students a starting point that is concrete; that is, in an external form.

Teacher: Conceptual Orientation

The teacher uses equipment to foster student thinking about mathematical ideas. Equipment choice is based on the teacher's purpose of the lesson. The choice of equipment varies in response to students' developing understanding and is limited by the teacher's own content and pedagogical content knowledge.

> Vicki to the students: "What I'm trying to do is to lead your thinking."
>
> Vicki to the students: "So why have I given you this set?"

In the above quotes, the teacher, in her use of equipment, has emphasised thinking. Her comments suggest that her choice of equipment is aligned to the overall purpose of encouraging students to think, but that this thinking is shaped by her choice of equipment.

Teacher: Dialogical Orientation

In a dialogical orientation, the teacher's focus is on extending concepts in response to students' actions and explanations. Essentially, this is a mediation role in which the teacher mediates student dialogue in justifying their mathematical thinking using equipment. Vicki, in the quote that follows, mediates by "decoding and rephrasing".

> They don't have the words to explain what they're doing … or put it out in a logical way that I can understand … I mean, you heard all that decoding and rephrasing … paraphrasing … I do that all the time because it's all jumbling out I guess … That's where the equipment fits for these kids. (Vicki, interview)

For this teacher, her use of equipment is to support the students' discussion of mathematical ideas.

Students: Algorithmic Orientation

The students' actions are to imitate exactly the teacher's actions with the equipment that shows the algorithmic operation. They observe the teacher's demonstration using equipment of the sequence to be followed. Typically, the students are expected to individually follow the teacher's procedure rather than do their own thinking as they answer a mathematical problem.

> Follow this procedure and you'll get the right answer at the end. (Nancy, interview)

The teacher here is emphasising that the way to get the right answer is to stick to the procedure that she has shown them.

Students: External Orientation

The students follow the teacher's models as they are introduced to the new ideas. It is important to note this teacher's reference to the students finding different sorts of equipment helpful in the initial stages of introducing a new idea.

> Well, I use the equipment for making the learning of new concepts much easier … If the kids are able to see that different equipment can be used in different ways … I think that maybe helps their thinking strategies as well and gives them more variety in the way they can work things out. (Nancy, interview)

The use of different pieces of equipment is important as it is unlikely that any single piece of equipment illustrates all aspects of a mathematical idea. However, this also assumes that teachers have knowledge on which to base their choice of equipment for students.

Students: Conceptual Orientation

In a conceptual orientation, students use equipment to represent their mathematical thinking.

> It's a visual thing and they can see their processes, they can work their processes through and manipulate through … Particularly those ones who don't think as quickly as the others, they can see and do it as they think it, they can get the … you know … the one and then change it to two and into three and into nine and then add one more for the ten. So they can manipulate the equipment and see their thinking at the same time. … It's like visualising what they're thinking and they can see it happening. (Nancy, interview)

The emphasis in this orientation is to offer the students a way of representing their thinking about a mathematical idea through the use of equipment.

Students: Dialogical Orientation

Students use equipment as a dialogical support for their explanations. The focus in the quote below is on Jerry's participation in the practice of using equipment to symbolise the mathematical ideas in his solution to the problem.

> Some of the time when they're talking … I struggle to understand how they've got there, but they know exactly how they've got there … Jerry got all the pirates off the island, but his system and his way of working it out was totally different to what I would have done … It was different to everyone else's too, but he got the answer at the end, he went though his own processes and explained it, although I didn't quite understand it … but he knew in his own mind and he showed me with the counters. (Nancy, interview)

Student explanations to mathematical problems can become complex and challenging for others to follow. The use of equipment is a way of supporting students as they work through their explanation.

Discussion

Equipment use is complex because it is dynamic in terms of the shifting balance of the teacher and student roles as well as its shifting place in the learning progression for each new idea from an external referent to a tool in the student's practice of symbolising (Cobb, 2002).

Any equipment has been designed with a purpose. Much of the equipment associated with the NDP has been chosen because the purpose of the developer of the equipment was to foster part–whole thinking. There is also the teacher's purpose in using the equipment. The purpose of the equipment designer and the teacher's purpose in using the equipment do not necessarily match. The teacher may adopt or reject or simply not understand the designer's purpose.

The function of a piece of equipment can be clarified in answer to the question "What does the equipment ask us to do?" For instance, a number line tells us to use the sequence of numbers to represent a mathematical idea; a tens frame tells us to use ten as a reference point; and a hundreds chart tells us to use groups of ten (up to ten groups of ten) as the basis for the solution strategies.

The choice of equipment for classroom use depends on a teacher's purpose, as exemplified in this paper through four broad orientations towards mathematics teaching. In the early stages of professional development in the NDP, where teachers are still shifting towards a conceptual and dialogical approach to mathematics teaching, their choice of equipment may be framed by their desire to model mathematical activities. Where teachers work from conceptual and dialogical orientations, their choice of equipment will be based on the number properties they are wanting to highlight to the students as well as the discussion of mathematical ideas they want to foster among students.

Teachers' knowledge of mathematics might also govern the range of equipment used in their teaching. Where teachers' knowledge is still developing, their choice of equipment is restricted and shaped by their understanding of mathematical ideas.

> So the equipment … you can't use one for one purpose only, one form … You need all of it because there's not one piece of equipment that I can think of that will show, particularly for a concept like decimals and fractions, that would show everything about it … yeah … I think that's the limit of something like … procedure type teaching, you don't show … you don't give the kids the versatility, the flexibility of it. (Vicki, interview)

Acknowledgments

My grateful thanks to the teachers and students who so willingly participated in this project.

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