Findings from the New Zealand Numeracy Development Project 2004

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Foreword
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Findings from the New Zealand Numeracy Development Project 2004

Foreword

There are at least two ways that reforms can be undertaken in education. One way is to provide set lessons for the teacher and set work for the students. The second is empowering the teacher through professional development. In implementing the Numeracy Development Project (NDP), New Zealand took the second of these options.

The motivation for the New Zealand reforms was undoubtedly the relatively poor showing of New Zealand students in the 1995 Third International Mathematics and Science Study (TIMSS). In that year, New Zealand and about 50 other countries participated in TIMSS. The published results identified the poor performance of New Zealand students in number (place value, fractions, and computation), measurement, and algebra concepts. These results were of sufficient concern that steps were taken that would ultimately lead to the NDP.

The NDP was first implemented in 2001, following a pilot project in 2000. The Number Framework, developed as the basis of the NDP, outlines the progress that most students follow in their development of number knowledge. Teachers are able to assess students' performances by using a diagnostic test in a one-to-one situation. Facilitators provide professional development for the teachers, using a pedagogy that is based around the individual contribution of students.

The NDP has developed rapidly and now includes the Early Numeracy Project (ENP) for children in years 1 to 3; the Advanced Numeracy Project (ANP) for years 4 to 6; the Intermediate Numeracy Project (INP) for years 7 and 8; the Secondary Numeracy Project (SNP) for years 9 to10; and Te Poutama Tau for students in years 1 to 8 in Māori-medium settings.

All the teachers involved in any aspect of the NDP return the results of each student's diagnostic test to a central data bank. This is done early in each year and at the end of each year. Consequently, a growing set of statistics is available to help monitor progress and suggest what might be expected by students at all levels. For further details of the numeracy projects, see www.nzmaths.co.nz/numeracy.

The success of the NDP is due to the soundness of its Number Framework, the strength of the teaching model, and the ability of the facilitators. Through the facilitators' work, teachers have been able to see and understand number in greater depth than before and are able to encourage students to use more open approaches to calculation than the traditional algorithm. But more than this, the teachers who are now confidently using the pedagogy of the NDP are doing so because of the progress that their students are making and the greater enjoyment that those students are exhibiting.

The eleven papers and an extract in this compendium look at a variety of aspects of the NDP. They can be roughly grouped under the categories of student achievement, effective teaching, and students' perspectives. A summary of these papers follows.

Jenny Young-Loveridge's analysis of the 2004 NDP data (p. 5) used results from approximately 70 000 students overall from ENP, ANP, and INP. As in 2002 and 2003, all groups benefited from their participation in the projects. While Asian students made greater gains than Pākehā/European students, followed by Māori and then Pasifika students, all ethnic groups moved to higher stages than they had in 2003. The results also showed a narrowing of the gap between most groups.

The evaluation showed that students from low-decile schools who started the NDP at stage 3 or lower on the Number Framework did better than corresponding students in medium-decile schools. This may be partly due to the additional resources that were provided for some of these low-decile schools through School Improvement initiatives.

One of the many ways that NDP data has been analysed is with respect to schools that have been involved over a period of time. The Longitudinal Study began in 2002 and aims to track the progress of students in schools involved in the NDP. In 2004, 31 schools were involved in this study, some of which had been in the NDP since 2000. Gill Thomas and Andrew Tagg's paper (p. 21) reports on aspects of this study.

They compared the data from 6099 students in the longitudinal schools with data from 70 000 students in the national database for achievement in additive, multiplicative, and proportional strategies. This was done in two ways: first, directly between peers, and secondly, between students at one year level in the longitudinal schools with students at the initial stage of the next year level. Students from the schools that had been in the NDP for some time were rarely outscored by the national cohort, and in many cases, their performances were significantly better.

Thomas and Tagg's data also provides evidence for the expected Number Framework achievement levels of students in the NDP. So, for instance, virtually all year 3 students might be expected to be at least at stage 4 (advanced counting), with 40 percent at stage 5 (early additive) or even higher.

One way to test overall productivity of the NDP is to compare the students against a known standard. This comparison is of much more interest and value if the standard is a recognised international one, such as the 1995 TIMSS. In their second paper (p. 35) Thomas and Tagg compare the performance of students who had been in the NDP for two years or more with both local and international TIMSS results from 1995 to test how well the NDP is succeeding.

A test was devised for year 4, 5, and 8 students in 31 longitudinal schools, using questions from the 1995 Grade 4 and Grade 8 TIMSS tests respectively. Of the 24 questions on Thomas and Tagg's test, the year 4 students performed better on 16 and equally well on six, compared with their compatriots in TIMSS 1995. In the case of the year 5 students, they did better on 19 questions and the same on two, while the year 8 students did significantly better on six questions and significantly worse on three. The outcomes of the year 4 and 5 students provide strong evidence for the success of the NDP.

Kay Irwin and Murray Britt's paper (p. 47) reports on the first year of a three-year study that is looking at students' development of algebraic thinking. In 2004, students from years 7 to 10 in four intermediate schools and four secondary schools were given the same tests on each of the four basic arithmetic operations. Some of the secondary school students had come from primary or intermediate schools that had not been involved in the NDP.

On the questions involving letters as algebraic symbols, the year 7 students outperformed all other students, with year 10 students being next best and the year 9 students the weakest. These results may be due to the fact that the year 8 and 9 students were being taught algebra in the conventional way. This is a study that will be worth following, as it will be of interest to see if the new SNP will help students at secondary level develop algebraic thinking skills.

Tony Trinick and Brendan Stephenson (p. 56) evaluated all the available data from the 33 Māori-medium schools that participated in Te Poutama Tau in 2004 and compared it to the corresponding results from 2002 and 2003. Apart from assessing overall performance and how it compared to previous years, the authors were interested in where students performed well and where their performance was weaker. It should be noted that there was very little difference between the proficiency in te reo Māori of students in 2003 and those in 2004.

Minimal student gains were made in the areas of numeral identification, multiplication, fractions, and proportion, while significant gains were found in the area of decimal knowledge. As with students in the NDP, advancement was more difficult at the higher year levels, probably because the difference between levels is greater at the upper end of the framework. However, there was still a slightly higher performance overall for the 2004 students.

Kay Irwin and Joanne Woodward (p. 66) analysed the mathematical discourse used by two teachers in upper primary classrooms.

In one class, there was particular emphasis on the use of enquiring discourse, where students were encouraged to explain their thinking and were given sufficient time to gather their thoughts. There is evidence that these aspects of discourse continued into discussions held between students when they were involved in their group work. The students in this class were Pasifika, and the gains they made on the Number Framework were significant when compared to the national average for Pasifika students. It is suggested that this improvement may be related to both the emphasis on language and to the mode of teaching.

Joanna Higgins's paper (p. 74) looks specifically at effective teaching in a particular Māori classroom, but the principles there would seem to apply equally to students of any ethnicity. In this classroom, the teacher uses the metaphor of the waka to describe the class: they are all heading in the same direction, but different members have different talents and are able to do different things. She also uses the koru as a metaphor to describe how all the students are growing with mathematics, emerge in different ways, and help others to emerge better than they would by themselves.

So the students know that it is all right for the groups of the mathematics classroom to have different levels of ability. In fact, the teacher of this classroom uses the abilities, particularly of the lead group, to help teach the less able students. It's something that she says is part of Māori culture. But she is sensitive to students' needs and will take care when pairing up a peer teacher to a student.

In the extract from her paper on the pedagogy of facilitation (p. 79)[1], Higgins considers two facilitating approaches: one following the guidelines of the teacher manual and the other from a standpoint that is more responsive to students. The teacher's manual, materials/activities, teaching method, and modelling practice are examined from the viewpoint of the two approaches. As a result of this analysis and of facilitator interviews, Higgins concludes that teachers are more likely to gain confidence and numeracy development is more likely to be sustained if facilitators introduce teachers to a framework of ideas rather than adhere to the design features of the NDP guide books.

Tony Trinick's paper (p. 80) reports on a study of two schools that had been in the Te Poutama Tau project in 2003 and whose student achievement data had shown positive mean stage gains. These schools were studied in order to identify key factors that might promote student achievement in Māori-medium schools generally. Data was collected using questionnaires and follow-up interviews that covered such things as socio-cultural features of

[1] Higgins, J. (2005). Pedagogy of facilitation: How do we best help teachers of mathematics with new practices? In H. L. Chick & J. L. Vincent (Eds), *Proceedings of the 29th annual conference of the International Group for the Psychology of Mathematics Education*, 3, 137–144. Melbourne: PME.

the school; relationships with the local community; the experience and attitudes of its management and teachers; and teachers' reflections on the Te Poutama Tau project.

Although the two schools were successful in the teaching of the Te Poutama Tau approach to pāngarau (mathematics), they differed significantly in a number of areas. However, they did exhibit a number of features in common that Trinick feels combine to promote successful achievement in pāngarau. These include the participation by both principals in the teacher professional development project, the setting of clear goals for the teachers, individual support for teachers where needed, and a focus on student learning.

Equipment has an important role to play in the NDP, and this is Joanna Higgins' focus in her paper on page 89. The teaching model of the NDP uses equipment to introduce new concepts, invokes imaging for students to visualise the concept, and then moves to internalising the idea and independently solving problems using the understanding of number properties. This reinforces the importance of equipment as the basis of a teaching model.

Equipment, especially in middle and senior primary school, may be used in mathematics to demonstrate the working form of an algorithm. However, Higgins shows how the use of equipment in the NDP should develop from a concrete manipulative reference point, to a representation of the thinking needed in the solution of a problem, and finally to a means to mediate discussion.

Higgins' paper amplifies the use and place of equipment through these three stages by referring to the tool itself and its use by teachers and by students. These comments are supported by teachers' quotes.

The paper by Jenny Young-Loveridge, Merilyn Taylor, and Ngarewa Hawera (p. 97) looks at how students feel about the importance of communicating their mathematical thinking and listening to the strategies of their peers. Despite a range of differences between the schools involved, most students saw an advantage in explaining their strategies to others, and both NDP and non-NDP schools were agreed on this. However, there was less agreement as to the importance of knowing other students' strategies, with more students in NDP schools seeing an advantage for this.

It is worth noting that the school in Young-Loveridge's other paper (p. 107) was much more positive in both areas than any of the other five schools in the paper above. Around three-quarters of the class in this school thought that these were important issues and were able to articulate their reasons for their responses. This reflects the fact that this school placed a strong emphasis on involving the students in their own assessment and making them aware of their learning.

As a result of the research that has been undertaken by the authors in 2004, it is clear that there are areas where progress is being made and there are areas of concern. Two points in this latter category are the progress of certain subgroups of students and how the overall progress to date can be sustained. These issues are already under consideration and are being focused on by the Ministry of Education and facilitators in 2005.

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Patterns of Performance and Progress: Analysis of 2004 data

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Data from approximately 70,000 year 1–8 students who participated in the Numeracy Development Project (NDP) in 2004 was analysed and compared with corresponding data from 2003 and earlier. As in previous years, all students seemed to benefit from participation in the project, but some groups made greater progress than others. Asian students were the highest performers, both in terms of percentages at the highest framework stages and in the progress they made relative to other students who began the project at identical starting points. Pākehā/European students did better than Māori students, who in turn outperformed Pasifika students. However, there was evidence to suggest that the gaps between groups may be getting smaller. As in previous years, students from high-decile schools did better than those from medium- and low-decile schools. However, students from low-decile schools who began the project at stages 0–3 made greater progress than comparable students from medium-decile schools. This may have been because certain low-decile schools were receiving additional support through another Ministry initiative. Analysis of data from adjacent year groups enabled the impact of the NDP to be separated from the effects of "normal" aging, and this showed younger students *after* the project to be significantly better than older students *before* the project. Effect sizes provided a measure of how practically meaningful the differences were. These were almost half a standard deviation for multiplication/division and proportion/ratio and about a quarter of a standard deviation for addition/subtraction.

Education systems worldwide have taken up the challenge to reform the teaching of mathematics in order to improve the mathematics learning of their students. The rhetoric that has accompanied such reforms has often justified them in terms of the need to produce citizens who are better able to cope with the demands of the twenty-first century (Bobis et al., 2005; British Columbia Ministry of Education, 2003; Commonwealth of Australia, 2000; Department for Education and Employment, 1999; National Council of Teachers of Mathematics, 2000; New South Wales Department of Education and Training, 2003; Ministry of Education, 2001). Recent evaluations of the reform process have highlighted the success of the reform efforts but drawn attention to some of the unintended consequences that indicate the need to modify approaches being taken (Earl et al., 2003).

New Zealand responded to the calls for reform in mathematics education by developing its Numeracy Development Project (NDP) approximately five years ago. This began initially with a small group of students in the early years of primary school (Early Numeracy Project [ENP] years 0–3), extended outwards to other schools, and then upwards into the senior primary years (Advanced Numeracy Project [ANP] years 4–6), the intermediate years (Intermediate Numeracy Project [INP] years 7–8), and most recently, the first two years of secondary school (Secondary Numeracy Project [SNP] years 9–10). By the end of 2005, it is expected that approximately 17 000 teachers and 460 000 students will have participated in the NDP (Parsons, 2005). It is predicted that by about 2007, virtually all teachers at years 0–6 and the majority of those at years 7–8 will have been given the opportunity to be involved in one of the professional development programmes as part of the NDP. Comprehensive evaluations have been undertaken of each of the professional development programmes that are part of the NDP (ENP: Thomas & Ward, 2002; Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004; ANP: Higgins, 2002, 2003, 2004; INP & SNP: Irwin, 2003, 2004; Irwin & Niederer, 2002). An analysis of the overall patterns of performance and progress across years 0–8 was undertaken for the years 2001–2003 (Young-Loveridge, 2004). A Māori-medium version of the NDP was also evaluated (Christensen, 2003, 2004). All of the evaluations have shown the NDP to be effective in raising mathematics achievement across primary and early secondary levels of the school system, and the benefits have been demonstrated in both Māori-medium and in English-medium settings.

A more fine-grained analysis was made possible by the aggregation of data across the three (English-medium) primary projects (Young-Loveridge, 2004). This analysis took students who began the NDP at the same stage on the Number Framework and looked at their progress over the course of the project as a function of gender, ethnicity, socio-economic status (as reflected in school-decile ranking), and year group. It became clear that not all groups benefited from the numeracy projects to the same extent. For example, Pākehā/European and Asian students made greater progress than Māori and Pasifika students, students at high-decile schools made greater progress than those at low- or medium-decile schools, boys tended to make greater progress than girls, and older students made greater progress than younger students. These findings showed that a "one size fits all" approach is not appropriate, and steps needed to be taken to tailor the projects to better meet the needs of particular groups of students. These findings were shared with all numeracy facilitators and consultants to reinforce the need for them to work with teachers in ways that helped schools shape the project to match the learning needs of their particular students.

This paper describes the findings from the analysis of data from year 0–8 students who participated in the project in 2004 and compares the patterns of performance and progress with those of students in 2003. The research question that guided this part of the project was:

> How does the performance and progress of students who participated in the numeracy projects in 2004 vary as a function of ethnicity, socio-economic status, and gender?

Method

Participants

Data from approximately 70 000 students who were assessed at the beginning and end of the NDP were included in the analysis. Just over one-third of the cohort was from ENP, almost half was from ANP, and the remaining students were from INP (see Appendix A). More than half of the students were Pākehā/European, about a fifth were Māori, a tenth were Pasifika, and the remainder were Asian or another ethnicity (see Appendix B). A third of the students were from high-decile schools, a quarter were from low-decile schools, and the remaining 40 percent were from medium-decile schools. The gender composition of the group was virtually identical. It was interesting to note that, compared to 2003, the 2004 cohort had slightly more Pākehā/European students and fewer Māori, as well as more students from medium- and high-decile schools and fewer from low-decile schools.

Procedure

Students were interviewed individually by their teachers at the beginning and end of the NDP using the diagnostic interview (NumPA), and the data was then sent to a secure website. Only students for whom there was complete data were included in the analysis for this report.

Findings

Patterns of Performance

The first part of the result examines students' performance, before and after the NDP and as a function of grouping variables such as age (reflected in year group), ethnicity, socio-economic status (reflected in school-decile band), and gender.

Differential performance as a function of year group

As in other years of the NDP, students tended to be assessed by their teachers as being at a higher framework stage after the project than they had been at the start (for details of the performance of each year group, see Appendix A). Performance improved steadily for each successive year group. By the end of the project, there was still substantial variation in performance across year groups. For example, on addition/subtraction, the percentage of students at the highest framework stage (stage 6, Advanced Additive Part–Whole) ranged from a fraction of a percent (0.1%) of year 1 students through to more than half of the year 8 students (55.1%). Some students, particularly the younger students, were not given the chance to show multiplicative thinking or proportional reasoning because Form A of the diagnostic interview (NumPA) was used, and the only operations assessed in Form A are addition and subtraction. By the end of the project, the proportion of students judged to be Advanced Multiplicative Part–Whole (stage 7) ranged from about four percent (3.7%) at year 4 through to a third of year 8 students (33.8%). The corresponding values for Advanced Proportional Part–Whole (stage 8) ranged from less than one percent (0.3%) at year 4 through to less than ten percent (9.3%) of year 8 students. The low levels of performance on the multiplicative and proportional domains have some important implications for the secondary schools that received these year 8 students into year 9 this year.

Differential performance as a function of ethnicity, decile, and gender

Appendix B shows the percentages of students at each framework stage on the various operational domains as a function of gender, ethnicity, and decile band. The most notable differences were evident at the highest framework stages. Consistently more boys than girls were at the highest framework stage, and this pattern held across all operational domains, both before and after the NDP. Out of the four main ethnic groups, Asian students performed the best, followed by European, then Māori, and finally Pasifika students. Again, the differences were very consistent across all domains and at both initial and final assessments. As a consequence, the relative differences among the various ethnic groups were maintained, and the gaps in performance between ethnic groups do not appear to have been narrowed appreciably by the project. When gender and ethnicity were examined together, the superiority of boys over girls was found consistently for all ethnic groups, though the magnitude of the gender difference varied somewhat from one group to another. Statistical analysis indicated that it was among European students that the gender difference was the greatest and most consistent. The tendency of Māori boys to outperform Māori girls in this project was contrary to the findings of many other projects. This suggests that individual diagnostic interviews, where both the presentation of tasks and students' responses to them are oral, may be a more valid assessment of the mathematical understanding of Māori boys than paper and pencil tests administered to large groups. In the past, Māori boys have done more poorly than Māori girls when mathematics is assessed using paper and pencil tests.

The variation in performance as a function of school-decile band was somewhat less consistent than the patterns for the other grouping variables. In previous years, performance tended to increase with decile band. However, in 2004 the medium- and high-decile bands were very similar in their performance patterns in many instances.

Comparison with 2003 data showed that students did better in 2004 than they had done the year before. This could be explained by differences in the composition of the cohorts, with the 2004 cohort including more students from medium- and high-decile schools. Another possibility is that the NDP facilitators have become more effective as they gain more experience with the project. Anecdotal evidence from conversations with facilitators indicates that they perceive themselves to be more effective now than they were in 2001 when the NDP first began. It is possible that both a cohort effect *and* the increased effectiveness of facilitators have contributed to better performance in 2004.

The impact of the NDP on students' performance: analysis of effect sizes

It is not immediately clear from the analysis of performance whether students would have made progress simply as a result of "normal" aging rather than because they had been part of the NDP. In a traditional experimental design, comparisons are made of the progress (as measured by the difference between pre-test and post-test scores) for the "intervention" group that received the "treatment" and the "control" group that did not. When the NDP began, evaluators made a deliberate decision not to have a control group because of the ethics of withholding the programme from teachers and students who could benefit from it. Another reason for not having a control group was that the logistical problems of training non-participant teachers to assess their own students simply for the purpose of comparison with students whose teachers did participate in the project would have been great. An important dimension of the NDP is that the assessment of the students is done by their own teachers as part of the professional development programme that comprises the "intervention", and hence is an essential component of the intervention process itself. Getting outside researchers to interview a control group of students for comparison purposes would also have been problematic, as it would have introduced another potentially confounding variable to the comparison.

One way around the lack of a control group is to use data from the students before they began the project as a comparison with the data after they had finished the project. Gill Thomas, for example, used a "reference group" to compare the "growth in each aspect of number learning that occurred over the duration of the project with the growth that would have been expected with age alone" (Thomas & Ward, 2001, p. 14). Thomas found that the gains made during the project were greater than the gains "that would have been expected in the students' previous classroom programmes" (p. 14). A bar graph (Thomas & Ward, 2001, Figure 3.1) showed the overall difference between the gains of the project students and their reference group on addition/subtraction strategies. However, no test was done of the statistical significance of this difference nor any analysis of variation as a function of age or magnitude of effect size.

For the purposes of this report, an analysis was undertaken of adjacent year groups to explore the differences between younger students *after* the project and older students *before* the project. The professional development programme took place over about three school terms (approximately three-quarters of a calendar year), so by the end of the project, the younger students were, on average, about a quarter of a year younger than the older students with whom they were being compared. This meant that the students at the end of the project were still at a slight disadvantage developmentally, compared with their older peers before the project. Hence, any statistically significant differences in favour of younger students after the project should reflect real and notable benefits to these students as a result of participating in the project.

Before presenting the comparison of younger students after with older students before, a simple comparison was done of each year group before the project to ascertain the pattern of "normal" development without intervention. This data provides a baseline for the other comparisons. Figure 1 shows the average framework stage for each year group before the project. In the early school years, there was a difference of about one stage between each adjacent year group. This decreased with age to about a fifth of a stage by intermediate because of a ceiling effect operating for addition/subtraction. In spite of a reduction in magnitude, all differences between adjacent year groups were statistically significant at or beyond the 0.001 level (see Appendix C). Effect sizes (based on the standardised mean difference between groups) are reported here in Table 1 and provide a measure of how practically meaningful the differences were, because it is well known that a large sample can yield statistically significant results that are not practically meaningful (see Fan, 2001). According to Fan, an effect size of 0.20 is "small", 0.50 is "medium", while 0.80 is "large". Table 1 shows effect sizes that were large initially (–0.83), but diminished to medium by about year 4 (–0.58 to –0.34), and small (–0.20) by year 8. It is likely that ceiling effects helped to reduce the magnitude of effect sizes in the senior primary and intermediate years.

Table 1
Average Framework Stages and Corresponding Effect Sizes for Younger and Older Students in Adjacent Year Groups (2004)

Addition/Subtraction						Multiplication/Division			Proportion/Ratio		
Year Groups	Younger **Before**	Older **Before**	**Effect Size**	Younger **After**	**Effect Size**	Younger **After**	Older **Before**	**Effect Size**	Younger **After**	Older **Before**	**Effect Size**
1 & 2	1.52	2.48	**–0.83**	2.54	**0.06**						
2 & 3	2.48	3.44	**–0.76**	3.50	**0.05**	4.18	3.73	**0.59**	4.17	3.75	**0.70**
3 & 4	3.44	4.13	**–0.58**	4.24	**0.10**	4.45	4.14	**0.36**	4.37	4.06	**0.42**
4 & 5	4.13	4.48	**–0.34**	4.69	**0.23**	4.83	4.51	**0.33**	4.69	4.34	**0.40**
5 & 6	4.48	4.69	**–0.22**	4.95	**0.30**	5.21	4.84	**0.35**	5.02	4.65	**0.38**
6 & 7	4.69	4.85	**–0.16**	5.17	**0.38**	5.57	5.12	**0.43**	5.40	4.95	**0.40**
7 & 8	4.85	5.03	**–0.20**	5.23	**0.23**	5.71	5.39	**0.31**	5.58	5.25	**0.28**
Average			***–0.44***		***0.19***			***0.40***			***0.43***

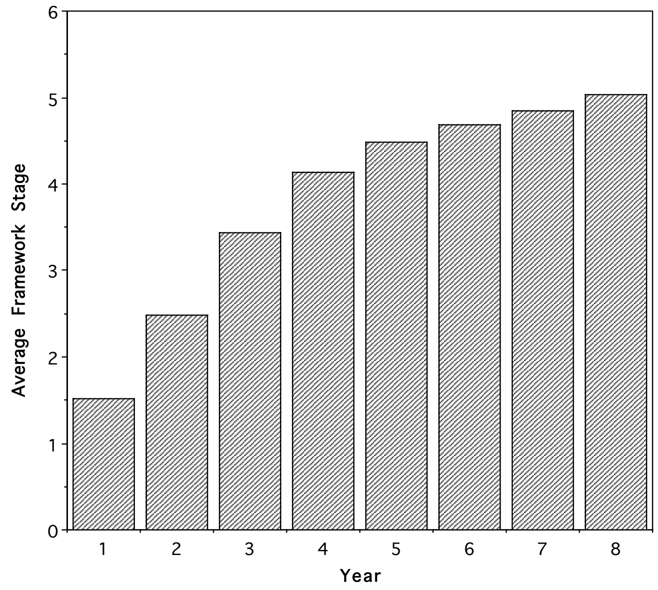

Figure 1. Average framework stage on *Addition/Subtraction* for each year group *before* participating in the project

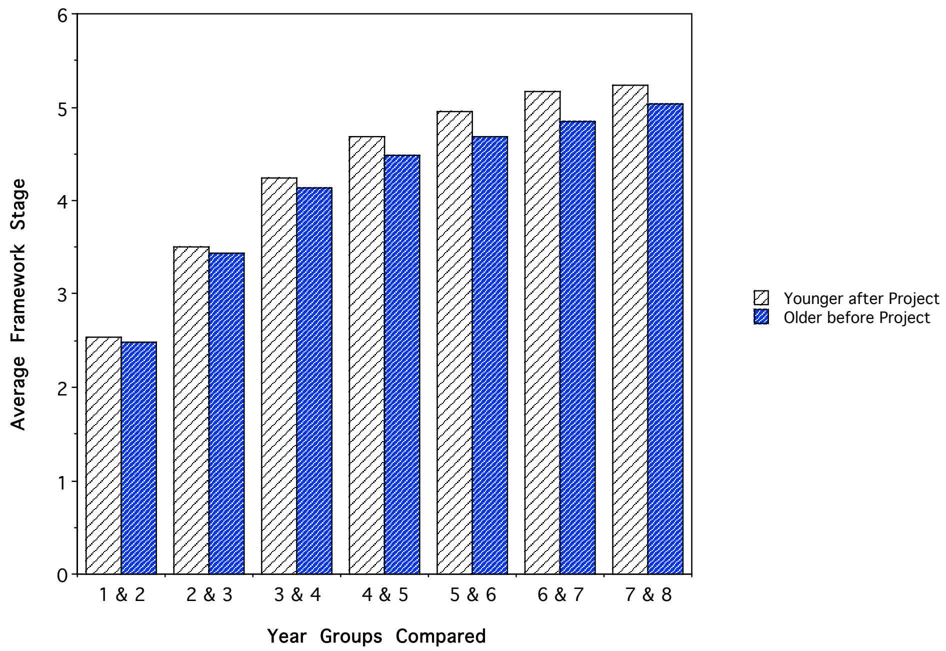

Figure 2. Average framework stage on *Addition/Subtraction* for younger students *after* the project compared with older students *before* the project

Figure 2 shows the average framework stages on addition/subtraction of adjacent year-groups for the younger students *after* the project compared with older students *before* the project (the 3rd and 5th columns of Table 1 show the framework stages used in the comparison, while the 6th column shows the effect sizes). All of the comparisons were statistically significant at the 0.001 level (see Appendix C). It is interesting to note that the magnitude of the effect size increased steadily with each year group up to a maximum of approximately a third of a standard deviation at year 6 and 7, then declined slightly at year 7 and 8, again possibly because students in year 7 and 8 are generally at the higher levels of the framework, producing a ceiling effect. This pattern of increase is consistent with the idea that older students benefit more from the project than younger students, despite anecdotal evidence suggesting that the project is easier to implement at junior primary levels than at the senior end of the school. The effect sizes for addition/subtraction ranged from very small (0.05) to moderate (0.38), with an average of 0.19. Part of the reason for smaller effect sizes at younger ages may have been the greater variability within the groups being compared because progression at lower framework stages is easier but may be less reliably assessed. At older age groups, there is a tendency to be at higher framework stages and progression to a higher stage is harder but more clear-cut and hence more reliably assessed.

A similar analysis was done for multiplication/division and proportion/ratio (see the 7th to 12th columns in Table 1). Figures 3 and 4 present the average framework stages on multiplication/division and proportion/ratio of adjacent year-groups from years 2 to 8 for 2004 (for details, see Appendix C). The effect sizes for multiplication/division and proportion/ratio were mostly within the moderate to fairly large range (0.28 to 0.70), with averages of 0.40 and 0.43 for multiplication/division and proportion/ratio, respectively. It was interesting to note that there was a particularly large difference at years 2 and 3 (larger than at years 3 and 4), then a steady increase up to years 6 and 7, followed by a slight decline. The large difference initially may have been the result of such a small proportion of year 2 students having been assessed on multiplication/division and proportion/ratio (approximately 16% of the year group), and the fact that these students must have been extremely good mathematicians for their age. This can be concluded from the fact that students must have impressed their teachers sufficiently to be assessed in domains not usually taught at their year level (using form B or form C of the diagnostic interview). As Appendix A shows, initially only 16 percent of year 2 and 53 percent of year 3 were given the chance to show multiplicative strategies or proportional reasoning. By year 4, virtually the whole year group was assessed on all three domains. Hence, the figures used to calculate effect size include students from the full range of mathematical abilities. It was interesting to note also that the benefits for students on multiplication/division and proportion/ratio were greater than on addition/subtraction, both in terms of the difference in average framework stage and effect size. The reason for the greater effect sizes may have been that there was much less of a ceiling effect operating for these domains than was evident for addition/subtraction.

Patterns of Progress

Patterns of progress were examined by looking at the proportion of students that moved up to a higher framework stage relative to a particular starting point. This analysis was done for ethnicity, decile band, and gender (see Appendix C).

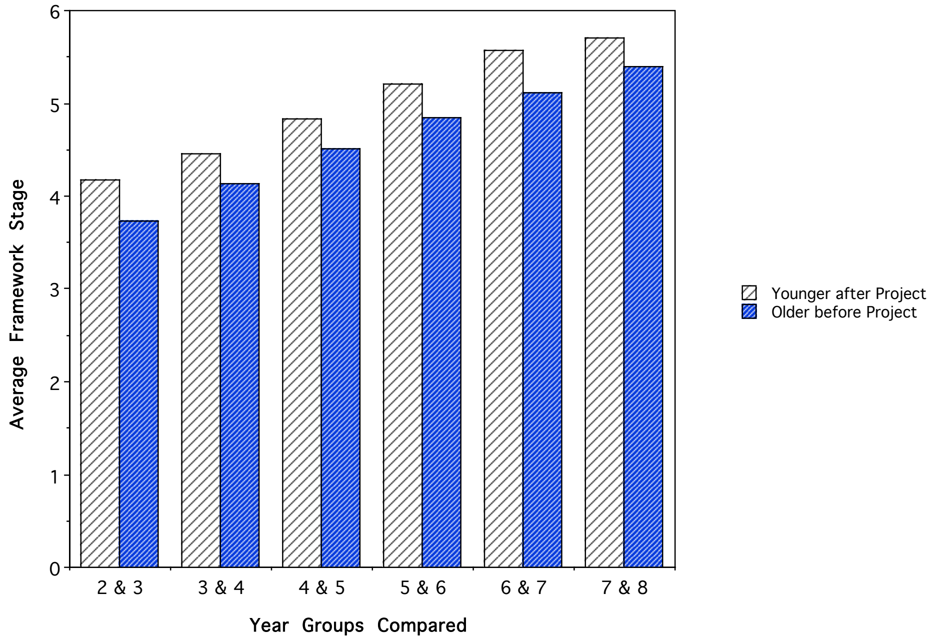

Figure 3. Average framework stage on *Multiplication/Division* for adjacent year groups

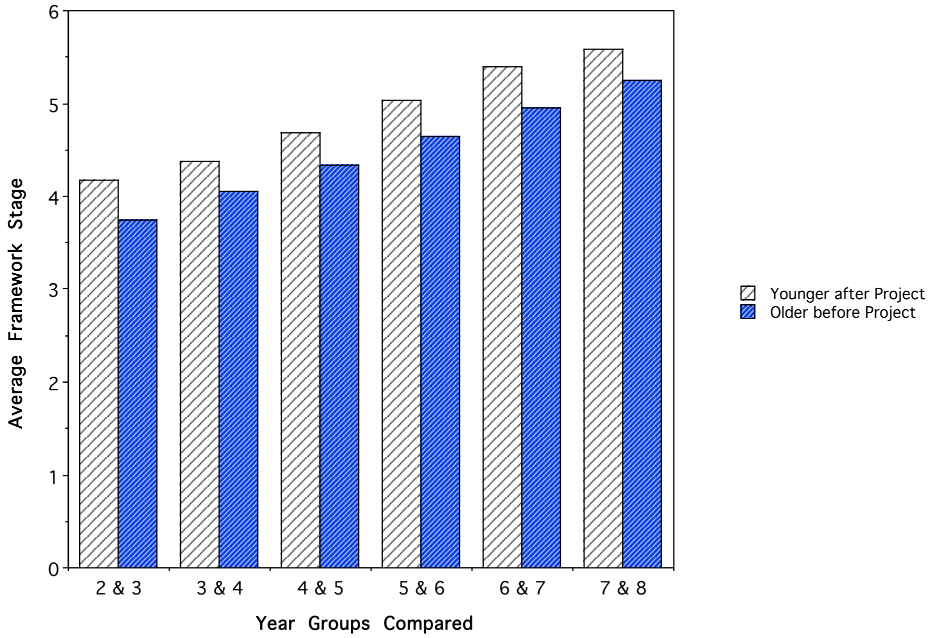

Figure 4. Average framework stage on *Proportion/Ratio* for adjacent year groups

The impact of ethnicity on progress

Figures 5 and 6 show the patterns of progress on addition/subtraction as a function of ethnicity. Asian students consistently made the greatest gains, followed by Pākehā/Europeans, with Māori and Pasifika students gaining the least. Progress to a higher stage was greater in 2004 for all ethnic groups than it had been in 2003, with the smallest increase for Europeans (1.9%) and the greatest increase for Pasifika (6.7%) (see Figure 6). The result was a narrowing of the gap between European and Māori students (6.1% to 2.9%) and between European and Pasifika students (8.4% to 3.6%). The gap between Māori and Pasifika students also narrowed (2.3% to 0.7%). Analysis of effect size was also undertaken (see Table 2 below). It was interesting to note that, although there were statistically significant differences in progress between ethnic groups, the magnitude of the effect size for comparisons of European students with Māori, and European students with Pasifika, was relatively modest (average = 0.13 and 0.17, respectively). The difference between the most successful group (Asian) and the least successful group (Pasifika) was, on average, just over a third of a standard deviation (0.36). The effect sizes for corresponding comparisons done in 2003 were identical for European with Māori, larger for European with Pasifika (0.25 vs 0.17), but smaller for Asian with Pasifika (0.31 vs 0.36) (see Table 2). This could be explained by an improvement in the progress of Pasifika students, accompanied by an even greater improvement in the progress of Asian students.

The impact of school decile on progress

Figure 7 (see page 15) shows the patterns of progress on addition/subtraction as a function of school decile. The most striking finding for this analysis is that students at low-decile schools who began the project at stage 3 or lower made greater progress than those from medium-decile schools (44.2% of low-decile students went up at least a stage compared to 39.9% of medium-decile students) and almost as much progress as students at high-decile schools (44.7%; see Appendix B, Table B7). This is very different from the pattern in 2003 when students at low-decile schools consistently made the least progress while those at high-decile schools made the most (see Figure 8 and Young-Loveridge, 2004). For students who began the project either counting on or using simple partitioning strategies, the pattern was more similar to the previous year, with students at high-decile schools making the most progress and those at low-decile schools the least. Table 2 shows effect sizes for the difference between the high- and low-decile bands in 2004 and 2003. It is clear from Table 2 that the difference halved from 2003 to 2004 (0.22 to 0.10, on average).

Table 2

Effect Sizes for Differences between Subgroups on Addition/Subtraction (2004 & 2003)

2004						2003			
Ethnicity				Decile	MEI*	Ethnicity			Decile
Initial Stage	Eur vs Māori	Eur vs Pasifika	Asian vs Pasifika	High vs Low	MEI vs nonM	Eur vs Māori	Eur vs Pasifika	Asian vs Pasifika	High vs Low
0	0.31	0.42	0.86	-0.44	-0.52	0.15	0.09	0.53	0.27
1	0.10	0.01	0.21	0.11	0.57	0.08	0.10	0.26	0.17
2	0.03	0.07	0.31	0.12	-0.06	0.13	0.17	0.21	0.18
3	0.10	0.13	0.20	0.13	0.24	0.13	0.25	0.25	0.26
4	0.09	0.19	0.25	0.17	0.18	0.12	0.38	0.31	0.23
5	0.14	0.19	0.34	0.15	0.14	0.14	0.49	0.31	0.21
Average	***0.13***	***0.17***	***0.36***	***0.10***	***0.05***	***0.13***	***0.25***	***0.31***	***0.22***

* Manurewa Enhancement Initiative

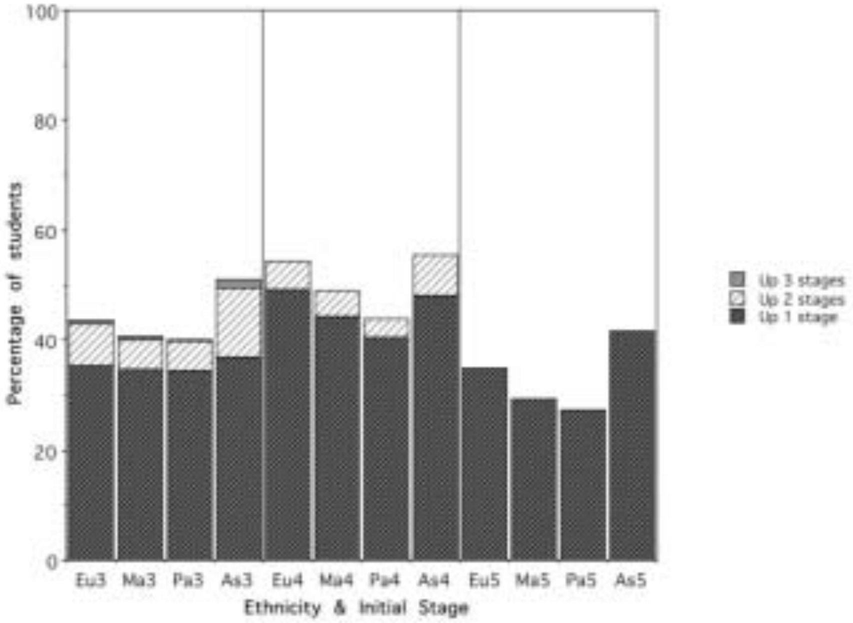

Figure 5. Percentage of students who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial stage and *ethnicity* (2004)

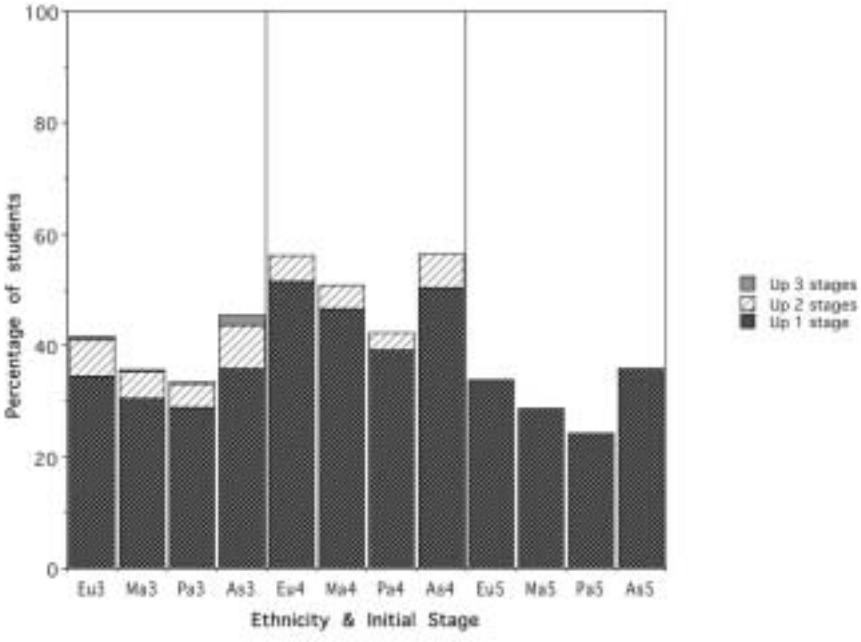

Figure 6. Percentage of students who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial stage and *ethnicity* (2003)

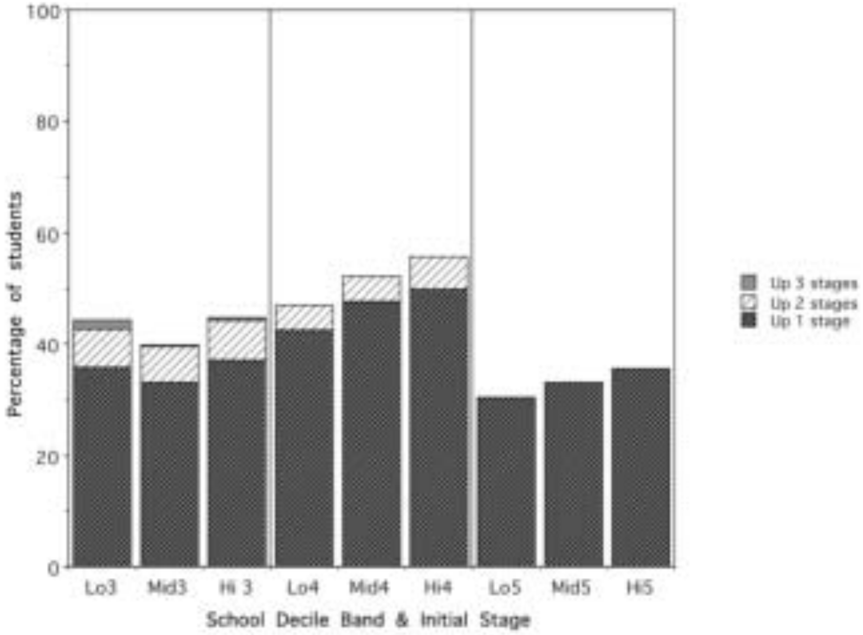

Figure 7. Percentage of students who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial stage and *decile* band (2004)

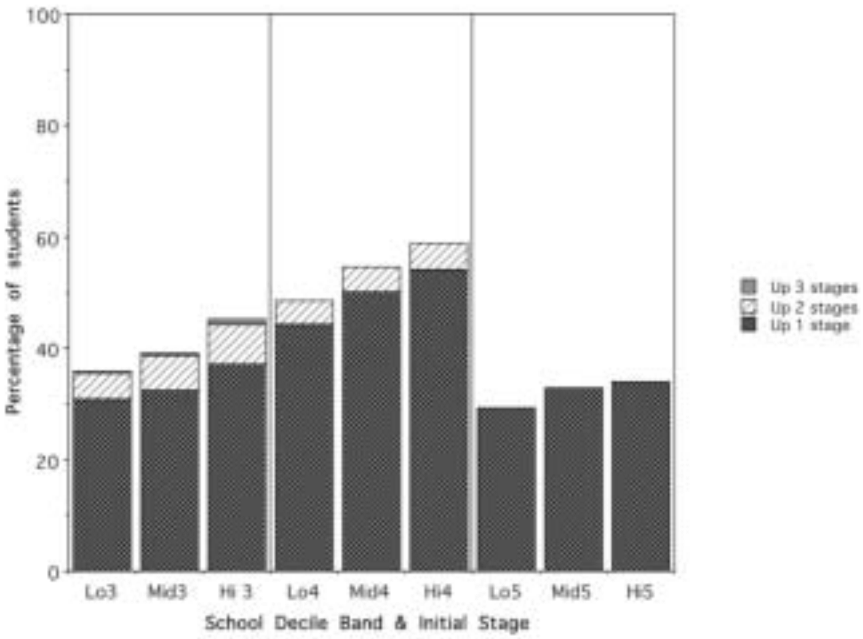

Figure 8. Percentage of students who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial stage and *decile* band (2003)

The possible impact of other initiatives on some low-decile schools

Other research has shown that students at low-decile schools have lower levels of achievement than those at medium- or high-decile schools (see Alton-Lee, 2003). The "decile" system that assigns a ranking of 1 to 10 to a school on the basis of census information about the income and educational levels in the mesh blocks in which its students reside was developed to enable more funding to be provided to more disadvantaged schools. Over the last few years, there have been various School Improvement initiatives operating quite independently of the NDP. An analysis done for this report indicates that some of the improvements for students at low-decile schools could have been the result of one of the special initiatives that was put in place in 2004 to provide extra support for schools in certain low-income areas. The Manurewa Enhancement Initiative (MEI) was a schooling improvement initiative focusing on integration and alignment with the NDP and had as one of its goals "added value", rather than just implementing the NDP in the normal way. Eight low-decile primary schools with complete data were identified from the list of MEI schools for this analysis. The patterns of progress for students at the eight MEI schools (n = 942) were compared with the corresponding patterns for the students at other low-decile schools (n = 17 329; see Table C5 in Appendix C). The patterns of progress were somewhat inconsistent and seemed to depend on students' starting points. For example, MEI students who began the project at stage 1 (One-to-One Counting) made greater progress than that made by other students at low-decile schools who also started at this stage [$t\ (58) = 3.62$, $p < 0.01$]. Those who started at stage 3 (Counting from One) also made significantly greater progress than that made by other comparable students [$t\ (67) = 2.08$, $p < .05$]. A similar pattern was evident for students who began the project at stage 4 (Counting On) [$t\ (437) = 3.64$, $p < 0.001$]. The opposite pattern was found at other starting points, including stage 5, Early Additive Part–Whole. Hence the effect sizes ranged from –0.52 to 0.57, averaging out at 0.05. The reason for the inconsistencies could be that the number of MEI students who began the project at stage 5 was relatively small compared with those in low-decile schools nationally. It may also be that the focus was more on developing increasingly efficient counting strategies and providing experiences with partitioning and recombining small quantities rather than working with multi-digit quantities.

The impact of gender on progress

Figure 10 shows the patterns of progress on addition/subtraction as a function of gender and initial stage. As in 2003, there appeared to be a small difference between boys and girls who started the project at a Counting All stage (stage 3) or below (favouring girls), whereas for those starting at stages 4 or 5 (counting on or simple partitioning strategies), boys made significantly greater gains than girls. Analysis of effect sizes for each starting point on each operational domain shows that the only significant gender difference that favoured girls was for those who were initially Emergent on addition/subtraction (stage 0). The average effect size on addition/subtraction was 0.05.

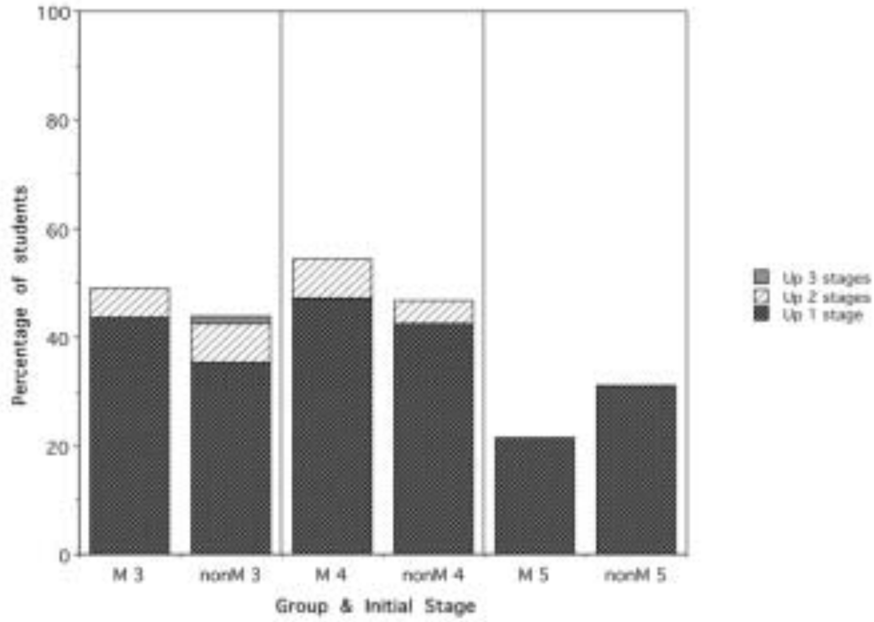

Figure 9. Percentage of students at low-decile schools who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial stage and *group* (schools involved in the Manurewa Enhancement Initiative vs others)

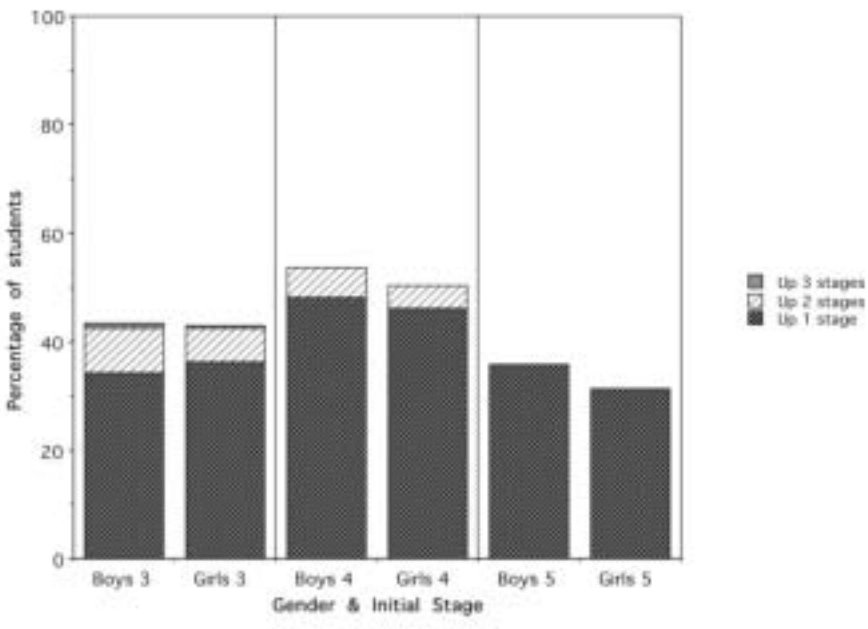

Figure 10. Percentages of students who progressed to a higher framework stage on *Addition/Subtraction* as a function of initial framework stage and *gender*

On multiplication/division, boys made significantly better progress than girls at each of the starting points, and the average effect size was 0.09. On proportion/ratio, boys again made significantly better progress than girls for all starting points except stage 7 (Advanced Multiplicative Part–Whole). The average effect size for proportion/ratio was 0.08.

Discussion

The analysis reported in this paper has shown that students who participated in the NDP in 2004 made significantly better progress on the Number Framework than would have been expected simply as a result of getting older. The advantage of being involved in the NDP was so great that it put younger students significantly ahead of slightly older peers who had not yet participated in the project. The calculation of effect sizes allowed the magnitude of differences to be examined for various different outcome measures and subgroups. The average effect size for addition/subtraction was 0.19, a relatively modest value, but very similar to that found for the National Numeracy Strategy in the UK (0.17 or 0.18; see Brown et al., 2003). However, average effect sizes for multiplication/division and proportion/ratio were more than double (0.40 & 0.43, respectively). It should be remembered that the effect sizes in the present study have been calculated using as a control group students who were, on average, a quarter of a year older than the students in the "experimental" group, and hence the effect sizes are very conservative measures of the impact of the "treatment" on students' performance.

There is some evidence in the 2004 data that shifts are beginning to occur in the patterns of progress found for some groups of students. Students from low-decile schools who began the project at stage 3 (Counting from One) or lower made significantly greater progress than students from medium-decile schools who also began the project at stage 3 or lower. Māori and Pasifika students made slightly better progress in 2004 than in 2003. These patterns (for low-decile and Māori/Pasifika students who began the project at or below stage 3) could be explained by the fact that additional support and resources were provided for low-decile schools in certain regions with a high concentration of low-decile schools (Ministry of Education, personal communication). A similar pattern was evident among students who began the project at the Counting On stage, although the progress of low-decile students was no greater than for medium-decile schools. The analysis of data for students involved in the Manurewa Enhancement Initiative shows how additional support and resources can make an even greater difference for students at low-decile schools. This is similar to the findings of a study that examined the impact of a major literacy initiative that succeeded in raising teachers' expectations of students' achievement in the early school years, and improved their students' literacy skills (*Picking Up the Pace*, see McNaughton et al., 2000, 2003; Phillips et al., 2002). Teachers in these schools have been able to see for themselves that it is indeed possible to change the educational outcomes for students from economically disadvantaged backgrounds, providing close attention is paid to meeting the students' particular learning needs in the classroom.

One of the strengths of the NDP is that it has evolved in response to feedback from the project evaluators and facilitators. Anecdotal evidence suggests that as the numeracy facilitators have gained more experience and understanding about the project, they have become increasingly effective in their work with teachers. At the beginning of the project, lower decile schools were given priority for inclusion in the project. However, it is the schools that have participated in the project in the more recent years that have benefited most from the accumulated wisdom of the facilitators. Ironically, more recent cohorts include disproportionately more high-decile schools and fewer low-decile schools. Teachers in low-decile schools, more often than those in medium- or high-decile schools, have additional issues to deal with on top of meeting the learning needs of their students in classrooms (for further details, see Ritchie, 2004). Although there have been efforts to provide further support for low-decile schools that participated in the project in earlier years (i.e., "sustainability" funding), it is not clear that these efforts have been sufficient to maintain the original impetus of the project. As the literature on educational reform shows, changing the ways that things are done in classrooms and schools is an extremely difficult and challenging process. However, it is to be hoped that the shifts beginning to occur in patterns of progress for students from lower decile schools will be sustained in subsequent years.

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Evidence for Expectations: Findings from the Numeracy Project Longitudinal Study

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Recent research in New Zealand and internationally suggests that a school-wide focus on using achievement information contributes to improved student achievement. Knowing acceptable rates of progress and appropriate levels of achievement enables school communities to critically reflect on achievement information. This paper reports on the numeracy achievement of students in 31 schools in the years following their participation in the Numeracy Development Project. The findings provide evidence for expected levels of numeracy achievement over time. The schools who reported extensive use of numeracy achievement data appeared to raise the achievement of their students more than schools with a lower reported use of achievement information.

Background

The Numeracy Development Project

The Numeracy Development Project (NDP) focuses on improving students' achievement in mathematics through strengthening the professional capability of their teachers (Ministry of Education, 2004). Several key components are considered central to the effective implementation of the project (Higgins, Parsons, & Hyland, 2003). At the core of the NDP is the Number Framework, which has been informed by research showing that there are identifiable progressions in how children develop number concepts (see Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Jones, Thornton, Putt, Hill, Mogill, Rich, & Van Zoest, 1996; Steffe, 1992; Wright, 1998; Young-Loveridge & Wright, 2002).

The framework, which has evolved over the duration of the project in response to student achievement information and feedback from teachers and numeracy facilitators, has a strategy and a knowledge section. The strategy section consists of a sequence of nine stages that describe the strategies students use to solve number problems. The first five stages (0 to 4) involve increasingly sophisticated counting strategies, while stages 5 to 8 involve the use of increasingly complex partitioning strategies. The partitioning strategies are based on using knowledge of number properties to break numbers apart and recombine them in ways that make the problem easier to solve. The knowledge section describes the key items of number knowledge that students need to learn, including number sequence and order, numeral identification, grouping and place value, basic facts, and written recording. The two components are viewed as interdependent, with strategies creating new knowledge, and knowledge providing the foundation for new strategies (Young-Loveridge & Wright, 2002). The Number Framework can be described as a pedagogical tool in that it provides teachers with "direction for responding effectively to children's learning needs" (Higgins et al., 2003, p. 166).

Another key component of the NDP is the Numeracy Project Assessment tool (NumPA), an individual, task-based interview designed to provide teachers with information about their students' number knowledge and strategy use. This number profile is aligned to the Number Framework.

A third component of the NDP is the professional development programme, which involves the participation of the whole school, usually over a two-year period. The programme involves a series of workshops and in-class visits by a facilitator, who provides feedback and support to the teacher in their implementation of numeracy practices. Teachers participating in the NDP are required to assess their students using the NumPA early in the professional development programme and again at its completion. The results of these assessments are submitted to a secure website for use by the project's evaluators.

The NDP was first implemented in New Zealand schools in 2001, following two pilot projects in 2000 (the Count Me In Too pilot for students in years 1–3, and the Numeracy Exploratory Study for students in years 4–6). Since then more than 300 000 students and 14 000 teachers have participated in the project. The project has been informed by annual evaluation reports that have examined the impact of the NDP on students' learning, as well as exploring the experiences and perceptions of the numeracy facilitators, teachers, and principals. Findings from the evaluations indicate that the project has had a positive impact on the quality of teaching and learning in mathematics (Christensen, 2003, 2004; Higgins, 2003, 2004; Irwin, 2003, 2004; Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004; Young-Loveridge, 2004).

Positioned within the evaluations commissioned by the Ministry of Education is the NDP Longitudinal Study. The overarching aim of the Longitudinal Study is to investigate the impact over time of the NDP on students' mathematics achievement. This paper reports on aspects of the findings from the 2004 Longitudinal Study.

Using achievement information to raise achievement

Recent research in New Zealand has linked the use of student achievement information to quality teaching practices that facilitate higher achievement (Alton-Lee, 2003; Timperley & Parr, 2004). Alton-Lee, in her evidential synthesis of quality teaching, states that:

> The gathering and analysis of high-quality student achievement data and the use of externally referenced benchmarks have been found to be powerful tools in bringing about changes in teacher practice that facilitates higher achievement for students. (p. 19)

Evidence from New Zealand and overseas suggests that a school-wide focus on using achievement information effectively helps to raise student performance (see Goddard, Hoy, & Woddfolk Hoy, 2004; Timperley & Parr, 2004).

> New Zealand research has shown that teachers and their curriculum leaders who worked together to examine the implications of evidence of student achievement for their teaching had higher achieving students. (Timperley & Parr, 2004, p. 11)

Further to this, Alton-Lee (2003) and Timperley (2003) contend that effective professional development initiatives are those that make explicit the kinds of teaching practices that support learning and link these approaches to student achievement information. Alton-Lee cites the NDP as an example of such a professional development initiative.

> The study [NDP] is particularly significant in our best evidence synthesis because it is one of the few New Zealand studies to trace increases in student achievement linked to professional development and teaching practice across a broad national sample of students. (Alton-Lee, 2003, p. 45)

NDP Longitudinal Study: Overview and Methodology

One of the aims of the NDP Longitudinal Study is to collect numeracy data from students in the years following their school's participation in NDP to help establish benchmarks or expectations for achievement. In addition to tracking the numeracy achievement in schools over time, the Longitudinal Study in 2004 sought to link achievement levels to the extent that schools reported they made use of achievement information.

Sample

The NDP Longitudinal Study began in 2002 and has focused on tracking the achievement of students on the Number Framework over time. Each year since 2002, further schools have been added to the study. In 2004, a total of 31 schools were invited to participate in the study. Eight of these schools first participated in 2000, seven first participated in 2001, and 16 began in 2002. Nineteen of these 31 schools submitted data on their students' strategy stages to the project website during November 2004. Table 1 shows the breakdown of these longitudinal students by ethnicity compared to the NDP 2004 figures. The longitudinal sample had a higher proportion of Māori students and a correspondingly lower proportion of New Zealand European students.

Table 1
Analysis of Students by Gender and Ethnicity

Ethnicity	Female		Male		Total	
	Longitudinal	NDP 2004	Longitudinal	NDP 2004	Longitudinal	NDP 2004
NZ European	52%	61%	51%	60%	51%	60%
Māori	27%	19%	27%	20%	27%	20%
Pasifika	11%	10%	11%	10%	11%	10%
Asian	7%	5%	6%	5%	6%	5%
Other	5%	4%	5%	4%	5%	4%
Total	3012	34423	3087	35875	6099	70298

Table 2 compares the longitudinal and NDP 2004 samples by year level and school decile.[2] There is a greater proportion of students from low-decile schools and a correspondingly lower proportion from medium-decile schools in the longitudinal sample that returned data than in the national sample from NDP 2004. There are similar proportions of high-decile schools. The analyses undertaken for the NDP evaluations since its implementation have consistently shown that students from low-decile schools are lower performing than students from medium- and high-decile schools (Young-Loveridge, 2004). In addition, the trend has been for Māori and Pasifika students to perform lower than New Zealand European and Asian students

[2] The Ministry of Education uses a decile rating system for school funding purposes. Each decile contains approximately 10% of schools. Schools in decile 1 have the highest proportion of students from low socio-economic backgrounds. Schools in decile 10 have the lowest proportions of these students. The low-decile band includes decile 1 to 3 schools, the medium band includes decile 4 to 7 schools, and the high-decile band includes decile 8 to 10 schools. A small number of schools in NDP 2004 did not return decile information and were excluded from Table 2.

(Young-Loveridge, 2004). Consequently, the impact of these two factors needs to be considered when the longitudinal results are compared to the national sample from NDP 2004.

Table 2
Analysis of Students by Year and School-decile Band

	Low Decile		Medium Decile		High Decile		Total	
Year	Longitudinal	NDP 2004	Longitudinal	NDP 2004	Longitudinal	NDP 2004	Longitudinal	NDP 2004
1	38%	20%	31%	37%	31%	40%	1119	7793
2	37%	21%	28%	35%	35%	40%	819	8196
3	37%	25%	26%	35%	37%	38%	912	8515
4	32%	26%	29%	37%	39%	34%	878	10012
5	41%	29%	33%	38%	26%	31%	925	9868
6	43%	29%	33%	36%	24%	32%	838	9959
7	44%	27%	28%	47%	28%	21%	326	8372
8	48%	29%	30%	45%	22%	19%	282	7306
Total	39%	26%	30%	39%	31%	32%	6099	70298

Methodology

The longitudinal schools were asked to submit the additive, multiplicative, and proportional strategy stages of their students on the Number Framework to a secure website during November 2004. In addition to the strategy stages, information was collected about each student's gender, date of birth, school year level, and ethnicity. The students were linked to schools, so their achievement can also be reported by decile. As previously mentioned, 19 of the 31 schools submitted this information; the remaining 12 schools failed to do so.

Questionnaires were sent to all teachers in the 2004 longitudinal schools in August to gain their perceptions on student numeracy achievement, details on the data they collect on student mathematics achievement, and their use of student achievement information. In addition, the researchers held a day-long meeting with the numeracy lead teachers in six of the schools that reported extensive use of student achievement information.

Results and Discussion

How do the longitudinal students compare?

This section details the levels of achievement, as measured by stages on the Number Framework, of students in the 19 longitudinal schools that submitted numeracy data. Figures 1–4 compare the achievement of the longitudinal students on the Number Framework with the before-project and after-project results of students in schools participating in NDP 2004. The bars show the percentage of students at each framework stage on the given strategy domain. The NDP 2004 students are labelled as "NZ", while the longitudinal are referred to as "Long". "Initial" and "Final" refer to the result of the NumPA interviews conducted at the start and conclusion of the NDP 2004. A description of the numbered stages is included in the appendices (Appendix D).

Figure 1 shows the performance of year 1 to 3 students on the additive domain. The additive domain examines the strategies that students use to solve addition and subtraction problems. These strategies can be categorised as counting strategies (stages 1 to 4) or partitioning strategies (stages 5 to 6). A student at stage 0 is classified as pre-counting or emergent. A comparison of the second and third bars within each year level shows that the year 2 and 3 longitudinal students have similar patterns of achievement to the NDP 2004 students. This suggests that, over time, the gains made during the project are sustained. The year 1 students in the NDP 2004 outperform the longitudinal year 1 students, with 12% of the NDP 2004 students at stage 0 or 1 compared to 29% of the longitudinal students. This is consistent with the findings of the 2003 Longitudinal Study (Thomas & Tagg, 2004) and is most likely the result of the almost exclusive focus on numeracy in the mathematics programme of teachers during the professional development phase of the project. It may also reflect the impact of the different demographics of the two samples as students in low-decile schools are more likely to start school at the emergent stage (24%) than students in high-decile schools (11%).

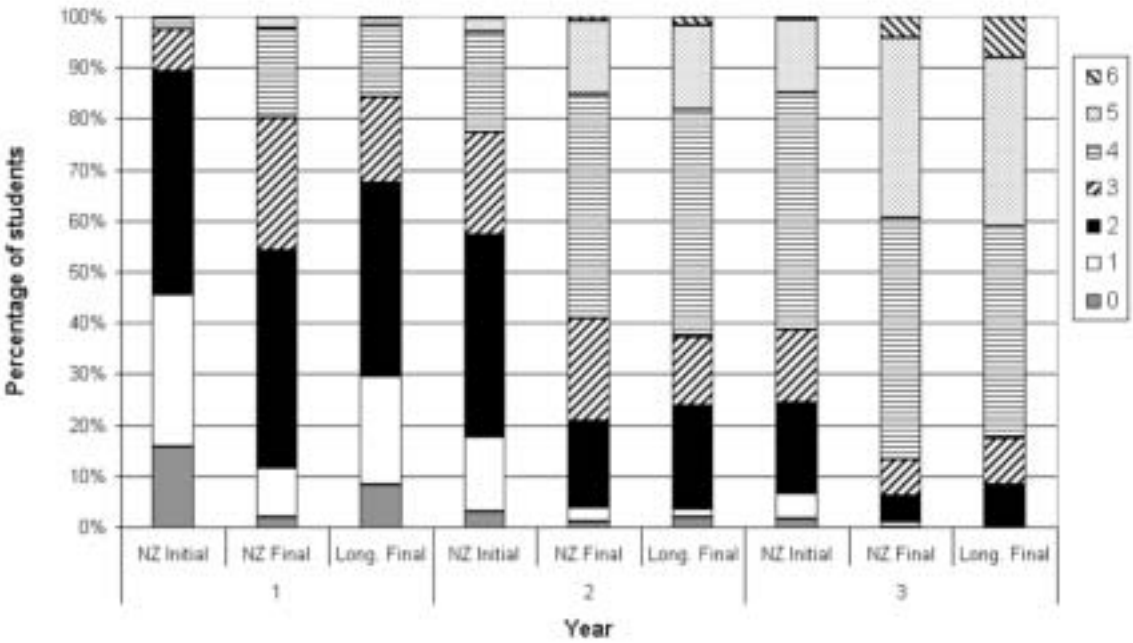

Figure 1. Additive strategy levels of year 1–3 longitudinal and NDP 2004 students

Figure 2 shows the percentage of year 4 to 8 students at each framework stage on the additive domain. The first two bars in each year level show the gains made during the project for the NDP 2004. These gains appear similar to those found in previous years of the project (see Young-Loveridge, 2004). A comparison of the second and third bars in each year level suggests that the gains made during the project have been extended over time. For example, 50% of the year 6 longitudinal students are at the highest stage on the additive domain (stage 6) compared to 37% of the NDP 2004 students. A comparison of the third bar at each year level with the first bar of the next year level provides an indication of the raised levels of numeracy achievement as a result of the project. At years 4 to 7, the longitudinal students have significantly higher levels of achievement than the before-project results of students at the next year level. For example, 78% of year 5 longitudinal students are using partitioning strategies (stage 5 or 6) compared to 62% of the NDP year 6 students prior to the project.

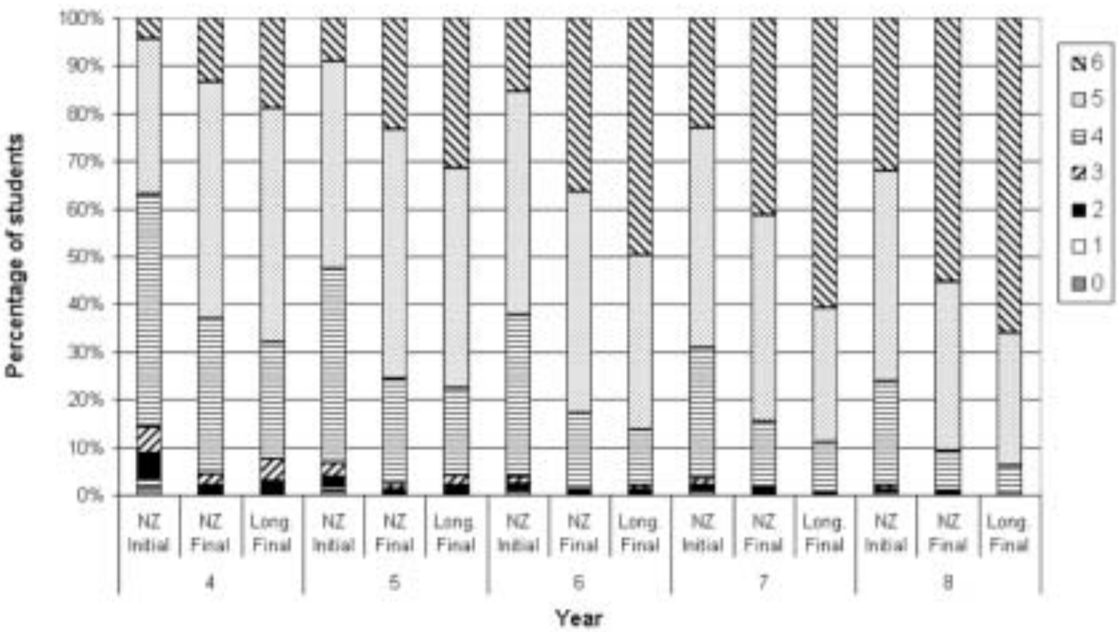

Figure 2. Additive strategy levels of year 4–8 longitudinal and NDP 2004 students

Figure 3 shows the percentage of year 4 to 8 students at each framework stage on the multiplicative domain. The multiplicative domain examines the strategies that students use to solve multiplication and division problems. The first two bars in each level show the gains made during the project for the NDP 2004. These gains appear similar to those made in previous years of the project (see Young-Loveridge, 2004). A comparison of the second and third bars in each year level indicates whether gains made during the project have been sustained over time. This comparison suggests that the gains made on the multiplicative domain develop further at years 7 and 8. For example, 47% of the year 8 longitudinal students are at stage 7 (advanced multiplicative) compared to 33% of NDP 2004 students. Similarly, 68% of the year 7 longitudinal students are at stages 6 or 7 compared to 61% of the NDP 2004 students. This improved performance over time is not apparent for the year 4 to 6 students on the multiplicative domain. For example, 32% of year 5 longitudinal students are at stages 6 or 7 compared to 38% of the NDP 2004 students.

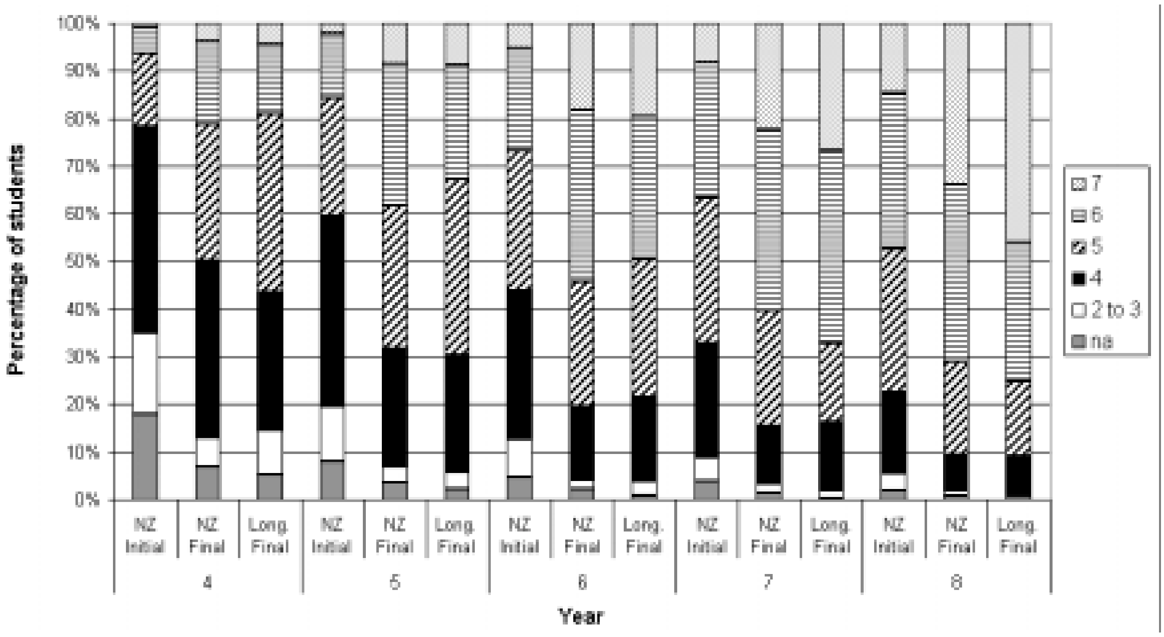

Figure 3. Multiplicative strategy levels of year 4–8 longitudinal and NDP 2004 students

Figure 4 shows the percentage of year 4 to 8 students at each framework stage on the proportional domain. The proportional domain examines the strategies that students use to solve problems involving rates and proportions. The gains illustrated by a comparison of the first two bars in each level are similar to those made in previous years of the project (see Young-Loveridge, 2004). A comparison of the second and third bars illustrates similar patterns of achievement at years 4–6. The longitudinal students appear to outperform the NDP 2004 students at years 7 and 8. As illustrated by Figure 4, a very small proportion of students are rated at stage 8. However, it is encouraging to note that there is a greater percentage of longitudinal year 8 students (19%) at stage 8, compared with NDP 2004 students (9%).

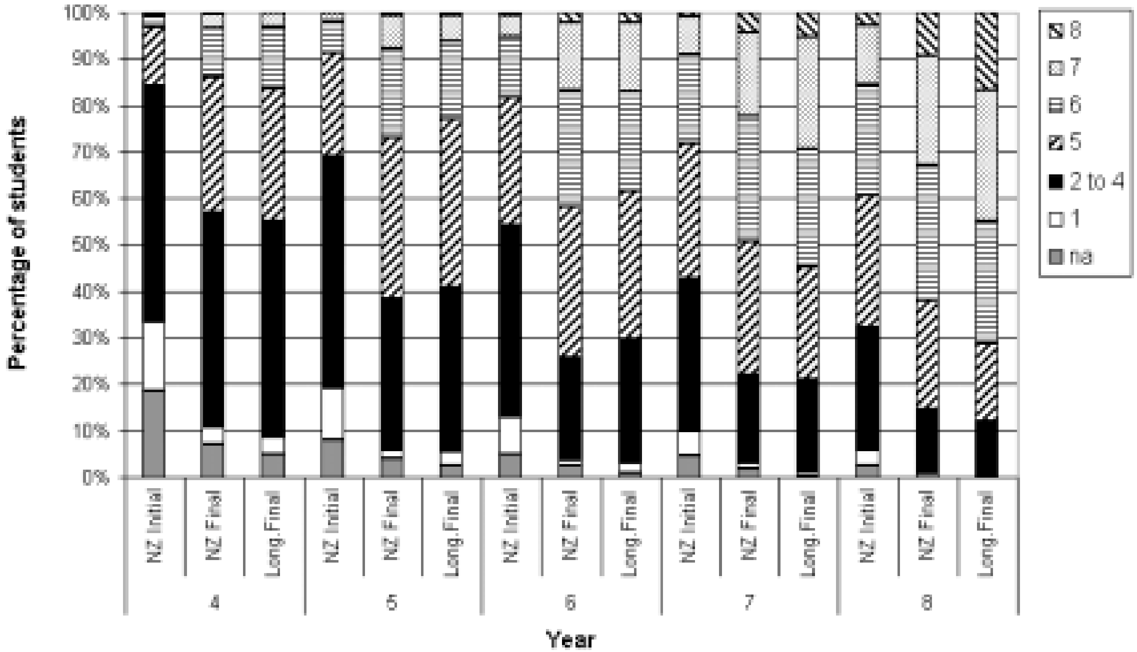

Figure 4. Proportional strategy levels of year 4–8 longitudinal and NDP 2004 students

Patterns of progress in the longitudinal schools: evidence for expectations

This section examines patterns of performance in the longitudinal schools since 2002. In 2002 and 2003, the schools were only asked to return information on the additive domain, so this is the only domain reported on in this section. There were also few year 7 and 8 students in the 2002 data return, so only year 1 to 6 students are discussed. The bars in Figures 5 to 6 show the percentage of students at each framework stage on the additive domain in the given year.

Figure 5 presents the percentages of year 1 to 3 students at each stage on the additive domain since 2002. The three bars at each year level illustrate a consistency in performance over time. The slightly higher levels in 2004 are surprising, given the skewed demographics of the longitudinal sample in 2004. It is pleasing to note how few year 2 and 3 students in the longitudinal sample are at the lowest stages of the Number Framework and how this percentage has continued to fall over time. For example, the percentage of year 2 students at stage 0 or 1 has dropped from 7% in 2002 to 3% in 2004. The consistent pattern of results illustrated in Figure 5 provides evidence for expected levels of achievement for year 1 to 3 students. For example, it seems reasonable to expect almost all year 3 students to be at least at stage 4 (advanced counting), with 40% at stage 5 (early additive) or higher.

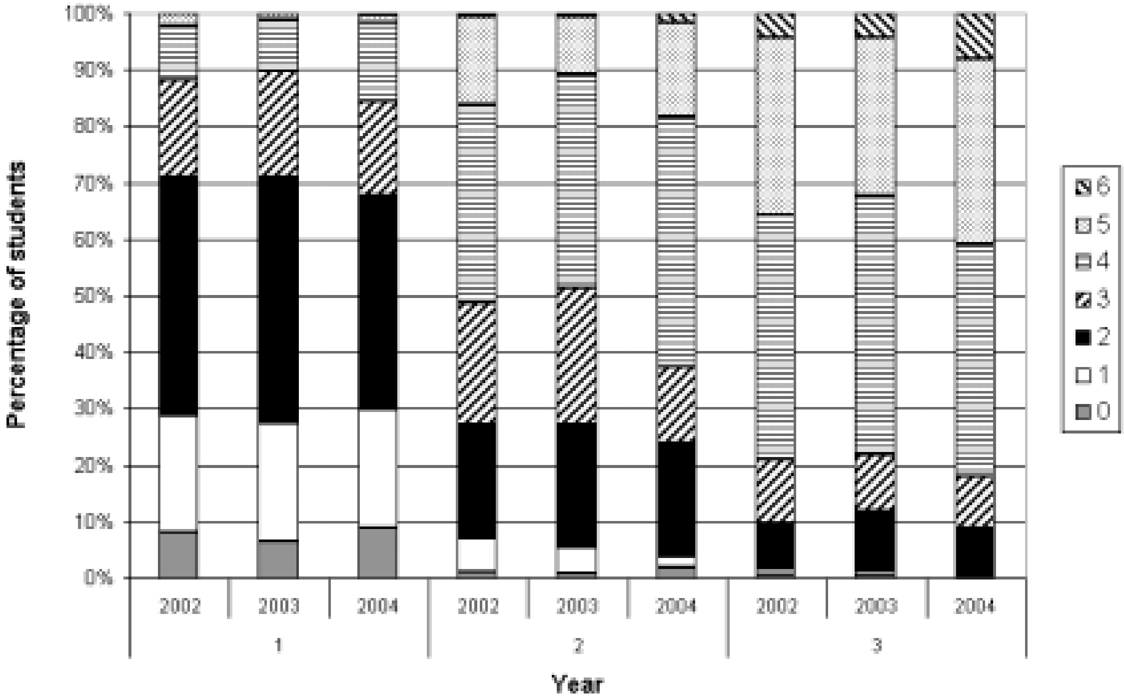

Figure 5. Additive strategy levels of year 1–3 longitudinal students

Figure 6 shows the proportions of year 4 to 6 students at each stage on the additive domain since 2002. There has been a decrease over time in the percentage of students using counting strategies (stages 1–4) at all three year levels. In 2002, 45% of the year 4 students used counting strategies compared with 32% in 2004. Fourteen percent of the year 6 students were assessed at the counting stages in 2004 compared with 22% in 2002. The percentage of students reaching stage 6 in 2004 is slightly lower than in 2003. This may be the result of the different demographic profiles of the two samples. Figure 6 provides evidence for expected levels of achievement for year 4 to 6 students who have been taught for at least two years by numeracy-trained teachers. It will be interesting to see if these levels continue to increase as students experience numeracy practices from school entry. Figure 6 suggests that it is reasonable to expect almost all year 6 students to be using additive strategies (stage 5 or 6), with 50% at the highest level of the additive domain (stage 6).

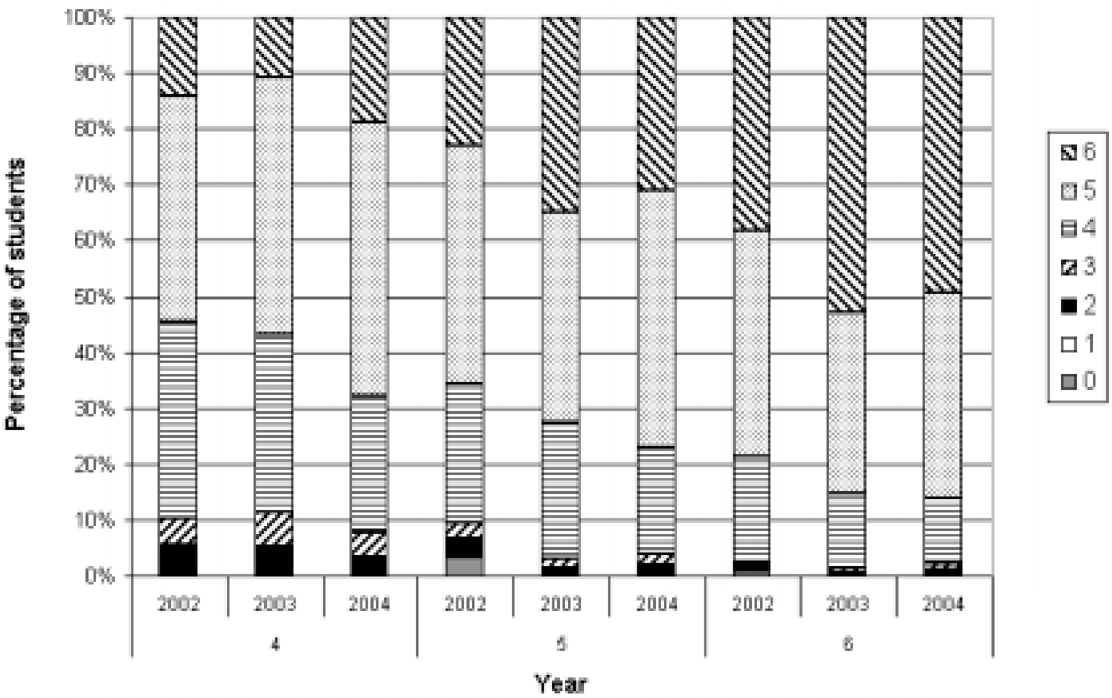

Figure 6: Additive strategy levels of year 4–6 longitudinal students

Perspectives of the teachers in the Longitudinal Study

This section summarises the key findings from the questionnaires returned by a total of 283 teachers from 30 of the 31 longitudinal schools. The questionnaire items asked teachers to respond using a five-point Likert scale. Each item included a space for teachers to provide written elaboration. Seventy-eight percent of the teachers had completed the NDP, with 89% of these undertaking training in their current school. The majority of those who had not completed the NDP reported receiving some level of numeracy professional development in 2004.

Two of the items asked the teachers to reflect on the impact of the NDP on their students' ability in number and in mathematics more generally. In general, the respondents believed that the project had an ongoing positive impact on students' ability in number, with 74% describing this impact as very positive or highly positive. One respondent who indicated a very positive impact explained that this impact would change to highly positive as her teaching strategies improved.

> I'm very impressed with how quickly the children are able to use numbers in their heads. I will circle highly positive impact when improvement in my teaching generates that. (Teacher, high-decile school)

The majority (66%) of teachers also believed that the project has had a very positive or highly positive impact on their students' mathematics achievement. Teachers ascribe the improved achievement to an improved attitude and enthusiasm for mathematics and an increase in number understanding, explaining that this understanding underpins the other strands of mathematics.

> Noticing an application of sound number sense in other areas of maths – stats, measurement. Greater enjoyment in maths, therefore progress is quicker. (Teacher, low-decile school)

Several questions were designed to gather information on the use of mathematical achievement information, including the establishment and use of achievement targets. Fifty-seven percent of the teachers indicated that their school had developed targets for achievement. While many of the teachers indicated that the targets were used for reporting progress to parents and the Board of Trustees, the focus in other schools was on the use of targets to identify students requiring additional support on or aspects of teaching practice that needed attention.

> Use targets to show strengths at certain year levels and any points of issue. To identify problem areas that are stopping us from reaching targets and develop these through professional development and targeted teaching. (Teacher, low-decile school)

> To measure teacher and whole-school and student effectiveness. Identify gaps/areas for improvement. Data collected yearly and children compared to targets. Those not reaching targets are identified for additional support. We raise teacher awareness of children not reaching targets. (Teacher, low-decile school)

Seventy-five percent of the teachers reported that they were happy with the achievement levels in their class. A similar proportion of teachers indicated that the majority of students in their class were on track to meet expected levels of achievement. Of those who were unhappy, the majority were able to suggest plans for addressing the problem. The plans ranged from a greater focus on the lower achieving students to more professional development for the teacher. A number of respondents also suggested a greater emphasis is needed on numeracy in early childhood education.

> My own professional development. As a beginning teacher, the professional development I received in 2003 didn't meet my needs. I would like to have seen more hands-on teaching and to have had more feedback on my practices. (Teacher, medium-decile school)

> Early childhood experiences/education should have a greater emphasis on early mathematics acquisition. Several children entering school have very limited experiences of number or language concepts such as above/below, more/less, big/small, etc. (Teacher, medium-decile school)

The impact of using achievement information

The data collected in the Longitudinal Study in 2004 allowed us to examine the assertion that a school-wide focus on using achievement information effectively helps to raise student performance. In order to examine this, we categorised the 19 schools that returned student numeracy results into two groups on the basis of their reported *use* of student achievement information. A school was categorised as having a focus on achievement data if more than 75% of the teachers reported that their school had established and used targets for student achievement in numeracy. Consequently, 13 of the 19 schools were classified as having a school-wide focus on numeracy. In all but one of the remaining six schools, fewer than 25% of the teachers said the school had established and used numeracy targets. In the remaining school, there was an almost even split between those teachers who said targets were used and those who said they weren't. It needs to be noted, however, that the six schools that were classified as not having a focus did at least collect achievement information for the longitudinal study. It was, of course, not possible to compare the achievement of students in schools where no achievement data was collected as the longitudinal achievement information is reliant on returns from schools.

Figure 7 shows the percentage of students at each strategy stage on the additive domain for the longitudinal schools, categorised by their reported use of student achievement information. The first bar in each year level shows the performance of students in schools whose teachers report a focus on the use of achievement information. The second bar shows the performance of students in schools where the teachers reported a lower or inconsistent focus on student achievement data. Table 3 shows the proportion of students in each decile group in the two groups of students. It is interesting to examine the figure for trends in the proportions of students who are at the lower levels of the Number Framework. There are fewer of these students in the schools that focus on student achievement information. For example, at year 3, 13% of the students in the focus schools are at stages 0 to 3 compared to 30% in the non-focus schools. Nineteen percent of the year 5 students in the focus schools are at stages 0 to 4 compared to 33% in the non-focus schools. There appears to be little difference in the achievement of students in year 4 and year 7 between the two classifications of schools. Another finding in support of a school-wide focus on achievement is the substantially higher proportion of year 6 students in the focus schools (54%) who are at the highest stage, compared to the non-focus schools (28%).

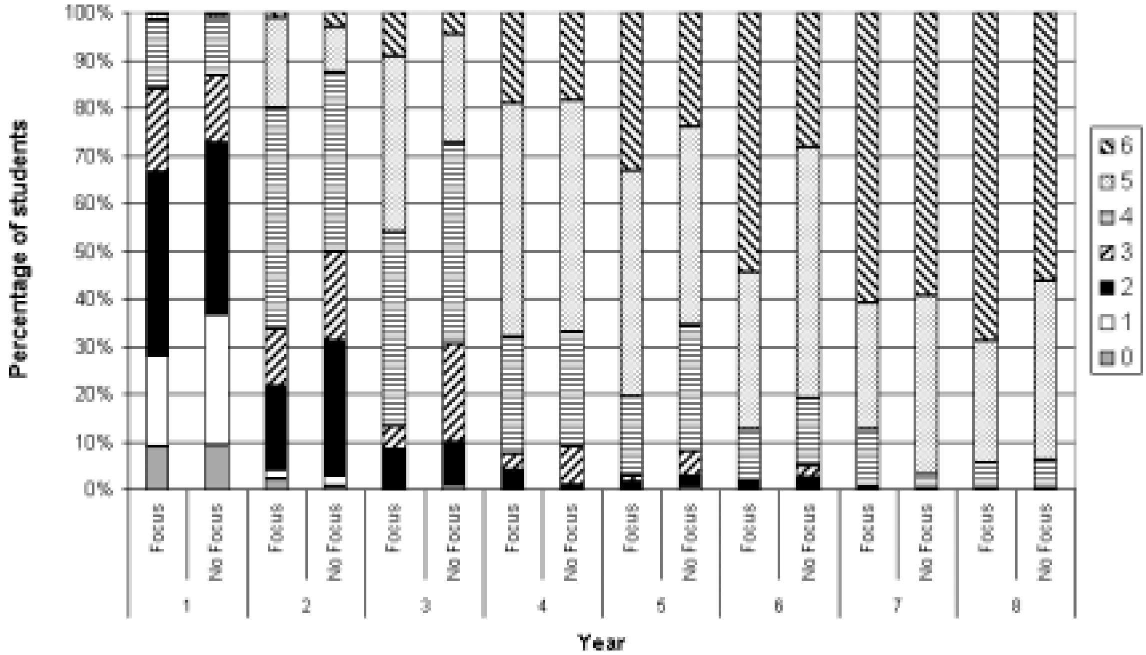

Figure 7. Additive strategy levels of longitudinal students by school focus on achievement information

Table 3
Number of Students by Decile Group and School Focus on Achievement Information

Decile Group	Focus	No Focus
Low	43%	22%
Medium	22%	60%
High	35%	18%
Total	4762	1337

Concluding Comment

The performance of students in the longitudinal schools on the Number Framework provides evidence for describing the levels of performance that can reasonably be expected from students in schools that have participated in the NDP. While there are slight fluctuations in performance over the three years of the longitudinal study, overall there is a consistent pattern. Students in schools that have implemented numeracy practices over several years consistently perform better than students of the same age did prior to the implementation of the NDP, and, at the higher year levels, also perform better than schools after their first year of involvement in the NDP. While demographic factors have been shown to influence the performance of cohorts of students, the representative sample of students involved in the longitudinal study means that the results obtained can reasonably be used as a starting point for goal setting.

The findings of the current research also provide further evidence in support of the importance of focusing on student achievement data as a means of raising achievement. The 13 longitudinal schools that set school-wide targets for numeracy and collected student achievement information outperformed the six longitudinal schools that did not set school-wide numeracy targets.

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The Impact of the Numeracy Development Project on Mathematics Achievement

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The central focus of the Numeracy Development Project (NDP) is to raise student achievement in mathematics by improving the professional capability of teachers. This paper reports on the mathematics achievement of year 4, 5, and 8 students in 31 schools that participated in the NDP prior to 2003. The findings suggest that the project has had a positive impact as the NDP students performed better on TIMSS items than did their same-age peers in 1995.

Background

The Numeracy Development Project

Much of the impetus for The Numeracy Development Project (NDP) came from the results of the 1995 Third International Mathematics and Science Study (TIMSS), which showed that New Zealand students performed poorly when compared to other education systems. Garden (1997), in the report on New Zealand performance, stated that "the most direct influence on student achievement in mathematics and science is the teacher" (p. 250). The government's Review of Teacher Education Green Paper (Ministry of Education, 1997a) further linked the relatively poor performance of students in the TIMSS mathematics assessment to their teachers' lack of knowledge in mathematics. In 1997, the Ministry of Education established the Mathematics and Science Taskforce in response to the reported difficulties of teachers (especially primary teachers) in implementing effective mathematics programmes. The report of the taskforce expressed concern about the professional skills and knowledge of teachers. They argued that "satisfactory learning of mathematics and science is strongly influenced by a teacher's own confidence" (Ministry of Education, 1997b, p. 3).

The focus of the NDP is to "improve student performance in mathematics through improving the professional capability of teachers" (Ministry of Education, 2004, p. i). The NDP was first implemented in New Zealand schools in 2001, following a 2000 pilot study of Count Me In Too, a numeracy initiative of the New South Wales Department of Education and Training. Since then, more than 300 000 students and 14 000 teachers have participated in the project. The project has been informed by annual evaluation reports that have examined the impact of the NDP on students' learning, as well as exploring the experiences of the numeracy facilitators, teachers, and principals. Findings from the evaluations indicate that the project has had a positive impact on the quality of teaching and learning in mathematics (Christensen, 2003, 2004; Higgins, 2003, 2004; Irwin, 2003, 2004; Thomas, Tagg, & Ward, 2003; Thomas & Tagg, 2004; Young-Loveridge, 2004).

Trends in International Mathematics and Science Study

TIMSS collects educational achievement data at the fourth and eighth grades to provide information about trends in performance in mathematics and science. Approximately 50 countries from all over the world participate in TIMSS. TIMSS is designed to help countries improve student learning in mathematics and science by identifying and monitoring areas of progress or decline in achievement. One of its most important features is that it has enabled the collection of information on the nature of teaching and learning at both international and national levels. Conducted on a four-year cycle, the first round of TIMSS was in 1995, the second in 1999 (eighth grade only), and the third in 2003. Preparations are underway for the next round of TIMSS, which will take place in 2007. TIMSS assesses the mathematics and science achievement of students in two target populations. Population 1 was defined as "all students in the two adjacent grades that contained the largest proportion of students in the age 9 cohort …" (Garden, 1997, p. 10). In New Zealand, this translated to students in years 4 and 5. Population 2 was defined as "all students in the two adjacent grades that contained the largest proportion of students in the age 13 cohort…" (Garden, p. 10). In New Zealand, this translated to students in years 8 and 9.

NDP Longitudinal Study: Overview and Methodology

As the impetus for the NDP was a desire to improve mathematics achievement, it is necessary to quantify such improvement. The student achievement data has predominantly come from the results of the Numeracy Project Assessment (NumPA) interview. The NumPA data has consistently shown that the project is successful in improving the number profiles of students (Young-Loveridge, 2004). To investigate the impact of the NDP on students' overall performance in mathematics, tests developed using 1995 TIMSS items were given to year 4, 5, and 8 students in 31 schools that first participated in NDP from 2000 to 2002. This paper reports on the performance of the students on these tests.

Sample

The Longitudinal Study began in 2002 with the participation of 20 schools that first implemented the NDP in either 2000 or 2001. In 2004, the number of schools in the Longitudinal Study was increased to 31 through the inclusion of 16 schools that first participated in 2002. Five of the original 20 schools withdrew from the Longitudinal Study at the start of 2004. The three tests generated from the TIMSS items were used to assess 2995 students at years 4, 5, and 8. Tables 1 and 2 show the breakdown of students by year level and gender and by year level and decile band. The low-decile band includes decile 1 to 3 schools, the medium-decile band includes decile 4 to 7 schools, and the high-decile band includes decile 8 to 10 schools.

Table 1
Analysis of Students by Year and Gender

Year	Female	Male	Total
4	663	684	1347
5	678	674	1352
8	148	148	296
Total	1489	1506	2995

Table 2
Analysis of Students by Year and School-decile Band

Year	Low	Medium	High	Total
4	392	405	550	1347
5	421	417	514	1352
8	103	106	87	296
Total	916	928	1151	2995

Table 3 compares the ethnic profiles of the TIMSS sample with the students in the Longitudinal Study. The ethnicity information for the longitudinal sample was obtained from the ethnicities given for students whose data was entered onto the numeracy website in 2004. For the 12 schools that did not enter data in 2004, the ethnicity information was taken from the most recent year they entered data. The proportion of Pasifika students in the longitudinal sample is higher than in the TIMSS 1995 sample for both populations. The difference is primarily balanced by a decrease in the proportion of New Zealand European students.

Table 3
Comparison of Ethnicities

	Population 1 (years 4 and 5)		Population 2 (years 8 and 9)	
	TIMSS	Long.	TIMSS	Long.
NZ European	63%	55%	68%	56%
Māori	26%	23%	19%	28%
Pasifika	6%	9%	7%	12%
Asian	3%	7%	5%	3%
Other	2%	5%	1%	1%

Methodology

Tests for each of the three year levels were developed from the pool of items released from the 1995 TIMSS. The tests comprised 24 items, which were selected to give coverage of all strands of the mathematics curriculum. The tests were piloted in a non-longitudinal school to check that they took approximately 40 minutes to administer. Tables 4 and 5 show the number of items by strand contained in the three tests. Each test was designed so that the average score in each test would be 50%, based on the percentage of New Zealand students answering each item correctly in TIMSS 1995.

Table 4
Analysis of Items in the Year 4 and 5 Tests

TIMSS Content Category	Year 4	Year 5
Data Representation, Analysis, and Probability	2	2
Fractions and Proportionality	2	5
Geometry	5	2
Measurement, Estimation, and Number Sense	3	3
Patterns, Relations, and Functions	5	4
Whole Numbers	7	8
	24	24

Table 5
Analysis of Items in the Year 8 Tests

TIMSS Content Category	Year 8
Algebra	1
Data Representation, Analysis, and Probability	6
Fractions and Number Sense	11
Geometry	4
Measurement	2
	24

Test scripts were sent to each of the participating schools in July. The classroom teachers administered the tests, following instructions adapted from those used with TIMSS. The tests were sent back to the researcher for marking during August. Once the scripts had been marked, a report was compiled for each of the participating schools. This report included details on the item responses of each student and their overall test score. The schools' average performance by item and overall was compared to the TIMSS 1995 performance for the same age peers.

NDP Longitudinal Study: Results and Discussion

All reporting of results in this section is based on the average percentage of items answered correctly by students in the stated sub-groups. For each question, the 95% confidence limits for the difference in mean proportion between the longitudinal sample and New Zealand TIMSS 1995 sample were calculated. This is the criteria used to define significant differences in the results reported below.

As shown in Table 6, the performance of boys and girls in the longitudinal schools was very similar at all three year levels.

Table 6
Average Score by Year and Gender

	Year 4	Year 5	Year 8
Male	56%	58%	53%
Female	56%	59%	52%
Total	56%	58%	53%

As shown in Table 7, year 4 and 5 longitudinal students performed significantly better overall than the New Zealand students in TIMSS 1995. The year 8 students performed at a similar level. The lack of improvement by the year 8 students may be explained by the fact that the earliest exposure they could have had to NDP practices was in 2001 as year 5 students. More than 50% of the year 8 students did not encounter these practices until 2002 or 2003 as year 6 or 7 students.

Low-decile students at all year levels performed not only lower than the medium- and high-decile schools, but also slightly lower than the New Zealand TIMSS 1995 sample (50%). There is no decile information available on the New Zealand TIMSS 1995 sample so no comparisons can be made by decile. The differences between high- and medium-decile students were not significant.

Table 7
Average Score by Year and Decile Level

	Year 4	Year 5	Year 8
Low deciles (1–3)	48%	47%	42%
Medium deciles (4–7)	59%	61%	58%
High deciles (8–10)	61%	65%	59%
Total	56%	58%	53%

Table 8 shows the average performance of students on items classified by mathematical content. The items classified as number for the year 4 and 5 tests included the categories of whole number; fractions and proportionality; and measurement, estimation, and number sense. The items classified as number for the year 8 test included: fractions and number sense; and algebra. The results show that the year 4 and 5 students performed significantly better than the New Zealand TIMSS 1995 sample on both number and non-number items. The year 8 students performed at a similar level on both categories of items.

Table 8
Performance of Students on Number and Non-number Items

	Year 4		Year 5		Year 8	
	Number	Non-number	Number	Non-number	Number	Non-number
TIMSS 1995	47%	53%	49%	53%	47%	53%
Longitudinal	55%	58%	56%	63%	49%	56%

Results of the Year 4 Students

Of the 24 questions in the year 4 test, longitudinal students performed better on average than the TIMSS 1995 New Zealand sample on 16 questions, and equally well on six questions. The two questions on which longitudinal students performed significantly lower than New Zealand students in TIMSS 1995 were question 3 (38%, compared with 47%), in which students were presented with the sum 6971 + 5291 in vertical form, and question 13 (43%, compared with 67%), in which students were asked to identify which of a set of shapes was "made with straight lines only". As shown in Figure 1, the numbers presented in question 6 are too large and, with three columns adding to greater than 10, too complicated to be readily added using a mental strategy. As written forms are not introduced until the higher stages of the Number Framework, few year 4 students would be expected to answer this question correctly.

6 Add

$$\begin{array}{r} 6971 \\ \underline{+5291} \end{array}$$

a 11 162
b 12 162
c 12 262
d 1 211 162

	Percentage
Longitudinal	38%
NZ 1995	47%
TIMSS 1995	67%

Figure 1. Item 6

The poor results for question 13 can possibly be explained by the quality of the reproduction of the question in the test. Although the image in Figure 2 was taken directly from the released items published on the TIMSS 1995 website, the corner at the top of the shape D appears slightly curved (http://timss.bc.edu/timss1995.html).

13. Which of these is made with straight sides only?

	Percentage
Longitudinal	57%
NZ 1995	67%
TIMSS 1995	66%

Figure 2. Item 13

Of the 15 questions on which the longitudinal students performed better than the New Zealand students in TIMSS 1995, there were 13 in which the difference in percentage of correct answers was greater than 5% and 10 in which the difference was greater than 10%. The largest difference was on question 22 (Figure 3), in which students were asked to draw the shape produced when a piece of paper that had been folded and then cut was opened out. Seventy-three percent of longitudinal students answered correctly, compared with 48% of New Zealand students in TIMSS 1995 and 45% of the TIMSS international sample.

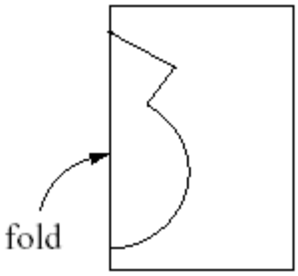


Figure 3. Item 22

The next three largest differences were for questions 9, 15, and 24, with differences in percentages of correct responses of 20%, 20%, and 18% respectively. These three questions are all based in the number strand, on which students in NDP schools have had a greater focus. Their strong performance on these questions is encouraging. Question 9 asked the students to write a fraction that was larger than $2/7$. Fifty-eight percent of the longitudinal students gave a correct answer, compared with 38% of the New Zealand and 41% of the international samples from TIMSS 1995. Question 15 asked the students to identify the missing number from a segment of a hundreds chart (see Figure 4). Seventy-seven percent of the longitudinal students answered this correctly, compared with 57% of the New Zealand and 64% of the international samples from TIMSS 1995. Question 24 asked the students to write the addition fact 4 + 4 + 4 + 4 + 4 = 20 as a multiplication fact. Sixty-three percent of the longitudinal students answered this correctly, compared with 45% of the New Zealand and 63% of the international samples from TIMSS 1995.

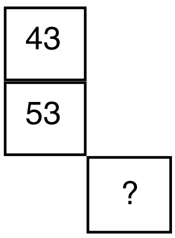


Figure 4. Hundreds chart (item 15)

Results for the Year 5 Students

The pattern of results for the year 5 students is very similar to that of the year 4 students. The year 5 students performed better on average than New Zealand students in TIMSS 1995 on 19 of the 24 questions and equally well on two questions. The three questions for which the percentage correct for longitudinal students was significantly lower than that for New Zealand students in TIMSS 1995 were questions 2, 5, and 19c.

Question 2, illustrated in Figure 5, was similar to the vertical form question found difficult by the year 4 students. Twenty-five percent of the longitudinal students identified the correct response, compared with 30% of the New Zealand students and 71% of the international sample in TIMSS 1995. It is confusing that the year 5 students performed relatively poorly on question 5 as this was the same hundreds chart problem (Figure 3) that the year 4 students performed well on. Fifty-six percent of the year 5 students answered this correctly, compared with 70% of the year 4 students and 73% of the New Zealand students in TIMSS 1995. Question 19c required the students to compare two rates (3 kilometres in 10 minutes with 1 kilometre in 3 minutes). Although significantly fewer of the longitudinal students (71%) answered this correctly, compared with the New Zealand students on TIMSS 1995 (75%), it was still one of the highest scoring items.

2. Subtract 6000
−2369

a 4369
b 3742
c 3631
d 3531

	Percentage
Longitudinal	25%
NZ 1995	30%
TIMSS 1995	71%

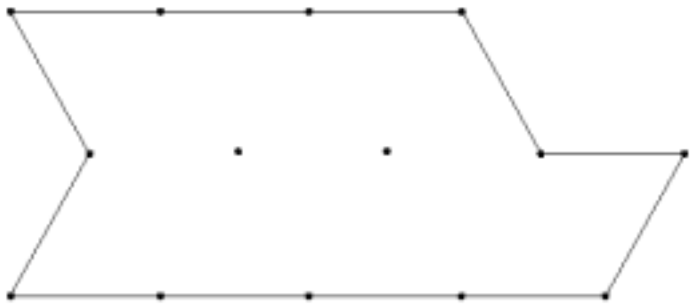
Figure 5. Item 2

Of the 19 questions on which the longitudinal students performed significantly better than the New Zealand students in TIMSS 1995, there were 10 on which the difference was greater than 10%. The two questions for which the difference was the greatest were question 17, the same folded paper question on which year 4 students also performed well (see Figure 3), and question 18, in which students were asked to identify the number of triangles required to fill a given shape (Figure 6). Eighty-four percent of the year 5 students answered the paper fold question correctly, compared with 64% of the New Zealand students and 59% of the international sample in TIMSS 1995. Sixty-three percent of the longitudinal students correctly answered the tiling problem, compared with 37% of the New Zealand students and 50% of the international sample in TIMSS 1995.

18. The triangle represents one tile in the shape of a triangle.

	Percentage
Longitudinal	63%
NZ 1995	37%
TIMSS 1995	50%

How many tiles will it take to cover the figure below?



Number of tiles: ________________________

Use the figure above to show how you worked out your answer.

Figure 6. Item 18

The next three greatest improvements were recorded for questions 9, 12, and 22, in all of which the longitudinal students outperformed New Zealand students in TIMSS 1995 by at least 15%. Two of these questions are directly linked to the number focus of the NDP. Fifty percent of the longitudinal students were able to correctly identify the pair of numbers in which the second number was 100 more than the first number (Figure 7). This was 17% more than the New Zealand students' performance on this item in TIMSS 1995. The longitudinal students also performed significantly better than the New Zealand cohort in TIMSS 1995 on item 22, which required the students to write the number that is 1000 more than 56 821. Fifty percent of the longitudinal students answered this correctly, compared to 32% of the New Zealand students in TIMSS 1995. It is interesting to note that for questions 9 and 22, whilst the longitudinal students performed considerably better than the New Zealand students in TIMSS 1995, they were within 2% of the international average.

9. In which pair of numbers is the second number 100 more than the first number?

a 199 and 209
b 4236 and 4246
c 9635 and 9735
d 51 863 and 52 863

	Percentage
Longitudinal	50%
NZ 1995	33%
TIMSS 1995	49%

Figure 7. Item 9

Results for the Year 8 Students

Although there was little difference in the overall performance of the year 8 longitudinal students and the New Zealand sample in TIMSS 1995, there were significant improvements on six items and significant declines on three. On the items in which the longitudinal students' performance was lower, one was a number item (7% lower), one involved the identification of an angle measure (8% lower), and one involved the rotation of a 3-D object (6% lower). The number item was a word problem that asked the students to find the difference between 61.60 and 59.72 metres (Figure 8). Fifty-nine percent of the longitudinal students answered this correctly, compared with 66% of the New Zealand and 67% of the international sample from TIMSS 1995.

1. In a discus-throwing competition, the winning throw was 61.60 m. The second place throw was 59.72 m. How much longer was the winning throw than the second place throw?

a. 1.18 m
b. 1.88 m
c. 1.98 m
d. 2.18 m

	Percentage
Longitudinal	59%
NZ 1995	66%
TIMSS 1995	67%

Figure 8. Item 1

Of the six questions on which the longitudinal students performed significantly better than the TIMSS students, there were four in which the difference was greater than 10%. Three of these items involved an understanding of fractions, so the improved performance of the longitudinal students is encouraging. Two of these items used fractions in relation to probability problems, while the third required the students to shade a fraction of a region (Figure 9). Fifty percent of the longitudinal students shaded the region correctly, compared with 35% of the New Zealand cohort in TIMSS 1995. The fourth item with a substantial improvement (13%) required students to read the temperature off a thermometer.

14. Shade in 5/8 of the unit squares in the grid.

	Percentage
Longitudinal	50%
NZ 1995	35%
TIMSS 1995	46%

Figure 9. Item 14

The longitudinal students performed similarly to the New Zealand TIMSS 1995 cohort on a total of 15 items, including nine number content items. One of these items was the same 4-digit subtraction posed to the year 5 students (see Figure 5). Sixty-nine percent of the longitudinal students identified the correct answer, the same percentage as for the New Zealand cohort in TIMSS 1995 and 17% lower than the international average. The longitudinal students also performed at a similar level on question 9, which required them to identify a list of equivalent fractions (Figure 10). Given their improved performance on the other fraction items, it is interesting that there was no comparative improvement on these items.

9. In which list of fractions are all of the fractions equivalent?

a. $\frac{3}{4}, \frac{6}{8}, \frac{12}{14}$

b. $\frac{3}{5}, \frac{5}{7}, \frac{9}{15}$

c. $\frac{3}{8}, \frac{6}{16}, \frac{12}{32}$

d. $\frac{5}{10}, \frac{10}{15}, \frac{1}{2}$

	Percentage
Longitudinal	55%
NZ 1995	53%
TIMSS 1995	62%

Figure 10. Item 9

There were a further two number questions for which the performance of the longitudinal students was as low as the New Zealand TIMSS 1995 cohort. One of the items involved proportional reasoning (Figure 11), and one was a word problem involving calculations of rate (Figure 12). Given that proportions and rates are not addressed until the higher stages of the Number Framework, many year 8 students would not be sufficiently advanced on the Number Framework to have experienced problems of these types.

12. Jan had a bag of marbles. She gave half of them to James and then a third of the marbles still in the bag to Pat. She then had 6 marbles left. How many marbles were in the bag to start with?
    a. 18
    b. 24
    c. 30
    d. 36

	Percentage
Longitudinal	55%
NZ 1995	353%
TIMSS 1995	62%

Figure 11. Item 12

13. A car has a fuel tank that holds 35 L of fuel. The car consumes 7.5 L of fuel for each 100 km driven. A trip of 250 km was started with a full tank of fuel. How much fuel remained in the tank at the end of the trip?
    e. 16.25 L
    f. 17.65 L
    g. 18.75 L
    h. 23.75 L

	Percentage
Longitudinal	38%
NZ 1995	36%
TIMSS 1995	35%

Figure 12. Item 13

The question with the poorest performance by longitudinal students was item 23, in which students were asked to order a set of numbers including decimals and fractions (Figure 13). Twenty-seven percent of longitudinal students answered correctly, similar to the performance of New Zealand students in 1995 but 11% lower than the international sample.

23. Which list shows the numbers from smallest to largest?

a. $0.345, 0.19, 0.8, \frac{1}{5}$

b. $0.19, \frac{1}{5}, 0.345, 0.8$

c. $0.8, 0.19, \frac{1}{5}, 0.345$

d. $\frac{1}{5}, 0.8, 0.345, 0.19$

	Percentage
Longitudinal	27%
NZ 1995	26%
TIMSS 1995	38%

Figure 13. Item 23

Concluding Comment

The performance of students in the longitudinal schools on the TIMSS items is encouraging. The year 5 longitudinal students performed on average 9% better than the 1995 New Zealand TIMSS cohort. Similarly, the year 4 longitudinal students performed 6% higher than the 1995 New Zealand TIMSS cohort. The year 8 longitudinal students' average overall test score was not significantly higher than that of the New Zealand students in TIMSS 1995, although they outperformed them on six questions and were lower on three. The comparatively low performance of year 8 students may be partly explained by fewer years of involvement in the NDP by both the students and their teachers. It may also be the result of requiring students to unlearn procedures and skills learnt prior to the NDP. Both of these possibilities require further investigation.

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http://timss.bc.edu/timss1995.html

Algebraic Thinking in the Numeracy Project: Year One of a Three-year Study

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The Numeracy Development Project (NDP) aims to help students develop the ability to split numbers in the most suitable way in order to carry out operations mentally. In doing this, they are using numbers as quasi-variables. This study will follow a cohort of students from years 7 or 8 through to 10 to note the ability of these students to move from using numbers as quasi-variables to using letters as variables. This paper reports on the first year of this study, which included a year 7 cohort. More year 7 students generalised from numerical to literal variables than other year groups. Year 9 students, who were being introduced to formal algebra, did the most poorly in generalising from numerical to literal variables.

Students who can think of numbers as made up of parts learn that these numbers can be broken up in a variety of ways. For example, 351 is made up of seven 50s and a 1, of 345 and 6, of 360 – 9, or of 350.7 + 0.3. When the students operate with these numbers, they can use whatever division into parts makes the operation easiest to perform. Students who apply these and other operational strategies to solve problems sensibly show an awareness of the relationship of the numbers involved in the problem. They see that numbers can be quasi-variables. In our view, they demonstrate that the strategy is generalisable and so are engaging in algebraic thinking. This connection between an awareness of generality in any mathematical domain and algebraic thinking is well supported by the views of Fujii (2003), Fujii and Stephens (2001), Kaput and Blanton (2001), Lee (2001), Mason (1996), and Steffe (2001). Fujii (2003) and Fujii and Stephens (2001) extend this link between number and algebraic thinking by arguing further that, within the strategies that students devise as above and in which generality of thinking is illustrated, the numbers themselves act as variables. They refer to these numbers as quasi-variables, which Fujii elaborates as:

> a number sentence or group of number sentences that indicate an underlying mathematical relationship which remains true whatever the numbers used are. (p. 59)

In 2003, we carried out a study to examine whether students in the NDP could generalise the use of quasi-variables, using whole numbers, more successfully than comparable students who were not in the project. They could (Irwin & Britt, 2005). In 2004, we examined whether or not different groups of students could demonstrate this algebraic thinking with decimals as well as with whole numbers. They could (Irwin & Britt, 2004). Again, those students who had been in the NDP were more successful than those who had not been in the NDP.

We are now investigating whether or not students who have had the NDP in intermediate school can demonstrate this algebraic understanding after they have moved to secondary school, where the usual manner of teaching algebra is different in that it does not build on students' understanding of using numbers as quasi-variables. We intend to follow individual students' responses to the same test items over three or four years to follow their patterns of achievement.

As 2004 was the first of our three-year study, we cannot make any statements about the longitudinal effect for students in the NDP. However, we can compare year groups. This also includes comparing students in year 9 who came from schools where the NDP was in use with students from schools that were not involved in the project.

Method

Participants

Students came from four intermediate schools and the secondary schools to which most of those students would go. All the intermediate schools had participated in the NDP. Two of the pairs of schools were in Wellington, and two of the pairs of schools were in Auckland. They were chosen because of the relatively close match of the decile ranking of the intermediate and the secondary schools that most students would attend. The ethnic composition of students at these schools is shown in Table 1.

Table 1
Characteristics of Schools in the Three-year Study of Algebraic Thinking

School Type	Decile ranking	Student roll	Asian	Māori	New Zealand European	Other	Pasifika	Date of ethnicity data
Intermediate	2	216	3%	30%	46%	-	21%	11.04
	3	528	8%	28%	28%	12%	24%	11.03
	5	628	-	17%	66%	15%	2%	6.02
	6	330	3%	15%	73%	5%	4%	5.03
Secondary	3	795	4%	29%	58%	-	9%	7.02
	4	1435	11%	23%	45%	-	21%	5.04
	5	1493	6%	14%	71%	6%	3%	11.04
	7*	1253	3%	18%	73%	2%	4%	8.02

*no tests given in 2004

For reasons that suited the schools, three intermediate schools gave the test to three or four selected classes, usually selected by the willingness of the teachers to participate. The fourth school gave the test to all of their classes. Since the classes in this school mixed year 7 and year 8 students, 98 year 7 students were assessed. One secondary school chose not to participate in 2004, but it is expected that they will in 2005 and 2006. Three secondary schools gave the test to their year 9 students, and two schools gave it to their year 10 students. Details of the sample are presented in Table 2.

Table 2
Number of Schools and Students Participating in Year 1 of the Three-year Study of Algebraic Thinking, with The Decile of the School

	Number of schools	Decile of the school	Number of students participating
7	1	2	98
8	4	2, 3, 5, 6	317
9	3	3, 4, 6	781
10	2	4, 6	549

Materials

The same test was given to all students. There were five similar items requiring compensation for the four arithmetic operations: addition, multiplication, subtraction, and division. Two exemplars were provided for each of these sections. For subtraction, students were to use *Kate's method*, illustrated with 37 – 18 being transformed into 39 – 20 and 71 – 43 being transformed into 68 – 40. The items for the students were similar to: 181 – 48, 16.1 – 5.2, $48 - d = 50 - \square$, $f - 9.9 = \square - 10$[3], and $a - b = \square - (b + c)$. The first item in each section involved whole numbers, the second item included decimal fractions, the third item involved whole numbers and one literal symbol, and the fourth item included one literal symbol and a decimal fraction. The fifth item required students to complete an algebraic identity with literal symbols only.

Method

The teachers administered the test towards the end of the term 4 in normal class time on a day that suited them. Students were instructed to read the section with the two exemplars carefully, to write the answer in the space below each question, and not to use a calculator. Graduate students, who had just completed their pre-service secondary mathematics teacher education programmes, marked the tests under the guidance of the authors. Responses were credited as correct if they followed the structure of the exemplars.

Results

Tables 3–5 and Figures 1–4 show the percentage of students in each year group that solved each item in the required manner. We were particularly interested in students' ability to use a technique on items that included letters that they had previously used only on items with numerals.

Table 3
Overall Percentage of Items Completed Accurately by Each Year Group

Year	Mean score	Modal score	Percentage of students with some literal items correct
7	4.95	1	46%
8	5.2	3	26%
9	3.9	0	18%
10	5.7	0	31%

We were surprised at the marked difference between year groups in the percentage of students with some literal items correct. These data are also shown in Figure 5.

A higher percentage of year 7 students were successful on these items than were other year groups, an issue that will be discussed later. The pattern of increasing difficulty within each page and operation that this year group demonstrated is the same for all year groups.

[3] See end note.

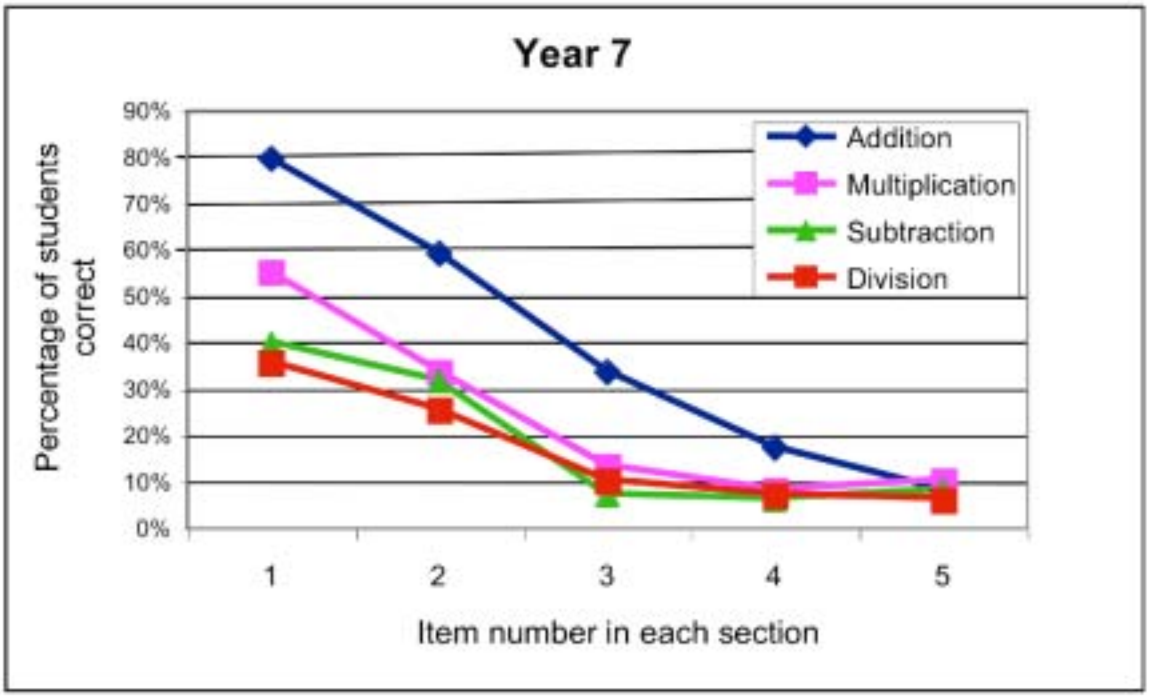

Figure 1. Results of year 7 students on the test of algebraic thinking (1 school, 98 students)

For year 7, unlike other year groups, division was the most difficult operation.

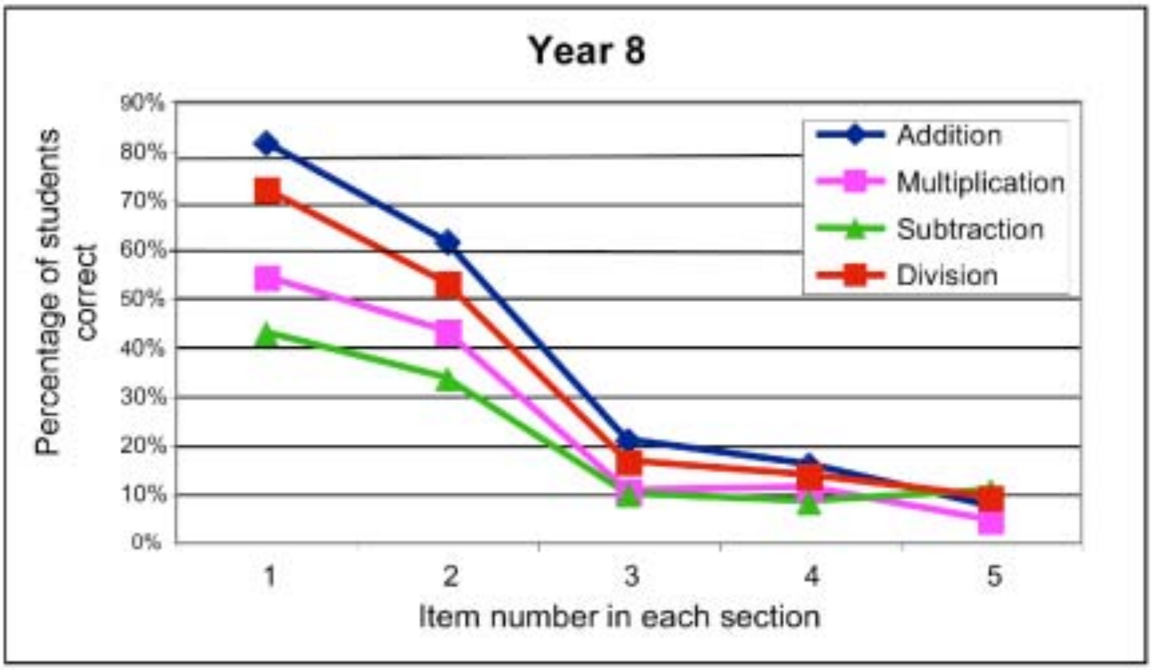

Figure 2. Results of year 8 students on the test of algebraic thinking (4 schools, 317 students)

The graph of the percentage of year 8 students who succeeded on these items showed a much sharper decline between item 2 and item 3 in addition than did the graph for year 7 students. It also showed subtraction to be the operation on which fewest students succeeded. Table 4 compares results for the four intermediate schools. As three of the schools chose which students to include, these data may not represent the whole school, except in the case of the decile 2 school.

Table 4.
Scores of Year 8 Students from Four Intermediate Schools

Decile ranking	Number of students	Mean score	Modal score	Percentage of students with some literal items correct
2	82	4.96	1	46%
3	66	4.68	0	37%
5	76	5.66	0	38%
6	93	5.61	3 and 6	19%

The differences between year 8 groups will not be important in future years of the study as each student will be compared against his or her own score in later years. However, there is some interest in the fact that the school with the lowest decile ranking, which did not select students, had a higher average score than any of the other schools in the percentage of students who were correct on some items that included letters. The students from this school appear to have done a better job at this than did the selected classes from other schools. Also, the school with the highest decile ranking had higher modal scores but few students who transferred this understanding of using numbers as quasi-variables to the use of letters as variables.

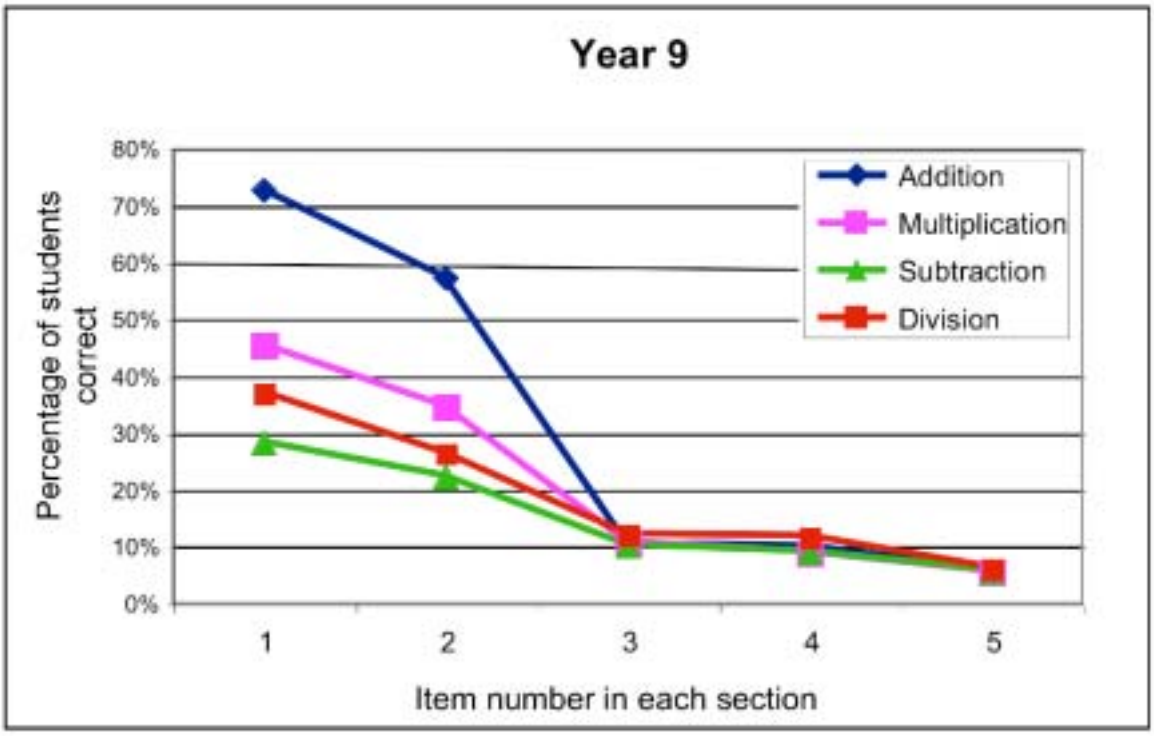

Figure 3. Results of year 9 students on the test of algebraic thinking (3 schools, 781 students)

Year 9 students did more poorly on average than did any other year group. We believe that this may relate to lack of continuity in the teaching of algebraic thinking between intermediate and secondary school.

We compared the scores of year 9 students who had attended intermediates that were using the NDP with those who had not, and there was no appreciable difference. Both groups had a mean score of 4.3 (NDP 4.31 and non-NDP 4.28). Both groups found addition to be the easiest operation and subtraction to be the most difficult. The main differences were on the first item in each section, where the students from NDP schools performed slightly better than those from non-NDP schools (see Table 5).

Table 5

Percentage of Year 9 Students Correct on the Initial Item for Addition, Multiplication, Subtraction and Division Who Had Attended NDP Intermediate Schools and Those Who Had Attended Non-NDP Schools

NDP participation	Number of students	Addition	Multiplication	Subtraction	Division
From NDP intermediates	402	75%	45%	29%	36%
From non-NDP intermediates	310	70%	45%	27%	35%

It is safe to assume that the small difference on the initial items in addition, subtraction, and division was due to some students having remembered doing items like these in the NDP. However, they failed to see the relationship of letter-based secondary school algebra to this use of quasi-variables.

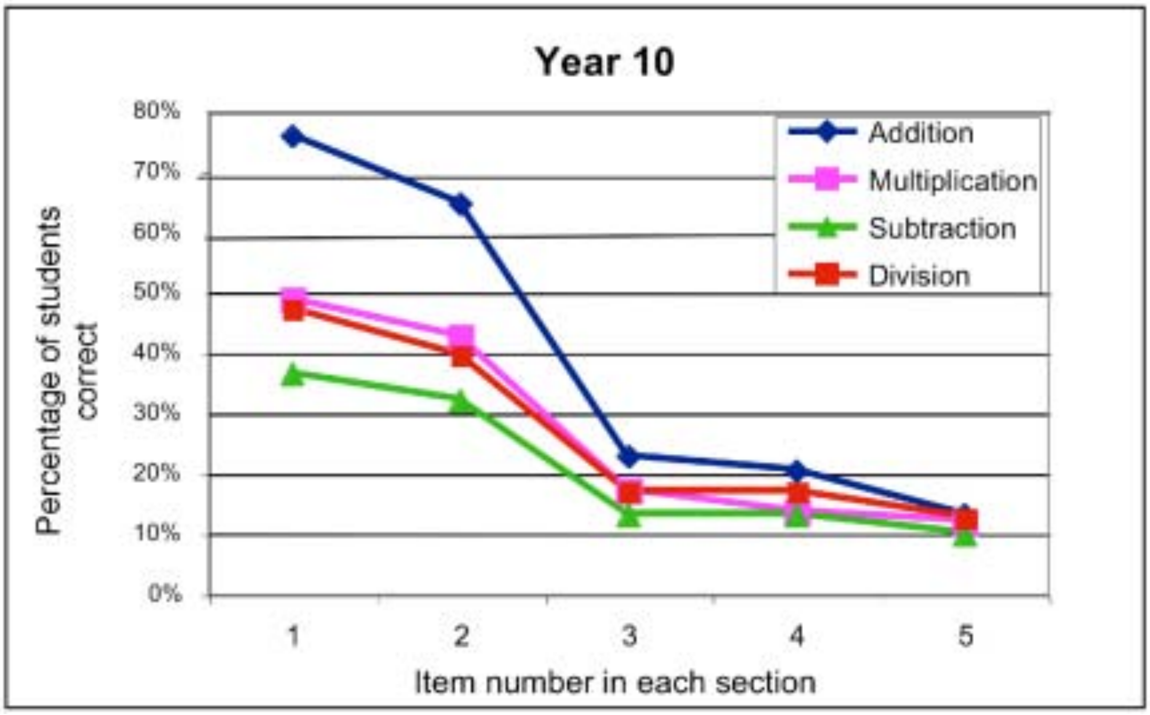

Figure 4. Results of year 10 students on the test of algebraic thinking (2 schools, 549 students)

Year 10 students did somewhat better than year 9 students on this assessment, but the pattern of achievement was similar to that for other year groups.

Our particular interest was in students' ability to generalise algebraic thinking from numerical items, something that they may have learned in the NDP, to expressing this algebraic thinking with letters as variables. Therefore we analysed the students who were successful on some numerical items and also on some literal items (see Figure 5). These figures also appear in Table 3.

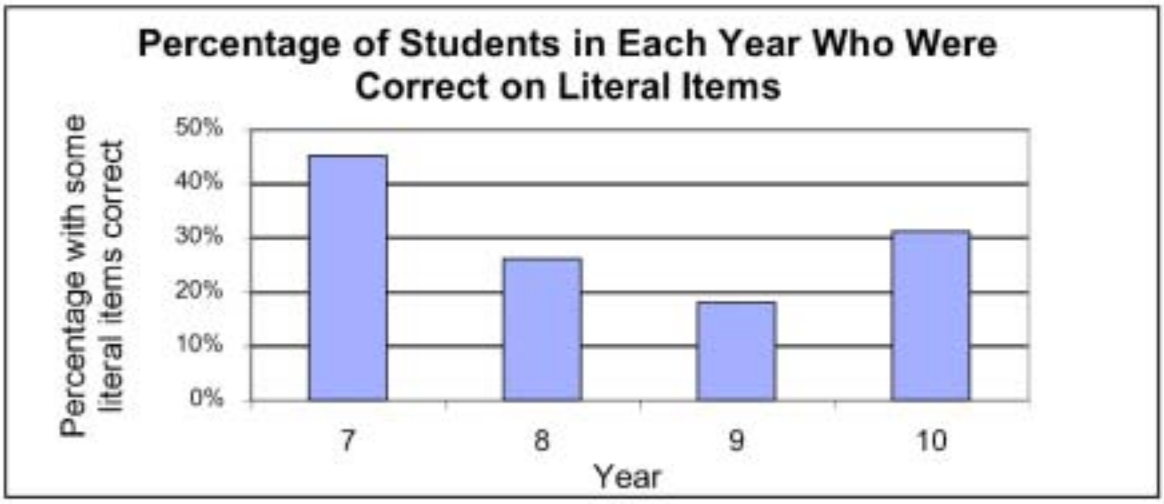

Figure 5. Percentage of students from all years who were correct on both numerical and literal items

This table shows most startlingly the superiority of the year 7 students in moving from the use of numerals as quasi-variables to using letters as variables. The year 9 students did most poorly in making this move. These year 9 students were being taught algebra in a conventional manner. This figure provides an excellent base line for determining if the new Secondary Numeracy Project will help students build new algebraic skills on their existing ones.

Discussion

In this discussion, we focus on two issues. One issue involves the students from all classes who appear to be in transition, that is, thinking algebraically on numerical items and beginning to transfer this algebraic thinking to literal items. The other issue we discuss involves possible reasons for the superiority of the year 7 group.

Students could complete up to 8 of 20 items correctly if they used algebraic thinking with numerals only. This did happen in the decile 6 intermediate school. In the decile 5 intermediate, only two students who scored 8 or less were correct on at least one literal item. However, in the decile 2 school, 19 students who scored 8 or less had some literal items correct. In the decile 3 school, 17 students who scored 8 or less had some literal items correct. These children were actively thinking in an algebraic manner. We do not know exactly what teaching occurred in their classes to encourage this thinking, but it would be worth exploring and fostering. Similarly, it would be useful to explore the teaching in schools where students could use numerals as quasi-variables but could not transfer this thinking to the use of letters. Most of the students who were accurate on some literal items scored at least a total of 5. We nominated students who scored a total from 5 to 15 as being transitional in the development of algebraic thinking. Those scoring from 16 to 20 were experts. There were 18 year 8 students scoring in this expert range.

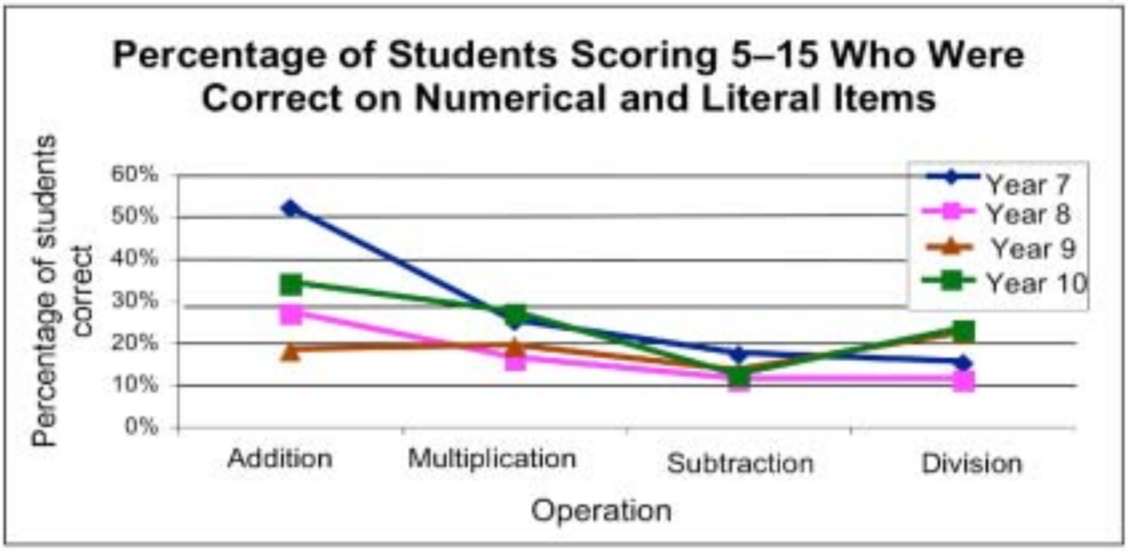

Figure 6. Percentage of students from all years who were correct on both numerical and literal items, given by operation

This figure demonstrates that addition was the easiest operation for transferring algebraic thinking from numbers to letters as variables in years 7, 8, and 10, but not for year 9, where success on literal items was generally low. It raises the question of whether or not year 8 students spend time on the use of quasi-variables in addition at the expense of the other operations. It again demonstrates the superiority of year 7 and year 10 students on this assessment of algebraic thinking.

These transitional students would appear to be those who need the opportunity to formulate the generalisation from operating with numbers as quasi-variables to expressing these operations with variables. Understanding variables, rather than unknowns, has long been difficult in secondary school algebra (Küchemann, 1981). These results suggest that students who score in this range are ready to express algebraic relationships as variables. The 19 experts found in this sample of year 8 students are already comfortable with this use of letters.

Why were the year 7 students so good? Again, we can only speculate. The fact that they were better than the year 8 students in their own school is another intriguing question. The facilitator for this school was asked for his views on this. He reports that when working with the teachers in this school he used the term “variable” from an early stage, representing it first with an empty square and then with a letter. He ran a workshop on developing algebraic thinking from number in the third term of that year and used examples from the second author in his workshops. Thus, the teachers may have introduced the term variable and the concept of moving from numbers to letters as variables in their classes. The year 7 students may have had a better understanding of this concept than the year 8 students in their own school because those older students, who were cross-grouped for mathematics, may have been introduced to algebra in a traditional manner that did not grow out of algebraic thinking with numbers as quasi-variables. We will watch this cohort in future years with interest and will be interested in the teaching that they receive as year 8 students.

Two of the secondary schools are involved in the Secondary Numeracy Project. This involvement will enable us to see if that project is able to avoid some of the drop in algebraic thinking noted in this year’s cohort of year 9 students. It will be an intriguing ongoing study.

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Note: These items were presented in the form given. We were aware of possible confusions if students tackled these items with an algebraic eye rather than from the structural point of view represented in the exemplars. In #3, we wanted students to try to figure out the adjustment involving the second number, in this case where d becomes $d + 2$. In #4 by contrast, the compensation adjustment involves the first number so that f becomes $f + 0.1$. We monitored this closely during extensive trialling in full classes from years 8, 9, and 10 in different schools and found that no difficulties arose as a result of the presentation. We also talked with the students involved in the trialling about their difficulties/misunderstandings, specifically in relation to these items. The crucial point here is that the tasks were related to generalising from the numerical examples, not recalling algebraic rules when dealing with a negative in front of an expression in parentheses. There was no evidence that students gave d – 2 in the empty box arising from an expansion of $-(d + 2)$. The graphical data for items 3, 4, and 5 in the full set of data across all three operations and year groups also shows, as reported, that these items were consistently poorly done and that #3 for subtraction is not idiosyncratic when compared with the others.

An Evaluation of Te Poutama Tau 2004

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Te Poutama Tau was initially developed in 2002 to support teachers in Māori-medium schools in the teaching of numeracy. It is based around the Number Framework developed for New Zealand schools. This paper analyses student data from Te Poutama Tau to examine students' progress on the Number Framework in 2004. Areas where students performed well and areas where progress has not been as positive are highlighted. The patterns of performance and progress of students involved in the 2004 project are compared with those of 2002 and 2003. The results of this study will inform the future implementation and foci of Te Poutama Tau in Māori-medium schools.

Background

For a number of years, there had been some discussion on the challenges teachers in Māori medium had in interpreting the learning outcomes of the Māori-medium mathematics curriculum statement. A possible solution to the problem was the development of a resource that would show more explicitly the content that students progress through. It was therefore of much interest to those working in Māori-medium education to observe the development of the Number Framework and its associated professional development programme, initially in the Count Me In Too and Early Numeracy projects (see Book 1: *The Number Framework*, 2003). However, concerns were raised in terms of the effectiveness of the professional development model, resources, and so on in relation to Māori-medium schools. Consequently, in 2002, a pilot numeracy project, Te Poutama Tau, was initiated as a component of a key government initiative aimed at raising student achievement by building teacher capability in the teaching and learning of numeracy.

Te Poutama Tau

Te Poutama Tau is based upon the Number Framework developed for New Zealand schools. The framework is divided into two key components, knowledge and strategy. The knowledge section describes the key items of knowledge that students need to learn. The strategy section describes the mental processes that students use to estimate answers and solve operational problems with numbers.

Teachers from 33 schools participated in Te Poutama Tau during 2004. Students were assessed individually at the beginning of the programme, using a diagnostic interview, and again at the end of the year.

The aim of this paper is to examine the following questions:

- What overall progress did the students make on the Number Framework in 2004?
- In which areas of the framework did the students perform well, and in which areas did they perform poorly, in 2004? Why is this so?
- How do the patterns of performance and progress of the students involved in the 2004 project compare with those for 2002 and 2003?
- What are the areas of the framework that they have performed well or poorly in over the three years? Why is this so?

Methodology and data analysis

The results for each Numeracy Development Project student, classroom, and school are entered on the national database (www.nzmaths.co.nz). The database shows the progress that students have made on the Number Framework between the initial and final diagnostic interviews. The time between the two interviews is about 20 weeks of teaching. Schools can access their own data on the national database to establish targets for planning and reporting purposes. Teachers can use the data to group students according to ability and choose activities that will support students in both strategy and knowledge development. The following summaries of the data were restricted to only those students who had both test and re-test results. In 2003, 1667 students completed both the initial and final diagnostic interviews, and in 2004, 1295 students participated.

Figure 1 shows that there was some difference in student numbers between the years 2003 and 2004, although there are insufficient numbers in the year levels 9 and 10 to make valid comparisons.

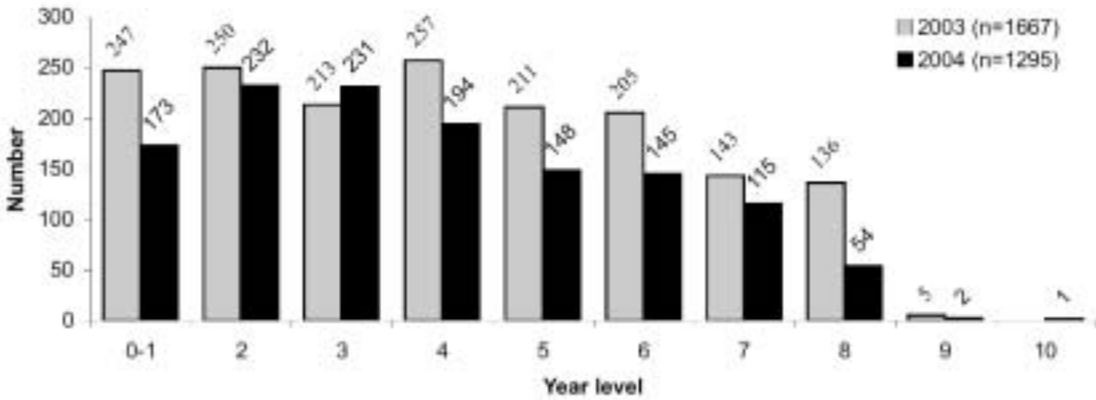


Figure 1. Distribution of Te Poutama Tau students across year levels

Overview of Student Progress 2004

Overall, the trend in student progress for 2004 was relatively consistent with the 2003 results (see Figure 6.1). However, there were minimal mean stage gains in numeral identification, multiplication, fractions, and proportion (see Figure 2). The complexity of the concept of proportion is closely linked to the building blocks of fractions (English & Halford, 1995) and multiplicative thinking (Behr, Harel, Post, & Lesh, 1992). Therefore, it is not surprising that students continue to struggle with proportion, considering the minimal stage gains in multiplication. Why there was minimal stage gain in numeral identification is not clear. Earlier NDP research (Irwin, 2003, and Young-Loveridge, 2004) suggested that such a result would be due to students entering at a higher level of the framework: the higher levels of the framework are more complex, and therefore progress is not as rapid as expected through the lower stages. However, a closer examination of the data shows that, in fact, the initial mean stage for 2004 Te Poutama Tau students was at stage 3, a fraction lower than 2003, in which the entry level was 3.2. (See Figure 6.2.) One explanation may well be that "big numbers", that is, those over 1000, are rarely used and heard in Māori outside of the classroom. As well, most big numbers are figured in the majority of resources, rather than spelt out as words. In the diagnostic interview, students are required to read and produce numbers before and after a number in a given range, in words.

There were significant stage gains made in decimals knowledge. This was recognised as an area of weakness in 2002 and 2003 and was the subject of major focus in both teacher and facilitator workshops during 2004. Understanding decimals is a multidimensional task. Students need to co-ordinate place value concepts with aspects of whole number and fraction knowledge. Making the transition to understanding decimals relies on having a thorough understanding of previous concepts, particularly base 10. Grouping and place value was one of the major focus areas for 2003. This has possibly had an influence on the positive mean stage gains for decimals in 2004.

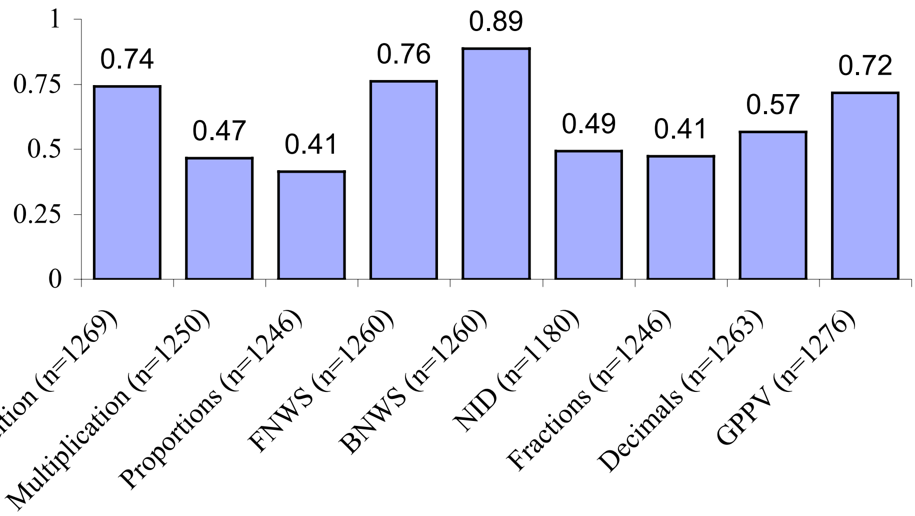


Figure 2. Mean stage gains across the Number Framework

Student Achievement and Year Level

The graphs below (Figure 3) show the variation in the mean gain for each aspect of the Number Framework across the year levels. There was no clear pattern common to all aspects of the Number Framework. Four of the aspects, addition, forward number word sequence (FNWS), backward number word sequence (BNWS), and numeral word identification decimals (NID), showed a "diminishing returns" pattern, where advancement was more difficult for students at successively higher year levels. The distribution for multiplication, proportions, and fractions resembled a more normal curve. Christensen (2004) explained that these aspects were "mainly connected with the higher stages of the framework" and so, when students at lower year levels were assessed, their progress would initially be limited due to the complex nature of these aspects. Students at year levels 3 or higher are quite possibly more able to work with these aspects and so were more able to advance. Generally, however, older students were at higher stages of the Number Framework (as was found and commented upon in the evaluation of Te Poutama Tau 2003 [Christensen, 2004]) and given that higher stages of the framework are larger and more complex, it would be more difficult for students at higher year levels to advance to the next stage of the Number Framework. The Number Framework aspects, decimals and grouping and place value (GPPV), while also related to higher stages of the framework, had a relatively flat distribution, with slightly smaller mean gains at higher year levels.

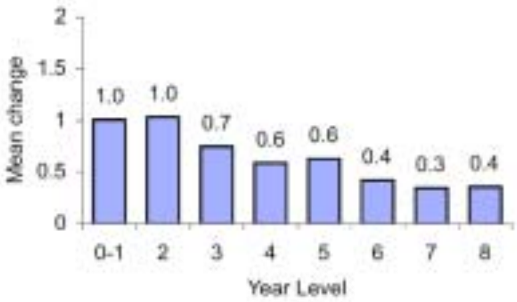


Figure 3.1. Mean stage gain for addition and subtraction and year level

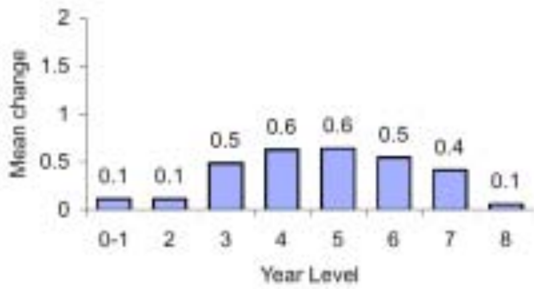


Figure 3.2. Mean stage gain for multiplication and division and year level

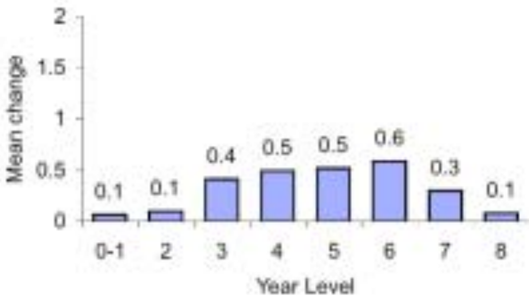


Figure 3.3. Mean stage gain for proportion

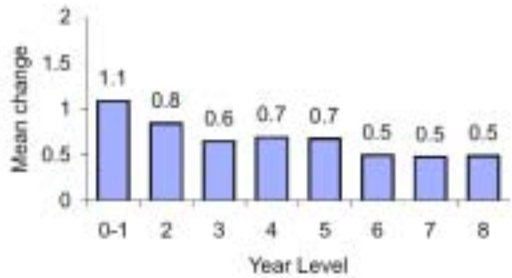


Figure 3.4. Mean stage gain for forward number word sequence and year level

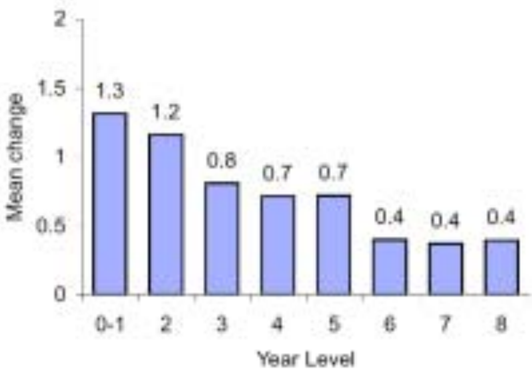


Figure 3.5. Mean stage gain for backward number word sequence and year level

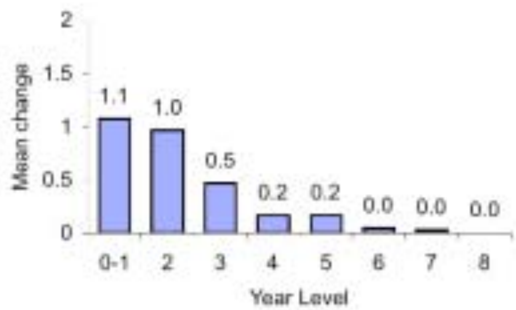


Figure 3.6. Mean stage gain for numeral identification and year level

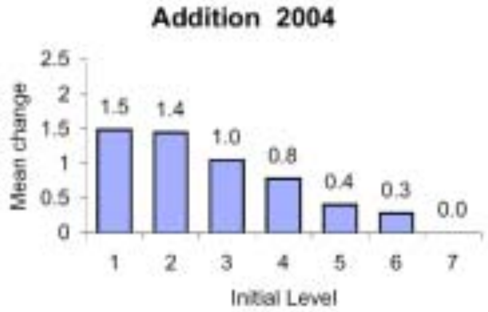
Figure 3.7. Mean stage gain for fractions and year level

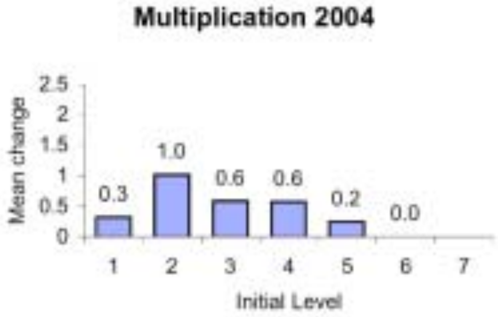
Figure 3.8. Mean stage gain for decimals and year level

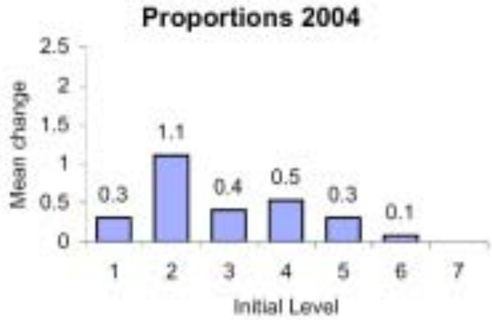
Figure 3.9. Mean stage gains for grouping and place value

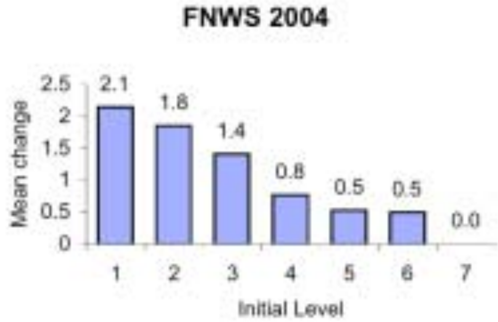
Student Achievement and Initial Stage Assessment

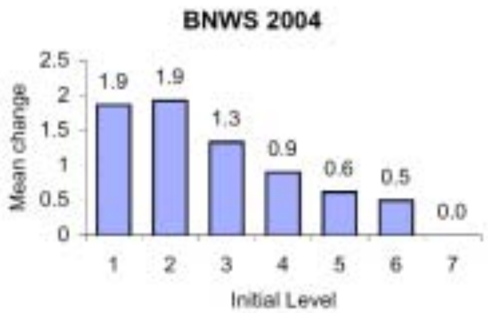
The following graphs (Figure 4) show how improvement in performance was related to the stage at which students were initially diagnosed. There was a consistent pattern across all nine aspects of the Number Framework, with improvements in performance being more difficult to achieve for those with higher initial scores. Students with an initial stage 2 level made the most gains, with this decreasing as the stages become higher. This can be attributed to higher stages of the Number Framework being larger and more complex, making it more difficult for students to advance to the next stage. However, Christensen (2004) made the point that this may also “indicate that teachers and facilitators were more effective at the lower levels” (p. 16).

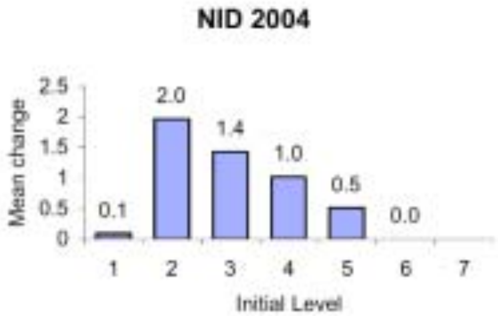

Addition 2004
Mean change
1.5 1.4 1.0 0.8 0.4 0.3 0.0
Initial Level

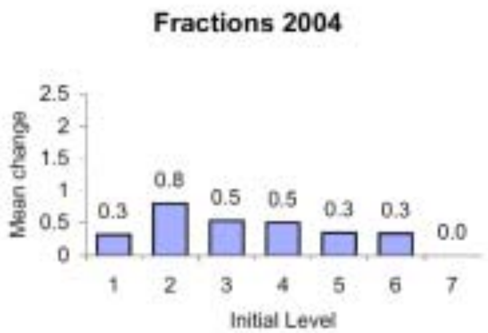

Multiplication 2004
Mean change
0.3 1.0 0.6 0.6 0.2 0.0
Initial Level

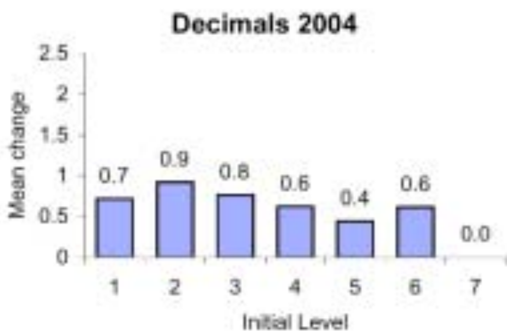

Proportions 2004
Mean change
0.3 1.1 0.4 0.5 0.3 0.1
Initial Level


FNWS 2004
Mean change
2.1 1.8 1.4 0.8 0.5 0.5 0.0
Initial Level


BNWS 2004
Mean change
1.9 1.9 1.3 0.9 0.6 0.5 0.0
Initial Level


NID 2004
Mean change
0.1 2.0 1.4 1.0 0.5 0.0
Initial Level


Fractions 2004
Mean change
0.3 0.8 0.5 0.5 0.3 0.3 0.0
Initial Level


Decimals 2004
Mean change
0.7 0.9 0.8 0.6 0.4 0.6 0.0
Initial Level

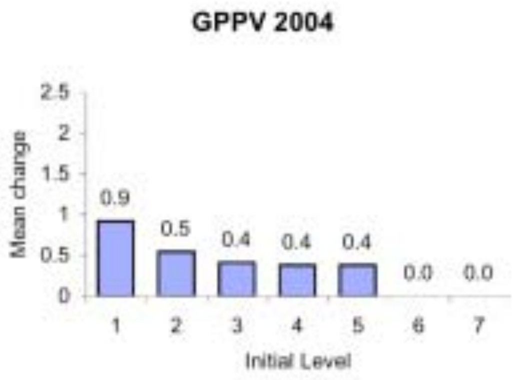


Figure 4. Student achievement and initial stage assessment

Student Achievement and Language Proficiency

There was little difference between 2003 and 2004 in how teachers rated te reo Māori proficiency of the students. The table below shows that 64% of the students were rated as either "proficient" or "very proficient", with 10% rated as "not very proficient" or having "poor proficiency".

Table 5.1
Language Proficiency of Students

	Language Proficiency				
%	Very proficient	Proficient	Reasonably proficient	Not very proficient	Poor proficiency
2004	13	51	26	8	2
2003	12	48	33	6	1

As in 2003, teachers were asked whether or not English was used during the diagnostic interview. The table below shows that for the great majority of interviews (90%), only Māori was spoken, with the remainder having little English spoken by the students during the interview and even less by the teacher.

Table 5.2
Language Used during Diagnostic Interviews

	Languages used				
%	Only Māori	Student used a little English	Student used quite a bit of English	Teacher and student used a little English	Teacher used a little English; student used quite a bit of English
2004	90	6	3	1	0.1

Longitudinal Patterns of Progress

This section examines patterns of performance over the three years of the implementation of Te Poutama Tau. 2002 was very much a developmental year, with considerable focus on the development of te reo Māori discourse, the supporting resources, the professional development models, and issues around the capacity of facilitators to support teachers. This is very much reflected in the 2002 results shown in Figure 6. As noted by Christensen (2003), the mean stage gains in grouping and place value, fractions, and decimals were disappointing. The results for fractions and decimals could be partly attributed to the fact that the majority of students in the 2002 project were years 1–4 and were being introduced to the lower stages of the Number Framework, where most of the focus is on whole numbers. The data showed that achievement in grouping and place value was a major concern (Christensen, 2003, p. 25). Consequently, this area was the subject of major focus for facilitators and teachers in 2003. The mean stage gains for grouping and place in 2003 in Figure 6.1 show considerable improvement as a result of the directed intervention.

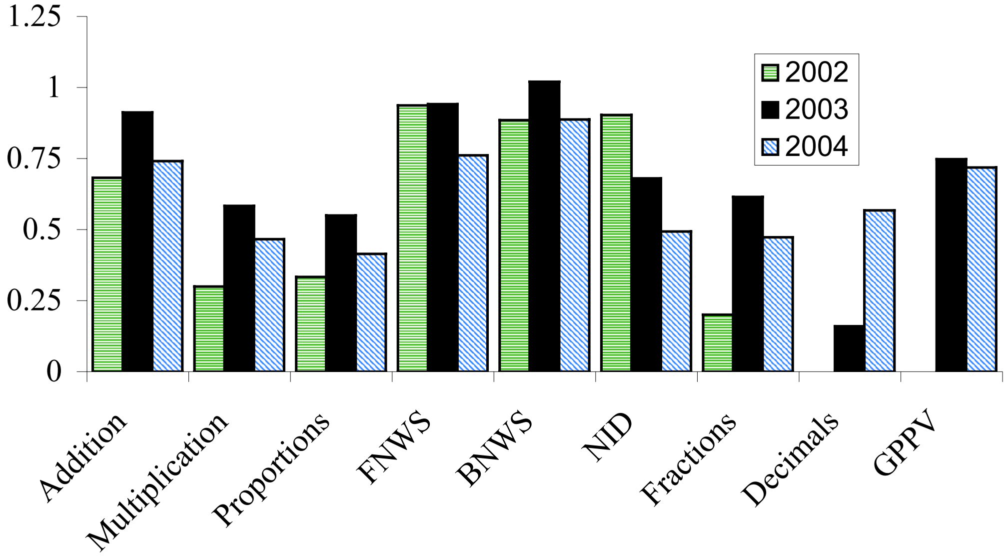


Figure 6.1. Longitudinal mean stage gains

In the table in Figure 6.2, two trends are immediately obvious: generally (with the exception of decimals), the mean improvement for 2004 was lower than for 2003, but the mean initial level for 2004 was higher than for 2003. This culminated in an improved mean final level result for 2004 when compared to 2003 for the Number Framework aspects of addition, FNWS, and BNWS, and in very large improvements made for decimals. There were small decrements in performance in the mean final assessment for multiplication, NID, and fractions, and a somewhat larger decrement for GPPV. There was a small decrease in performance for the proportions aspect of the Number Framework in 2004 when compared to 2003.

	Mean	2003 (n = 1667)			2004 (n = 1295)		
		Initial	Change	Final	Initial	Change	Final
Strategy	Addition	4.0	0.85	4.77	4.1	0.73	4.82
	Multiplication	2.1	0.57	2.60	2.1	0.45	2.52
	Proportions	1.9	0.54	2.42	2.1	0.40	2.41
Knowledge	FNWS	4.6	0.88	5.36	4.7	0.74	5.43
	BNWS	4.3	0.96	5.14	4.4	0.86	5.22
	NID	3.2	0.62	3.52	3.0	0.45	3.10
	Fractions	1.8	0.60	2.36	1.9	0.46	2.25
	Decimals	1.2	0.16	1.28	2.6	0.71	3.24
	GPPV	3.1	0.82	3.86	2.5	0.55	3.04

Figure 6.2. Comparison of mean gain for the nine strategy stages across years 2003 and 2004

Figure 6.3 shows how the average for the final result for all tests varies across year levels for 2003 and 2004. There was a small improvement in students' overall performance in the final tests in 2004 compared to 2003 across year level, apart from a very small dip for those in year 8. Results for years 9 and 10 should be disregarded due to the small sample size.

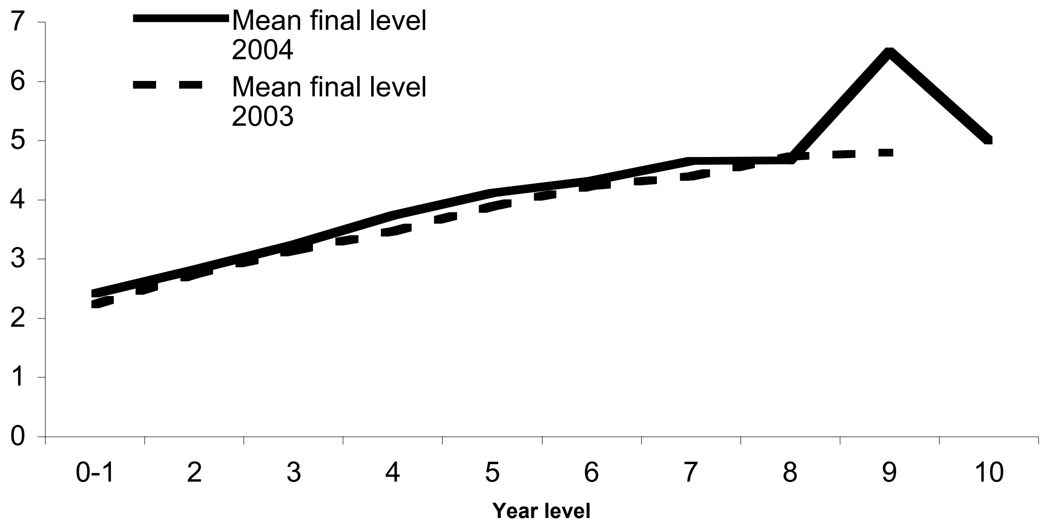


Figure 6.3. Comparison of average for final test across years 2003 and 2004

Recommendations

The following recommendations arise from the research that has been discussed in this report. A stronger emphasis for teachers' and numeracy facilitators' professional development in 2005 should be on:

- Providing more resources and activities to show numbers in word form and numbers up to and over a million. As well as assisting with numeral identification, this may well also help students to understand the part–whole concept. Syntactically, numbers in the word form in te reo Māori are written and said with their parts differentiated, that is, rua rau wha tekau mā whā (2 hundreds, 4 tens, and 4 ones).
- Maintaining an emphasis on grouping and place value. This concept underpins many of the mathematical concepts associated with numerical thinking.
- Developing multiplicative thinking (see Mulligan, 2002) and the appropriate Māori discourse.
- The higher stages of the framework – multiplication and division strategies, proportion and ratio, and knowledge of fractions.
- The relationship between te reo Māori and mathematical thinking. For example, which te reo Māori linguistic structures support or hinder students' ability to learn mathematics? How do students represent mathematical concepts linguistically?
- Improving the outcomes for those students who make little or no stage gains.

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A Snapshot of the Discourse Used in Mathematics where Students Are Mostly Pasifika (a Case Study in Two Classrooms)

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This report concentrates on the discourse that one teacher used with her students who were predominantly Pasifika. It shows that the teacher used questioning and response styles in her interviewing and class lessons that were consistent with the pedagogy of the Numeracy Development Project (NDP). There is some evidence that students used some characteristics of this discourse with one another when working on problems together. This discourse emphasises explaining one's thinking rather than merely providing correct answers. There is less evidence that the NDP places emphasis on the use of correct mathematical terms and on presenting complete evidence of the forms that guide thinking in more advanced mathematics. It is suggested that an emphasis on terms and logical explanations in mathematical discourse be added to the NDP to increase the likelihood of all students having an equal chance of success in mathematics.

The specific register of mathematics often receives less attention in New Zealand classrooms than it requires. The term "mathematics register" covers both the terms that are specific to mathematics and the ways of presenting mathematical arguments. Some studies of the mathematics register have concentrated on classroom discourse (e.g., Cobb & Bauersfeld, 1995; Khisty & Chval, 2002; Moschkovich, 2003; Pimm, 1987). Other researchers concentrate on specific terms (e.g., Cowan, 1991; Riley & Greeno, 1988) or linguistic forms (e.g., Presmeg, 1997). There is also a considerable amount of literature on mathematical discourse and learners of English as an additional language (e.g., Hofstetter, 2003; Moschkovich, 1999).

There are many interrelated issues that may affect the achievement of Pasifika students. In South Auckland, these may include family income, health, stability of housing and schooling, number of people in the household, and parental understanding of what is required to succeed in mathematics in New Zealand schools. Many of these issues are not under teachers' control. However, the language used in mathematics classrooms can be strongly influenced by teachers. A very striking report of a teacher's ability to improve the mathematical achievement of students in her classroom through the use of the accurate mathematics register is that reported by Khisty and Chval, 2002. The teacher in that report took a class of students with English as an additional language from being one or two years behind grade level in achievement to being one or two years above grade level. Students were reported to leave her class "smart in mathematics …" (p. 157). That report portrays how she modelled sophisticated mathematical terms and discourse and then encouraged her students to use this same language in their discussions. The teacher appeared to have a good knowledge of mathematics and its terminology, but language was the avenue through which she let her students gain this knowledge and ability to discuss mathematical objects and relationships.

Our study looked at two teachers in different schools who taught classes in which nearly all students were Pasifika. Both teachers had participated in NDP training in past years. They turned out to be markedly different in their use of language, both in assessing their students and in their classroom discussions. The teacher who is the focus of this report was picked up late in the year when the other teacher left her school. This was serendipitous as it provided a useful contrast. We concentrate here on the teacher picked up later in the year, who used language more effectively, with comments on the other teacher and her students for contrast. Because this focus teacher and her class were videotaped only in the final term, it is not possible to indicate changes in students' language during the year. This teacher has agreed to be observed over three terms of 2005, in which we hope to be able to observe development of students' mathematical discourse.

Method

Participants

The teacher who is the primary focus of this report is a New Zealand European with relatively few years of teaching experience. We will call her Ms Connor. She taught a class of 25 year 5 and 6 students, 21 of whom reported that they were of Tongan, Niuean, Cook Island, and/or Samoan descent. Three were Māori and one was Australian. Some of these students were born in New Zealand, and some had come to New Zealand within the past month. The school was classified as decile 1, as was the school of the comparison teacher and class. We will call the teacher used for comparison Ms Regal. She taught a year 4 class that had only one non-Pasifika or Māori child in it. Both teachers appeared to be popular with their students.

Method

The teachers were videotaped while giving four or five individual assessment interviews and while teaching at least one whole class. For both teachers, a period of whole-class teaching was videotaped, followed by videotaping of small groups of students carrying out assigned mathematical tasks. By chance, both teachers were teaching a unit from the Statistics strand. In both cases, the first author observed classes before videotaping and spoke informally with the students to allow them to become familiar with her and to get their consent to be videotaped.

Analysis

Digital videotapes were transferred to DVDs, transcribed, and then analysed. Intensive analysis was done on similar sections from each teacher and any patterns checked with the full transcripts to see whether or not they were representative. The interview that appeared to have the most input from the student was analysed for each class, and the patterns found in this interview were compared with other interviews to see if they were representative. The entire teacher-led portion of the lesson and selected student dialogues were analysed. These selected dialogues were also compared with other dialogues in each class to see if they were representative. Categories used for analysis included type of question, wait time as evidence of listening, language patterns and utterance type of both teachers and students, focus of the discourse, expected audience, relative number of words spoken by teachers and students, and mathematical vocabulary.

Findings

Discourse in interviews and in class

Classroom discourse has several components that can be distinguished. Although this study separates some of these components, this is for analysis only. Dialogue or conversation is an integrated whole, particularly wherein it involves expectations for each party's contribution.

From the data available, we can identify characteristics of the teacher's questioning, the length of time that she waited for students' answers, her expectations of the students as seen in her response to students' responses, and the focus of the dialogue, especially whether the focus was on answers or on the thinking process. From our data, we can examine these in both the Numeracy Project Assessment (NumPa) or Global Strategy Stage (GloSS) interviews that set the style for mathematical discourse, in the one class lesson transcribed for each teacher and in one conversation between students while working on a set of problems.

The teacher's questioning, wait time, and responses to students

The NumPa sets the model for how the teacher is to ask questions and the expectations for responses. In the knowledge section, these are closed questions that require one answer. For the strategy questions, they are relatively open questions that request a student to explain his or her thinking. Focus here is only on the strategy questioning.

Ms Connor's questioning was exactly as prescribed in the NumPa document, although she had memorised the script and presented it in a conversational tone. Her questioning made it clear that she was interested in how students thought rather than in particular answers.

She waited for long periods for students to answer. Several of these wait times were over 30 seconds and one was 48 seconds. An example was:

T: At the car factory they need 4 wheels to make each car. How many cars could they make with 72 wheels?
S: (after 41 seconds) Not sure.
T: Not sure. You don't want to just give it a try?
S: (after 48 seconds) Oh, I lost my count.
T: OK, do you want to tell me how you were working it out so far?
S: I was using my four times table and 4 wheels is one car, 8 wheels is 2 cars.
T: Working it out that way.

This passage also provides evidence for the teacher's expectations of the student. The student's responsibility was to think how to do the problem and to explain his thinking rather than just come up with an immediate and accurate answer. Pimm (1987) wrote of "allowing the students thinking time" and giving students "control of the spoken communication channel" (p. 51), both of which assume that teachers will wait for answers. Ms Connor's response indicated that she appreciated the way that the student was working out the answer and that his explanation of his thinking was adequate for her to understand the strategy used for this item. Another indication of the importance that this teacher placed on the student doing the thinking in the interviews was the ratio of words that she used in comparison to those used by the student. Examination of other interviews by Ms Connor showed this to be a typical pattern of questioning, listening, and responding. The ratio for the interview analysed for Ms Connor was 3:2. In comparison, the ratio of words used by Ms Regal to that of her students was 3:1 in the interview in which the student had said the most. Many of the student's words in that interview were the result of Ms Regal asking him to read the question. That teacher had very few periods of silence. In another of her interviews, the student spoke 6 words during 4 minutes and 5 seconds of interview while the teacher spoke 405 words. This interview would have a ratio of 68:1. It appeared that if Ms Regal thought that the student was not going to succeed, she reworded the question, presented materials to help the student work the problem out, and sometimes talked over the student in her eagerness to have the student succeed. This characteristic of teachers, to have their students succeed, preferably by telling them because the teacher knows the answer, has been called "teacher lust" (Maddern & Court, 1989). It is a characteristic that all teachers need to be aware of and control if they want their students to do the thinking.

A teacher who adopts the pedagogy of the NDP will have some of the same questioning and response techniques in her class teaching, although class teaching will also have some instances of instruction when appropriate. The students working in groups should also adopt some aspects of the same discourse, in that they should be interested in each other's thinking and ask for it to be explained when necessary. They also need to be able to evaluate their own answers. In the portion of the class that was led by Ms Connor, she uses a similar pattern of acknowledging but not immediately evaluating students' responses. She asks for other students' responses and then asks them to evaluate. The following transcript comes from the introduction to a probability lesson about playing cards in which students were asked "Can you tell me using 'likely', 'unlikely', and 'impossible' that she would pick a card that would be less than ten."

T What do you think, Chris?
S1 Unlikely.
T Unlikely. OK, what do you think?
S2 Likely.
S3 Likely.
S4 Likely.
S5 Likely.
T Is there any way we can prove this?

The balance of teacher and student talk in a classroom is a good index of whose job it is to do the thinking, as indicated in the quotation from Pimm (1987) given above. The ratio of teacher to student talk is usually much higher in the period in which the teacher is working with the whole class. For Ms Connor it was 5:1, and for Ms Regal it was 7:1. The pattern of classroom discourse is usually that of T, S1, T, S2, T, S3, and so on, and can be pictured as a star, with the teacher at the centre. This pattern is traditionally that of teacher's initiation, students' response, and teacher's evaluation (IRE) (see Cazden 2001). This pattern assumes that teachers are asking questions that they know the answers to and the students' task is to find the answer that the teacher has in mind. Frequently the teacher does the vast majority of the talking and presumably of the thinking.

While Ms Regal used this pattern for most of the whole-class session, Ms Connor rarely used an IRE pattern of discourse in the lesson analysed. The ratio of teacher's words to students' words in this instruction period was 5:1, with many of the teacher's words being ones that showed that she was listening, like "OK" or "yes". Her students asked questions of one another in the whole-class session, sometimes spontaneously and sometimes when prompted. She often revoiced the student's answers. This is also a technique evident in the lesson scripts of the NDP. Revoicing provides a second opportunity for students to hear a good model of speaking (Khisty and Chval, 2002). O'Connor & Michaels (cited in Forman, Larreamendy-Joerns, Stein, & Browns, 1998, p. 531) believe that it may also help students to "see themselves and each other as legitimate participants in the activity of making, analysing, and evaluating claims, hypotheses, and predictions". The teacher's discourse is the same regardless of students' ability. This high expectation of quality thinking means the students are not restricted by the "discourse of the 'less able'" as Brown, Eade, and Wilson (1999) phrase it. Figure 1 shows the discourse pattern of a section of this class lesson.

In this analysis, claims are answers without justification and warrants are explanations (Krummheuer, 1995).

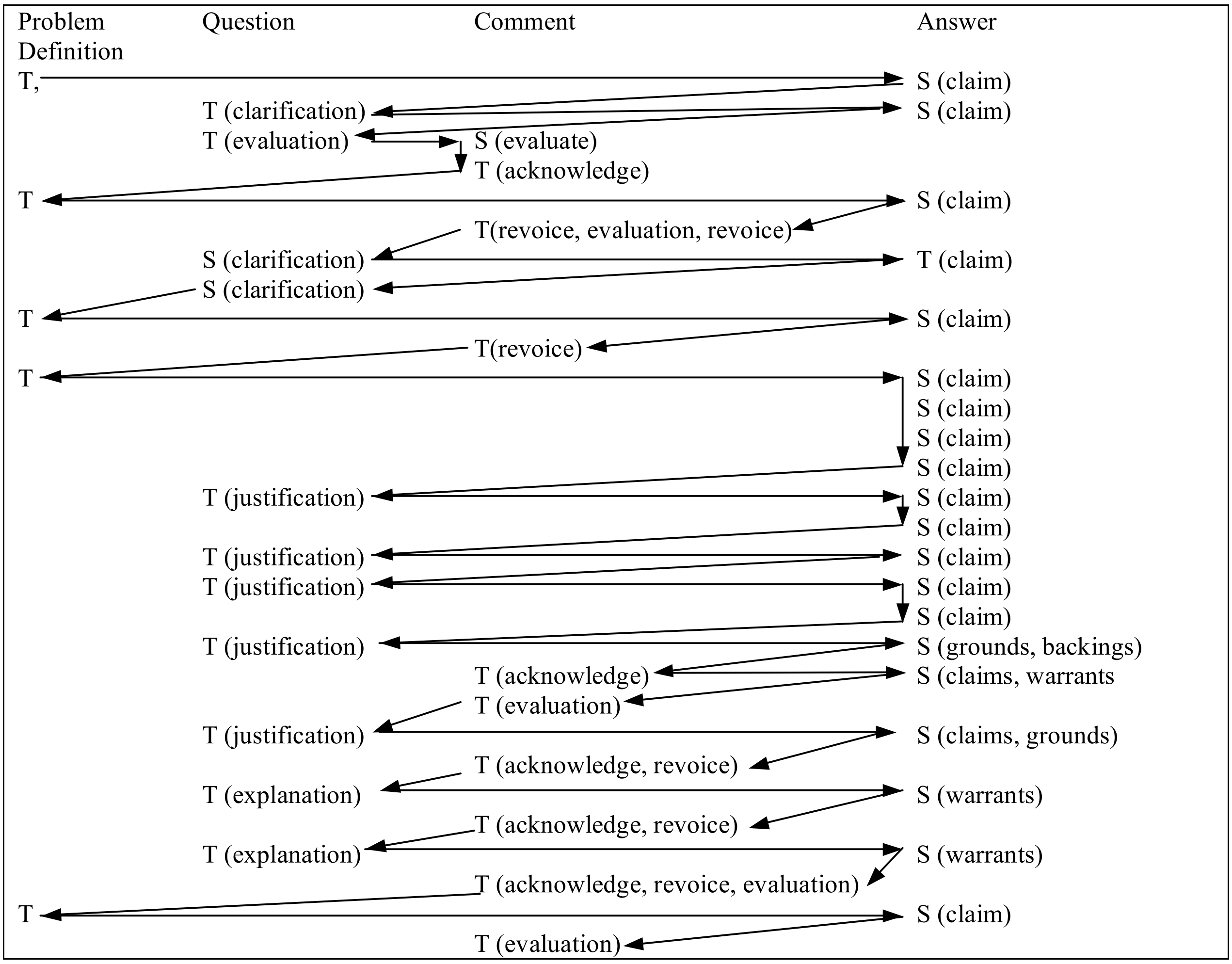

Figure 1. Discourse pattern of whole class portion of Ms Connor's lesson on probability

Note that in Figure 1 most of the teacher's follow-up questions are for justifications or warrants. She does give explanations when she believes they are needed. Only at the end of the extended dialogue does she evaluate the students' work.

Audience

The students use self-talk (talking to themselves as they work out a problem) in the whole-class section of the lesson. While the teacher does not think aloud herself in this lesson, she gives the students time for this, saying, "Give him time to think." One student mumbles his answer to himself and then faces the class and explains his thinking to them. Twice in the lesson the students face the class to explain their answers. The student takes the role of the teacher.

Student-to-student discourse

The real test of how well students understand this pattern of discourse is whether or not they use it among themselves. There are some examples of use of similar discourse in the student-to-student dialogues captured on tape. More examples would be needed to claim that students had adopted the discourse that emphasised how one got one's answers rather than what the answers are. However, analysis of one pair provides a framework for further analysis of such dialogues. The discussion analysed here involved a discussion of the mathematical equation for the probability of drawing a red card from a pack. This discussion included claims, warrants, challenges, counter claims, and agreements. The identifiers in this analysis combine the naming of arguments used by Krummheuer (1995) with those related to the relationship of members of the pair (see Irwin, 1997; Piaget 1965).

S2 I think I know how to work it out … And 10 (places a 10 on top of another 10). CLAIM
S1 So see, there's 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 22, 26, so there's 26 packs. WARRANT
S2 No, cause it's 2 … DISAGREEMENT
S1 26 cards. RESPONSE TO DISAGREEMENT, RETURN TO WARRANT
S2 No wait, 2 x 13 and that's 26. The 2 stands for there's two suits and there's 13 altogether. Thirteen in the red cards. See. CLAIM, WARRANT, REQUEST FOR AGREEMENT
S1 Yip and that equals 26 … 36. ACKNOWLEDGMENT, CLAIM, REVISED CLAIM
S2 26. DISAGREEMENT
S1 I mean 26. ACKNOWLEDGMENT
S2 No, we've got to make times 13 to make 13. DISAGREEMENT, CLAIM
S1 Thirteen is an odd number – you can't divide 13. (unintelligible) Only an even number like 12 or 18. CLAIM
S2 Yeah, but we … (PARTIAL – not scored)
S1 See, 13 x 2 equals 26 and 26 is an even number. REPEAT CLAIM
S2 Oh yeah… that is right. ACKNOWLEDGMENT
S1 And then the 2 the 13 in it [referring to two suits of 13 cards]. CLAIM
S2 And there's 26. CLAIM
S1 That two suits and 13, 13 … 13, 13 is … (partial CLAIM – not scored)
S2 No, it's 13 altogether in red. DISAGREEMENT
S1/S2 Thirteen altogether in red. (The students say it in unison as they write it.) AGREEMENT

This passage has some of the characteristics of joint problem solving between peers who want to understand each other but lacks other aspects (for example, see Irwin, 1997; Piaget, 1965). They are actively engaged in making sense of the mathematics of their task. S1 makes more claims than her partner (S1: 5 claims, S2: 3 claims); both students offer two warrants; S2 disagrees three times while S1 does not disagree but asks for agreement once. They have the characteristics of a pair that listen to and respect each other's contributions, but they are not equal in giving and asking for explanations. As is often the case with classroom tasks, these students appear to see their job as getting an answer, writing it, and moving on to the next question on the sheet.

In contrast, in Ms Regal's class, students were not seen to engage in discussion with one another in any attempt to explain or convince. Students at the same table worked on similar problems and talked to themselves but did not mimic the form of mathematical questioning and explaining modelled in the strategy questioning in the numeracy assessments.

Use of mathematical vocabulary

An emphasis of the Khisty and Chval (2002) paper is the fact that the teacher in that study, who was so successful in raising the achievement of her students who had English as an additional language, introduced mathematical terms early and expected her students to use these terms. Thus, very early in the year, she introduced the term "inverse" and told students of its importance in relating operations like multiplication and division. This is only one of many words that allow students to think about mathematical relationships in ways that are useful for further mathematics. There appears to be relatively little emphasis in NDP materials on the use of advanced mathematical language or the mathematics register. The advice given in *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) may compound difficulty with use of the mathematical register as it instructs teachers to use everyday language with their students before introducing mathematical language. Use of children's language versus the use of mathematical language is an issue in classrooms where students do not have a firm grasp of English.

Some of the students in this class did have difficulty with terms used in mathematics. The first author observed two classes in which students were struggling to sort out the meaning of "likely" and "unlikely". The teacher gave additional instruction on the meaning of these terms, and students helped each other with them. It was apparent from one of the interviews that the English word for half was unfamiliar or forgotten, but in this setting, the teacher appropriately acknowledged his confusion but did not instruct. There were occasions in class where she and the class used the colloquial terms "sum" for equation and "timesing" for multiplying. All of these language difficulties and uses are understandable. A teacher wants to be understood and it is easiest to use the common language of students, such as "timesing". The use of students' terms follows the rules of conversation (Sacks, 1992) if not those of the mathematics register. One recommendation that could be made on the basis of this brief analysis of teacher's and Pasifika students' language is that there is a place in the NDP for emphasis on using mathematical terms that will enable students to master more complex mathematics rather than relying on the students' everyday language.

Relation of language to success in numeracy

We have inadequate evidence to show a direct relationship between appropriate mathematical language and progress, but it is of interest that this class was relatively successful by the criteria of the NDP. At the start of the year, 14, or 51%, of the 27 year 5 and year 6 students in this class were assessed as part–whole thinkers. By the end of the year, 21 students, or 78%, were using part–whole thinking on at least one of the strategy scales. These figures are based on a student's top stage in any of the three strategy scales, as previous evidence shows that students perform differently to the challenges of these different scales (Irwin & Niederer, 2002). The closest comparison to this is the percentages for Pasifika students in the national sample of Pasifika students in 2004. That sample showed 54% of year 5 Pasifika students and 65% of year 6 Pasifika students to be working at the part–whole level in addition at the end of the year.

This move to part–whole thinking is seen as the crucial step for any students, and one in which Pasifika students lag behind other ethnic or linguistic groups. Ms Connor's class appears to be a successful class by numeracy criteria. The teacher's and students' language may be a contributing factor to this success.

Summary and Suggestions

The focus teacher, Ms Connor, uses many aspects of discourse in her teaching that are similar to the language of the NDP interviews in her teaching. Her emphasis is on the students' thinking and learning, not on telling students the answers. She displayed very little "teacher lust", the natural enemy of enquiry teaching. The students appear to respond by using similar discourse structures themselves. The contrast between her discourse and that of Ms Regal highlights the different way in which she promotes mathematical thinking in her classroom. Although the language reported here is only a small snapshot of the language in either class, it was analysed intensively. We believe that Ms Connor presents the NDP in the manner intended by its authors and the Ministry of Education.

Our one suggestion for improving the use of the NDP with Pasifika students would be to put more emphasis on the use of the mathematics register, both terms and the discourse of premise and consequence, rather than colloquial terms and conversational conventions. As said in the introduction, this is one of the factors that might affect the achievement of Pasifika students that teachers can influence. The paper by Khisty and Chval (2002) provides an example of the way in which the correct use of advanced mathematical terms helped the development of mathematical thinking in a group of students who had English as an additional language and had not been doing well at school.

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Effective teaching strategies for Māori students in an English-medium Numeracy Classroom

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Ma te tuakana ka totika te teina,
Ma te teina ka totika te tuakana.
From the older sibling, the younger one learns the right way to do things.
From the younger sibling, the older one learns to be tolerant.

This paper focuses on groups as a pedagogical strategy. The teacher sets up a "lead group" to mediate the learning of other groups. From a socio-cultural perspective, the teacher appropriates the contributions of a lead group to advance the understanding of other groups. Using a group to mediate other groups' learning appears to be an important difference to the ways in which groups have been typically used for instructional purposes in mathematics.

It is important that we understand critical aspects of teachers' actions that best support Māori student learning. Bishop, Berryman, Tiakiwai, and Richardson (2003) challenge us to examine dominant teacher-centred "monocultural pedagogies developed in New Zealand on the basis of unchallenged metaphors" (p. 23). They suggest "we need a pedagogy that is holistic, flexible and complex, which will allow children to present their multiplicities and complexities and their individual and collective diversities" (p. 13).

The category "Māori" may be unhelpful to teachers as it suggests Māori as a homogeneous rather than a diverse category (Bishop et al., 2003; McKinley, Stewart, & Richards, 2004). This leads teachers to consider simplistic pedagogical strategies such as the use of Māori contexts and "Māori learning styles" (McKinley et al., 2004). What follows is intended to unpack the common practice of group work in order to expose the complexities surrounding its definition as well as its use for instruction in mathematics.

Background

Recent studies investigating Māori in English-medium schools have focused on secondary schools (Bishop et al., 2003) and primary schools across all subjects (Tuuta et al., 2004). The literacy intervention in decile 1 schools, *Picking up the Pace* (Phillips, McNaughton, & MacDonald, 2001) reported on changes to pedagogy that resulted in improved outcomes for Māori and Pasifika students. An earlier study into Māori pedagogies was undertaken in 1996 (Hohepa, McNaughton, & Jenkins, 1996). While this is encouraging, the literature on effective teaching highlights the importance of subject and pedagogical content knowledge, suggesting that investigations into pedagogy should be discipline-specific (Alton-Lee, 2003). To date, there has been little investigation into classroom pedagogy in numeracy for Māori in English-medium schools.

Raised student achievement and improved teacher practice have been reported in evaluations of the Early and Advanced Numeracy Projects (Higgins, 2001, 2002, 2003, 2004; Thomas, Tagg, & Ward, 2003, 2004; Thomas & Ward, 2001, 2002). While this progress is irrespective of decile rankings and ethnicity, students in lower decile schools and Māori and Pasifika students have made lower gains in numeracy than students in other groups. This mirrors the trend across the school system for Māori and Pasifika students (Alton-Lee, 2003). There are, however, some schools with high Māori student populations and with low-decile rankings in which Māori students' achievement in numeracy is above that for Māori students as a whole group. More work is needed on investigating the reasons behind these results and identifying ways of reversing the overall trend in future years.

In the Numeracy Development Project (NDP), the Number Framework and its associated diagnostic interview provide a structure for teacher practice by enabling teachers to identify student knowledge and strategy stage. This information is then used to group students for instruction. Typical practice is for teachers to work specifically with a group of students to develop knowledge and strategies at their stage on the framework. The role of the teacher when working with a group has been informed by Fraivillig, Murphy, & Fuson's (1999) model, in which emphasis is given to eliciting, supporting, and extending concepts in response to students' actions and explanations.

To examine the practice of group work in more depth, it is necessary to identify the underlying assumptions shaped by different theoretical orientations. There appears to be confusion about group work in New Zealand teacher-support documents (Higgins, 1998). For instance, several theoretical frames underpin suggestions about group work. These include those reflecting a child-centred approach, those reflecting the co-operative learning movement, and those from a socio-cultural approach. Further, the purpose of group work ranges from its use as a management tool to its use as a tool for instruction.

A socio-cultural perspective is helpful to understanding group work. The inquiry-based approach to group work of the NDP is aligned with Bishop et al.'s (2003) description of discursive classrooms in which power-sharing interactions between teachers and students are promoted, where the culture of the child rather than the culture of the teacher are central to interactions, where learners are taught to critically reflect on their own learning, and where there is active engagement of students. The rest of the paper reports a case study in which groups were conceptualised as tools of instruction for Māori students in English-medium classrooms.

The Waka Metaphor for Classroom Group Work

Fundamental to the effective strategies in the case study classroom was the way in which the teacher conceptualised and used groups as an instructional tool. This appeared to be an important difference to the ways in which groups are typically used for instruction in numeracy classrooms. In this study, the teacher saw the class as a collective of interconnected groups rather than as a collection of separate instructional groups. The teacher described this as thinking about the class as a waka. She elaborated on her "waka" metaphor by explaining that it is about "groups within a group". She was thinking about all the groups simultaneously rather than focusing exclusively on the group with which she happened to be working at that time. From her comments below, she appeared to be concerned with the dynamic between the groups.

> It's regarding everybody … yeah. I mean the waka is the focus of its own. It's always within the group [the class], it's never you people are doing this and you guys just go away … you know … I do make them go away, but come back and see what these guys doing … how they're doing it. It's always involved … I mean everyone knows what everyone else is doing, that's … you know … that's a whānau thing.

In the following excerpt, she talked about "building up the lead group" as the starting point of the co-construction of understanding, that is, that "knowing" and understanding of learning can be "passed on".

> That's the one key strategy that I've learnt … is building up that lead group. That is the key to it … because they set the model for thinking … They become the leaders.

The key point is that this teacher was not just thinking of the expert as an individual but the expert as being a collective – the group as a whole.

> I could have one group … helping out another group, so you can expand it … not just a one-on-one … That's a concept that Māori students are comfortable with … They understand that it's my responsibility to help someone … Tuakana teina is because of age, but maths is because of knowledge and strategy. So it's a responsibility thing … They're quite happy to take it on and they like it because from the learner, the less able learner, I mean they get a new version of it.

In this class, the "lead group" became a tool of instruction not only for the teacher but potentially also for the students. One might think of this as a co-construction dynamic by which the class's understanding of mathematical ideas is shared.

> Even though they accumulate knowledge for themselves, it's never "I learnt this", it's "we learnt this", "we had a good day at maths" … So I regard the class in doing maths as being all on the same waka, but they don't have the same skills. Some of them are not paddlers, and I'll actually say this to the kids because they like to see analogies, they can see that it brings them all together and we all help each other to get to the end.

Working with the Groups as Tools of Instruction

The establishment of classroom norms is critical to using groups as tools of instruction. The next section of this paper identifies some of the ways in which this teacher set up the learning environment so that she could use the groups in this way. The teacher saw her role as having responsibility for the classroom learning environment.

> In the Māori sense I sort of see it as this little koru growing … part of a whole tree, some are further ahead and shelter the winds … but they are part of the whole … Very much a tool that I have is the kids themselves and … they're all growing with maths and they emerge in different ways. Some kids help others emerge better than I would, although I create the environment for that to happen … I create this common language for the kids.

It is important for the teacher to ensure that the students understand her strategy by being explicit about their role.

> I'll have them all in different groups, but I'll make sure that the group that knows less than the others … [say] "You guys listen because this is where you're going to."

It is important that the teacher defines the nature of the activity that will occur in the mathematics classroom. This includes the teacher conveying her expectations about the nature of the activity. A common theme running through this teacher's expectations was that "maths is a thinking activity". Time was also spent explaining to the students what maths is about, how school mathematics exercises "work".

The ways in which teachers set up group responsibility for the group's achievement was played out in a number of ways in this classroom. For instance, the teacher checked with all members of the group even when the teacher didn't expect a response from the student. The students also retained the right to "tell the teacher" what they needed. The ways in which teachers protect the mana of students when taking a risk of being wrong appears to be a pivotal point for many students. The teacher's actions are critical.

> I think it's about … being very precise with them, so that when they give … a possible answer, that that is the best that they can do and quite often they don't feel good. They know they're not right and so the things I do are just … "that's ok, we can start from there" and use what they know, so it's not putting them down … but it's still working towards that whole thing of being precise about what we're doing in maths … So they don't know and they don't know how to get there and I don't put them down for trying to get there.

There were lots of instances of emphasising thinking about maths.

> It's things like within a small group you see one kid that's … really lost, so what I often do is … obviously realising that I'm not getting through … hand the teacher role over to another kid and Māori kids love that, they love having that role, they love being able to work with other kids, [I say] "Can you show that person how to do it?" … So the kid shows the other kid how to do it and then I'll re-phrase what that "teacher kid" was saying … It's a whole language thing.

Using some students as experts needs to be carefully managed by the teacher to ensure that this is acceptable to the students as a group.

> Everyone knows who the experts are, that's a whānau thing … I guess what I do is not create barriers between all of them … but I don't put them too high on a pedestal … you know … they're just … they're learning, we're all on the same waka.

> It's usually friends that will team up … They're pretty picky … Someone won't teach that person because they don't get on … so it relies on their relationships.

> Someone might be like T having trouble with something … There were a number of people I could have got to help him … But I would choose someone that T has a good relationship with and is respectful of that person and understands that person. So … it's not just teaming him up with someone who's able, it's teaming him up with someone he gets on really well with, but won't be silly, but that person is responsible … I judge the responsibility, but I also judge the relationships between the kids and use that to help him.

> Yeah, because they don't like being wrong … If they know they're wrong they won't say anything, so they won't try, they're not very good risk takers … so you've got to break down that boundary of risk-taking, let them go for it … because … they like to do it well, but the whole learning thing … [requires you] take a risk and get it wrong.

The teacher needs to ensure that the mana of all students is protected in the classroom learning situation.

Discussion

From a socio-cultural perspective, the teacher, in order to advance the understanding of other group members, appropriates the contributions of those who are knowledgeable. These become the norms of group work, and these norms are mirrored in the student group in which peers interact with each other. These interactions are framed in the context of the classroom setting, not as a separate peer culture operating at odds to the adult culture. Alton-Lee (2003) described "the peer culture [as having] been developed by the teacher to support the learning of each member of the community … Caring and support is integrated into pedagogy and evident in the practices of teachers and students" (p. 89). Alton-Lee challenges us in "making student learning processes and understandings transparent" (p. 90).

The role of the teacher is in being responsive to student learning processes that are inclusive of Māori students.

> You have to think about it … So everything they say has to reflect what's going on in their head. The Numeracy Project asks us to change … Māori kids can do it but it's the style you do it in … It's the way you do it, you can't isolate them, you can't make them feel bad for not knowing … That's the trick … you know … keeping them on board.

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PEDAGOGY OF FACILITATION: HOW DO WE BEST HELP TEACHERS OF MATHEMATICS WITH NEW PRACTICES?

In this approach, the assumption is that the help or guidance is built into the teachers' handbook and their literal knowledge of the materials-based activities. Facilitators often, in respect to material designers, emphasise the materials' attraction for children and the need to follow the handbook sequencing.

By contrast, a second approach to facilitation is to emphasise guidance for teachers centering on structural components so that they gain skills needed for flexibility in classroom use.

Characteristic of new practice	Orientation of facilitator's actions A: Facilitation disposed towards **design adherence**	Orientation of facilitator's actions B: Facilitation disposed towards **contextual responsiveness**
Teacher's manual or handbook	Emphasis is given to adhering to the programme design and the handbook.	Emphasis is given to using structural elements to interpret the handbook.
Materials (activities)	Emphasis is given to engaging students actively with the materials.	Emphasis is given to teachers' understanding of the mathematical purposes and concepts underlying the materials.
Teaching method	Emphasis is given to the experiential effect of activities.	Emphasis is given to students' representations of their mathematical understandings.
Modelling new practice	Emphasis is given to students' "proper" use of the materials.	Emphasis is given to extending concepts in response to students' actions and explanations.

Design Adherence compared to Contextual Responsiveness

That's what we're doing, we're giving teachers a structure without giving them **"You will do page this or that"** … I'm in favour of resources … whatever you might be using, [but] it's which bit are you going to choose. (Emma, Facilitator, ANP 2001)

[We need] **more activities/resources** that could be just picked up and used. (Teacher, ANP 2001)

I have changed my style of teaching maths. … Just trying to bring out more language from them whereas **before it was a lot of bookwork.** (Teacher, ANP 2002)

Practical demonstrations are much easier **to follow than the manual**. (Teacher, ANP 2001)

The emphasis of the **design adherence orientation** is focused on procedural classroom practices. The expected procedures are usually unambiguously stated in the teachers' handbook. In essence, the activity is viewed as paramount.

It's about giving simple, understandable, credible, reasonable structures for teachers to use. (Roger, Facilitator, ANP 2003)

You are able to talk with the teacher … not just show what the activity is, but talk about underlying concepts because we still have a lot of teachers teaching an activity without any concept. (Emma, Facilitator, ANP 2001)

I now encourage children to share their strategies with the class – the focus in teaching maths has changed to "How did you find out the answer?" not, "What was the answer?" (Teacher, ANP 2001)

I've been watching Emma take lessons … Like they give her an answer, but she'll always come back and ask them that extra step … You then start realising what your own kids are capable of … We were stopping children because I think we were afraid that our own knowledge wouldn't go that far. (Teacher, ANP 2001)

In contrast, the emphasis of the **contextually, responsive orientation** is focused on students' strategies, meaningful activities, and multiple representations. In essence, the students' understanding and thoughtful investigation is paramount.

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Te Poutama Tau: A Case Study of Two Schools

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This paper reports on two schools that have made positive mean stage gains in the Number Framework. The Number Framework is a key component of Te Poutama Tau, which aims to lift teacher capability and raise student achievement in Māori-medium education. The results of these case studies may help to inform schools, numeracy facilitators, and policy initiatives in order to support the future implementation of Te Poutama Tau projects in Māori-medium and kura kaupapa Māori.

Introduction

Te Poutama Tau is a professional development programme focusing on numeracy for teachers in Māori-medium schools and classrooms. It is a component of a key government initiative aimed at raising student achievement by building teacher capability in the teaching and learning of numeracy. Te Poutama Tau is based on the Number Framework developed for New Zealand schools. The framework is divided into two key components, knowledge and strategy. The knowledge section describes the key items of knowledge that students need to learn. The strategy section describes the mental processes that students use to estimate answers and to solve operational problems with numbers. It is important that students make progress in both sections of the framework.

Students are assessed individually at the beginning of the programme, using a diagnostic interview, and again at the end of the year. The diagnostic interview is designed to provide teachers with quality information about the knowledge and mental strategies of their students and to assist in locating each student's position on the Number Framework. The results for each student, classroom, and school are entered on the national database. The database shows the progress that students have made on the Number Framework between the initial and final diagnostic interviews.

Te Poutama Tau Case Studies

This study examines two schools that participated in Te Poutama Tau in 2003 and that achieved positive mean stage gains on the Number Framework. Students involved in the programme were expected to make a mean stage gain of at least one stage. This was based on the calculations for the English-medium numeracy projects. (See Thomas & Ward, 2002, p. 13.)

Māori-medium mathematics education is still very much in its infancy, so it is important to identify key factors that promote student achievement. Studies in Māori-medium education (Hohepa 1993; Smith, 1999; Bishop & Glynn 1999; Bishop, Berryman, & Richardson, 2001) note the key role of culture and that effective teachers create caring relationships and structured, positive, and co-operative environments. Glynn, Berryman, & Glynn (2000) focus on the impact of home and school relationships on learning and achievement. Studies by Christensen, 2003 and 2004, note the issue of language fluency on achievement. Christensen (2004) argues that there is a strong link between students' proficiency levels in te reo Māori and their progress through the stages in the Number Framework. He notes that there is significant correlation between language proficiency and performance in the diagnostic interview (Christensen 2003, p. 27). However, considerable research is still required to assist in improving student achievement.

Methodology

For the purposes of this research, schools that participated in Te Poutama Tau in 2003 were selected on the basis of the results of their students' performance in the diagnostic interview. This was done by ranking schools that entered initial and final data in the national database in terms of their mean stage gain overall.

There was a cluster of five schools and/or classrooms that achieved similar results. Schools who participated in Te Poutama Tau did so either as whole schools (in general, these were the kura kaupapa Māori) or as Māori-medium units (classes in English-medium schools). It was decided to concentrate on only two schools. This was based first on manageability of the data and the process. If there were more schools, there would be less time given to investigating the results of each one. Secondly, two schools provided the opportunity to identify the common factors that may have contributed to their positive mean stage gain. It was decided to examine two of the schools, kura A and kura E, using the bigger data samples to minimise the chance of "one-off" spikes and dips. The advice of the Te Poutama Tau numeracy advisers for the two schools was also sought, in order to gain their perspectives on the implementation of Te Poutama Tau in these particular schools. In both cases, the numeracy facilitators confirmed the schools' positive attitudes and commitment to the programme.

Once the two schools were selected, the principals were sent a letter explaining the nature of the research contract. This was followed up by a phone call to establish the identity of the researcher, to establish positive relationships, and to confirm the schools' willingness to participate. The principals and teachers from the case study schools who were involved in the 2003 Te Poutama Tau project were sent questionnaires (see Appendix E), followed by an interview. The interview questions focused on the following areas:

- The socio-cultural and demographic features of the school and its community
- Relationships between the school and its local community, including links to the local iwi and hapū
- The experience and qualifications of management and teachers, particularly in relation to pāngarau
- Attitudes and involvement of school management and teachers in Te Poutama Tau
- The effect of the Te Poutama Tau programme on classroom practice
- Teacher reflections on the implementation of Te Poutama Tau.

Each teacher was interviewed for 15 to 20 minutes. This was followed by an interview with the principal. The interview responses and the reflections of the staff involved in Te Poutama Tau are discussed in the first case study summary.

As a key component of the research methodology, consideration was given to relevant approaches to working with Māori-medium and kura kaupapa Māori. Principals were initially sent a letter informing them of the rationale and aims of the project. This was followed by a phone call to organise a visit to the school and to establish the identity of the researcher. Māori-medium schools tend to be resistant to research projects that do not benefit the school directly or Māori-medium education in general. However, the principals were very positive about being involved in the project and recognised the positive outcomes for students in relation to the Number Framework.

Most of the interaction with the interviewees was carried out in the medium of Māori to validate and to establish the commitment of the researcher to the importance of te reo Māori to the kura. This was followed up by a personal visit (kanohi ki te kanohi) to discuss issues relating to mutual benefits of the project, to outline the research process, and to establish cultural legitimisation. Before the interviews could begin, the researcher had to be formally welcomed onto the school grounds. In one of the cases, this involved whole-school participation in a formal pōhiri and in the other case, the researcher had been welcomed on in a formal pōhiri on a previous occasion at the launch of a Māori-medium mathematics dictionary. This process recognises the mana (power) and the turangawaewae (identity) of the school and the community.

Results

Case Study 1: Kura A

Kura A is a rural, full primary, decile 2 school with 245 students, of whom about 94% are Māori. For teaching and learning, the students were fairly evenly split between the English- and Māori-medium units. However, this study focused entirely on the Māori-medium component of the school, Te Whānau Reo Māori. Despite the dual medium of instruction of the school, the school identifies itself very closely with the local iwi and hapū. High numbers of children have links to the local iwi and a high percentage are also bussed each day from the nearby city to the school. Many of the parents attended the school as students themselves and feel a need for their children to be immersed in the reo and traditions of their own iwi. In some cases, children live with grandparents who still reside in the local area so that they can go to this particular school. The concept of a strong and vibrant iwi identity flows strongly throughout the culture of the school. The school therefore plays a vital role in the maintenance of tribal identity and is the heart of its community.

A few of the parents work in professional fields, but the majority of parents/caregivers are in the low socio-economic category. Some of the students who reside with their grandparents do so to enable them to attend this school, but others are a "mokopuna whangai"; that is, they are being formally brought up by their grandparents, a relationship not uncommon in rural Māori communities. Most of the Māori students have regular contact with Māori-speaking whānau at home or in their whānau whānui or marae experiences. This intergenerational language flow is a vital component in the development of Māori language proficiency (Chrisp, 1998) and the maintenance of te reo Māori.

The school actively promoted community involvement in the project, particularly in sharing students' progress. A significant number of whānau attended the introductory hui to Te Poutama Tau and were fully supportive of the initiative. The Te Poutama Tau facilitators had a significant role in this process. The Board and parents were regularly informed of student progress on the Number Framework. Parents/caregivers were invited to participate in the development of aspects of the teaching and learning programme, particularly in the area of te reo and tikanga. The principal and other senior staff visited the local contributing kōhanga to discuss with staff and parents topics such as classroom routines. The principal felt that this strategy assisted in preparing kōhanga graduates for entry into primary school. Staff at the local kōhanga, who were often parents or whānau of students from the school, were also introduced to the Number Framework. The principal felt this had positive outcomes, with a number of kōhanga graduates entering the kura beyond the emergent stage of the Number Framework.

School Leadership

The principal is a Māori woman who has had 26 years in the teaching profession and 11 years in this particular school. She has a diploma of teaching and a diploma in bilingual education and is close to completing her Bachelor of Teaching degree. She is aware of the need to keep up to date with the range of initiatives designed to raise the achievement of students. The school also had been involved in a number of previous professional development contracts, so the principal and the senior teacher of the Te Wh nau Reo M ori syndicate were thus well experienced in managing professional development initiatives. The senior staff argued that the success of the initiatives was in part due to collaborative leadership and a common commitment to maintaining currency in national initiatives, particularly those that raised student achievement. As a consequence, the school was often called upon to trial local and national educational initiatives, including the pilot Te Poutama Tau project in 2002. The principal felt it was critical for the success of the programme that she be directly involved in the programme by attending professional development and progress meetings with the numeracy facilitators and by providing release time and financial support for staff involved, particularly the lead teachers. In collaboration with the staff, clear goals and expectations were developed. The programme was continuously monitored by the Te Poutama Tau facilitators. The principal felt strongly that the role of the senior teacher of Te Wh nau Reo M ori was essential to the success of the implementation of Te Poutama Tau in the school.

Teachers and Classrooms

As a group, teachers in kura A were also considerably experienced in teaching at the primary school level. Levels of experience ranged from 7 years' to 26 years' teaching, with the majority over 10 years. All the teachers involved in the project had taught for a number of years in the school, thus providing a relatively stable workforce. The principal noted that her staff created structured and positive learning environments, with excellent classroom management and routines. She believed that the focus on cooperative learning fitted very well with the teaching and learning philosophies of Te Poutama Tau.

All the teachers interviewed identified pāngarau as one of their favourite subjects, along with either literacy or te reo Māori, although a few admitted this was not always so. This response may have been due, in part, to the belief that this was the response the interviewer sought.

Class sizes ranged from 15 to 30 students across a number of age levels. Some of the teachers rated the language fluency of their students as the majority of their students being fluent, while other teachers rated their students as being "somewhat fluent". There was also an expectation that teachers would rate the Māori language proficiency of their students for Te Poutama Tau. This rating may well be influenced by each teacher's own level of fluency.

All teachers in kura A rated their students' attitudes to mathematics as now being very positive, with some seeing major shifts in attitudes during the implementation of Te Poutama Tau. Many felt that the effective strategies, particularly the classroom organisation and teaching pedagogy, they had developed in Te Poutama Tau could be transferred to other learning contexts.

Implementation of Te Poutama Tau

One of the key strategies of both the principal and the lead teacher of Te Poutama Tau was to keep the board and parents well informed of progress and targets for future development. The Poutama Tau facilitators attached to the school supported this process by also attending the meetings.

The principal, the staff, and the Te Poutama Tau facilitators interviewed felt there was total school commitment to the project. The principal worked in collaboration with the lead teacher, who played a critical role in implementation, in the setting of clear goals and expectations, and in establishing an appropriate time commitment for successful outcomes. Teachers did acknowledge some of the struggles, particularly understanding the content of the framework, the time needed to test students, and the linking of their classroom programmes to the outcomes of the diagnostic interview.

All the participants felt that they had made significant shifts themselves in a number of key areas in the teaching and learning of numeracy. Some of the shifts were in their own teaching pedagogy, while other shifts were in their attitudes and beliefs about how children learn numeracy. All the teachers felt their attitude towards the teaching of pāngarau was much more positive, and the outcomes were positive for students. This is consistent with previous Te Poutama Tau research (Christensen, 2004) in which facilitators were unanimous about the potential benefits of the programme in lifting teacher professional capability and student achievement.

Case Study 2: Kura E

Kura E is a relatively newly established urban primary school located on the outskirts of a medium-sized city. It is classified as a decile 1 school with a roll of 245 students who learn through the medium of Māori, with the overwhelming majority being of Māori descent. All those interviewed felt there was strong whānau involvement in school. However, the principal felt that the links to local iwi and hapū were not as strong as they could be. There had been significant urban migration during the 1950s and 1960s in the area, and, consequently, the local iwi were sometimes swamped by the infusion of other iwi and hapū. Therefore, the kura E population was still in the process of establishing its identity and relationships with the local hapū. This is not unusual with Māori-medium and kura kaupapa Māori located in cities where there has been significant urban migration.

As a consequence of urban migration, there was also a considerable Māori language shift in the migrating Māori community (Benton, 1981). "Language shift" refers to the change from one language to another as the primary language (Crawford, 1996). Many of the school's local community members migrated from the outlying rural Māori communities to the city, where economic opportunities for employment and commerce tended to be open only to those who are fully proficient in the dominant language, English. Consequently, there has been a decrease in the number of Māori speakers and limited opportunities for the language to be spoken.

This school was created in resistance to the dominant culture's disregard for the language and cultural aspirations of Māori in the area and as a means to revitalise the language and to establish a school based on the centrality of tikanga.

School Leadership

The principal is a Māori woman who has been teaching in primary schools for 36 years. She spent 18 years of those years as a principal in a variety of schools, including 5 years in a kura kaupapa Māori. She is passionate about the teaching of mathematics and has maintained a keen interest in curriculum developments in mathematics education over a number of years. She has also been significantly involved in the implementation of Te Poutama Tau in her kura and has attended all the numeracy workshops organised by the local Te Poutama Tau and numeracy facilitators. She felt that her active involvement in the professional development workshops assisted her greatly in the successful implementation of the project in her school.

The principal believes that her school has made significant shifts in the teaching and learning of numeracy. She believes this was due in part to the involvement of several beginning teachers who tended to be more receptive to new ideas and approaches than some of the more experienced teachers.

The principal felt that the involvement in the Numeracy Development Project has had a positive effect on other areas of pāngarau. Students were more motivated to learn, with the majority having developed positive attitudes to mathematics.

She used a number of techniques to closely monitor the progress of Te Poutama Tau and played a key role in the data analysis. Some of the techniques used included regular meetings with the lead teacher and numeracy facilitator, regular reports from teachers and syndicates, and the setting of targets with the teachers. When individual teachers required additional support, she either provided direct support herself or organised support from the lead teacher of Te Poutama Tau in the school.

Teachers and Classrooms

Teaching experience in primary/kura kaupapa ranged from year 2 to 15 years. However, the majority had taught for 2 to 3 years and a number were teaching that particular age group for the first time. The classes ranged from year 1–8, with the class sizes mainly around 15–20. The majority of teachers had a Diploma of Teaching, with no specific qualification in mathematics education. There was no overwhelming preference for any one curriculum area that teachers preferred to teach, although pāngarau did feature a few times.

Teachers felt that the majority of students were reasonably fluent in te reo Māori, with a level of proficiency that allowed them to interact in the medium of Māori. They noted that many of the students had developed more positive attitudes to pāngarau in general, and for many teachers, this was one of the most positive aspects of the programme.

A number of the teachers admitted that prior to the commencement of the programme, they had negative feelings towards pāngarau, but their involvement considerably changed their attitudes. The Number Framework enabled them to see progression through number much more effectively. The diagnostic tests and follow-up snap tests allowed teachers to clearly identify the stages at which the students were achieving. The structure and nature of the programme enabled them to see the content more explicitly in comparison to the pāngarau curriculum statement. Some saw the marautanga pāngarau as not being very “user friendly”, but the hands-on nature of the programme appealed to many of the teachers. The teachers who were interviewed all felt they had been well supported by the school management in the implementation of Te Poutama Tau.

The implementation of Te Poutama Tau

In general, all those interviewed in kura E shared a common commitment to the implementation of Te Poutama Tau. The principal and the lead teacher managed most of the organisation tasks associated with its implementation. They were responsible for the data analysis (with the support of the numeracy facilitator) and for the setting of goals.

Results and Discussion

While it is difficult to isolate individual items, the outcomes of this study suggest that the following key points that the two kura have in common contributed to the positive progress of the students in the Number Framework. It would also seem that the following points cannot be seen in isolation from each other, but in combination.

- Teachers and principals felt there had been significant change over the duration of the project in teacher and pupil attitude to pāngarau. Previously, a number of teachers and students felt negative about the subject. For students, the way in which numeracy was taught in the project eased many of their anxieties and increased knowledge and confidence. This is consistent with the results from the studies by Christensen in 2002 and 2003.
- The principals participated in the professional development programmes with the teachers. They worked alongside staff to develop a shared sense of purpose and direction. By modelling desired dispositions and actions, principals enhanced the rest of their staffs' belief in the project and in their own capabilities and their own enthusiasm for change.
- The principals and lead teachers closely monitored the school performance during the year, setting clear goals for teaching staff. The goals were evaluated throughout the year.
- For teachers, the framework provided a much more explicit picture of the required content and how students progress through the content. This point is closely associated with the setting of goals and the monitoring of performance.
- Individualised support was provided for a number of the teachers. The principals recognised that some teachers needed support and guidance in order to make changes.
- The lead teachers played a significant role in the implementation of the project and were well supported by the Te Poutama Tau facilitators.
- As a result of the programme, the principal and staff focused on student learning, not only the knowledge and strategies of the Number Framework but also the development of positive attitudes.
- Both kura had a commitment to teaching and learning in the medium of Māori, but it is not clear from this research what impact the level of proficiency of the students had on progress through the Number Framework.

There may well be features of the two case study schools that are unique to them and that contributed to the positive results. Although the two schools are classified as decile 1, they are different in their histories, their staff, and their relationships with their local communities. For example, kura A had been established for a significant number of years, with families having connections to the school for a number of generations. On the other hand, kura E was recently established and is still developing relationships with the local community. The teaching qualifications of each group of teachers are also quite different. One group of teachers held predominantly diplomas of teaching, while the other group of teachers have Bachelor of Education degrees and other post-graduate qualifications. Neither of the two groups of teachers has specialised qualifications in mathematics or mathematics education. Teaching experience is also quite different between the two groups of teachers. The teachers in kura A who participated in Te Poutama Tau have a mean of about 10 years' teaching experience, in comparison to kura E, in which the mean teaching experience is approximately 3 years.

Future Research

There has been research done in New Zealand on "successful schools" (Poskitt, 1993), on highly successful teachers in low-decile schools (Carpenter, McMurchy-Pilkington, & Sutherland, 2002), and on quality teaching for diverse students in schooling (Alton-Lee, 2003), but there has been little research on Māori-medium schools, as noted in the introductory section. Therefore, this report recommends:

- developing a set of criteria to identify successful Māori-medium schools and profiling a range of successful schools. This report is limited in that it focuses only on the mean stage gains on the Number Framework as a success indicator, when in fact there are a considerable range of success indicators.
- extending the case studies to an additional two schools from the 2004 Te Poutama Tau and comparing the results with the two schools from this study. The schools' numeracy data should also be analysed to develop a more detailed picture of the students' progress.
- examining effective strategies that teachers used in the various curriculum areas and the quality of the relationship between student and teacher. This study is limited in that it does not probe into identifying the effective teaching and learning strategies used by teachers to improve achievement.

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Equipment-in-use in the Numeracy Development Project: Its Importance to the Introduction of Mathematical Ideas

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Teachers' orientation to equipment use in the classroom is an important factor in the opportunities afforded to students to discuss mathematical ideas. The opportunities arise through the extent to which the initial focus of student activity is on using equipment when introducing a new mathematical idea. This paper examines the ways in which teachers use equipment in the Numeracy Development Project (NDP) for new concepts. Particular attention is paid to the extent to which they orient students' use of equipment to promote student thinking and discussion. These teaching strategies are tools for equipment-in-use.[4]

The Teaching Model

The NDP is structured around the use of equipment to teach students mathematical concepts.[5] The teaching model describes the progression from the features and functions of the equipment, to imaging, and then to number properties in students' representations of number ideas. The uses are in contrast to an experiential "hands-on" orientation common in so-called child-centred approaches where the use of equipment is to keep students actively engaged (Higgins, 2001). Experiential use does not necessarily lead to students developing mathematical understanding, nor does it necessarily imply that students talk about what they are doing.

Previously, the use of equipment in New Zealand schools has been associated with teaching mathematics to younger students, with the expectation that older students progress to book-based studies. The NDP emphasises the use of equipment as the starting point for any new idea. The levels of abstraction described in the NDP teaching model are referred to as a progression from materials, to imaging, and on to number properties, and are followed each time students are introduced to new learning, irrespective of their stage on the Number Framework. This progression can also be thought of as a shift for students from an externalised representation to a visualised idea and then to an internalised representation.

The teacher's role is one of leading the focus in these initial stages of learning new ideas. However, that role lessens as students' knowledge and strategies develop so that they are ultimately able to independently solve problems abstractly, using mathematical properties rather than equipment. This progressive cycle is repeated when a new mathematical idea is introduced. The teacher's focusing strategies can be described as shifting from an emphasis on demonstrating using materials to mediating using number properties.

As shown in Figure 1, the initial teaching–learning sequence presents the teaching model described in the NDP materials. It depicts the integration of student activities and the associated teacher strategies. The traditional, experiential use of equipment is not depicted.

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[4] The term *equipment-in-use* has been coined to emphasise the purpose of equipment use from a socio-cultural perspective. This paper describes equipment use in terms of four orientations.

[5] One of the key Ministry of Education goals is to reduce disparity. To help address this goal through the NDP, supplementary funding was provided to lower decile schools to purchase equipment and provide ancillary support.

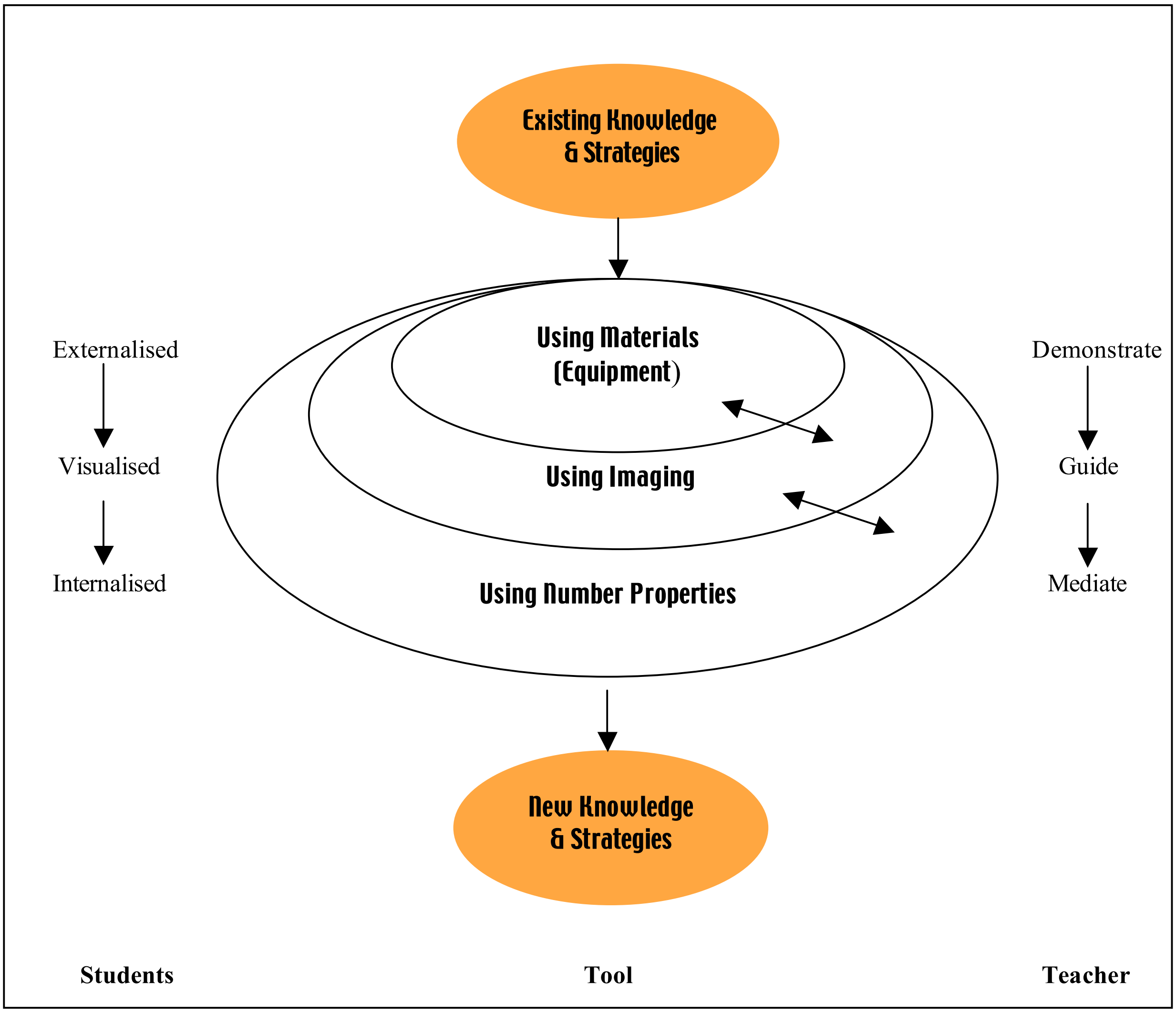

Figure 1. The initial teaching–learning sequence when introducing new mathematical ideas[6]

A Comparative Analysis: Four Orientations to the Use of Equipment

The use of equipment in early mathematics is a well-established practice for New Zealand primary school teachers. Its use can vary, according to the age of the students, from an experiential focus on mathematics operations with younger students to a more formalised, structured approach with older students.

The schematic, Figure 1, identifies the three orientations of equipment use: externalised, visual/conceptual, and internalised. These contrast in significant ways with the traditional experiential approach focused on algorithms. The differences between all four orientations are summarised in Table 1. The table sets the distinction across the orientations according to three elements: the equipment-in-use, the teacher's focusing strategies, and the student's progression to intellectual independence and peer discussion.

[6] This diagram is a modified version of The Teaching Model (Ministry of Education, 2004).

Table 1
Comparison of orientations of actions towards equipment use

Element	Algorithmically oriented activity	Externally oriented activity	Conceptually oriented activity	Dialogically oriented activity
Tool (equipment-in-use)	The physical action on the equipment demonstrates the "working form" of the algorithm.	Equipment provides a concrete manipulative reference point.	Equipment represents the thinking in solving a mathematics problem.	Equipment mediates discussion.
Teacher	The teacher gives directions to the students about the actions to take with the equipment to complete the algorithm.	The teacher demonstrates the elements of the problem using equipment.	The teacher guides the students to use the equipment to show their thinking.	The teacher mediates student dialogue in justifying their mathematical thinking using equipment.
Students	Students imitate the teacher's demonstration of words and actions of the algorithm with the equipment.	Students follow the teacher's demonstration or model.	Students represent their mathematical thinking through equipment.	Students use equipment as a dialogical support for participating in a mathematical discussion.

In the first orientation (algorithmically oriented activity), equipment is frequently used in middle and senior primary classes in a more structured way to support the teaching of addition, subtraction, multiplication, and division algorithms. Used in this way, equipment is portrayed as an external representation of the procedures of the computational algorithm.

In contrast, a second orientation (externally oriented activity), arises from students' experiences. From this perspective, equipment is used as a tool for the support and guidance of students' thinking sequences in solving a mathematical problem. Equipment can be used to provide a concrete manipulative reference point for students in the introduction of new ideas. This strategy uses the equipment as an external representation of the thinking process, not just for the operations of calculations. Such use of equipment highlights the elements of mathematical concepts rather than the procedural stages of algorithmic operations as in the first orientation.

The third orientation (conceptually oriented activity), supports students conceptually to understand the structural elements of a mathematical idea.

The fourth orientation to equipment use (dialogically oriented activity) centres on the structural elements of an idea by encouraging discussion and explanation. The equipment becomes a reference point in the justification and negotiation of meaning of mathematical ideas as students work towards a shared understanding.

Across the three orientations of the NDP (excluding the algorithmic), the focus shifts from individual students (with the external) to collective students (with the dialogical). Similarly, the responsibility for the ways in which the equipment is used progresses from wholly the teacher (with the external) across to wholly the students (with the dialogical).

The comparison of the orientations draws on excerpts from interviews informed by classroom observations in three mainstream classrooms across two regions in New Zealand. These interviews were part of the evaluation of the NDP in 2004 aimed at creating an image of each orientation to equipment-in-use.

In Table 1, each element is discussed in terms of its orientation to use. The excerpts illustrate the element for a particular orientation.[7] This is followed below by a discussion of the impact of the NDP on equipment use.

Comparison of Orientations of Teachers' Actions Towards the Use of Equipment

Equipment: Algorithmic Orientation

Use of equipment from this orientation is limited to representing the working procedures of a mathematical algorithm. This algorithmic approach uses equipment as a tool for procedures. It mirrors the steps in the working form of an algorithm. This orientation is associated with a transmission model of teaching where emphasis is given to knowledge of the procedures to be followed. The teacher's quote below highlights the restriction of the use of equipment to being steps to follow.

> The procedural way of doing it doesn't ask them to think about what they know. (Vicki,[8] interview)

The choice and use of equipment is therefore based on how well the piece of equipment represents the procedures of the algorithm,[9] rather than how well aspects of a mathematical idea are represented.

Equipment: External Orientation

The equipment provides a concrete manipulative reference point when introducing new learning to students. The quote below is from a NDP teacher who is trying to help her son, whose school has not yet participated in the project.

> He had not seen the equipment to understand the idea … so you know the equipment is essential for the kids … it's not to keep their hands busy. (Vicki, interview)

At this stage, equipment is useful as a concrete representation introducing elementary aspects of a mathematical idea and, as such, might be seen as a tool for "getting started".

Equipment: Conceptual Orientation

In a conceptual orientation, equipment-in-use becomes a tool to show aspects of mathematical ideas rather than a tool for showing the procedures to be followed in solving a mathematics problem. Gravemeijer (1994) compares the shift in equipment use from a *working* to a *thinking* model. The distinction he draws is not about the equipment per se, but about its use as a tool for thinking mathematically. He suggests that actions with equipment should provide a "frame of reference" or *thinking model* when doing recording. The degree and ways in which equipment is structured will shape but not necessarily limit its use. In the NDP, much

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[7] Excerpts to illustrate the algorithmically-oriented activity are comments by teachers that are in contrast to their current use of equipment guided by the NDP.

[8] Teachers' names have been changed.

[9] The renaming method for subtraction replaced the equal additions algorithm because place value blocks widely available in New Zealand schools in the 1980s were useful for demonstrating the steps in renaming, but not for equal additions.

of the equipment is structured around groups of five or ten because the emphasis is on part–whole schema.

> So you need all the equipment to show the different parts of a number, the different properties of a number. (Vicki, interview)

Teachers' knowledge of the design of equipment gives a basis for its usefulness in developing an understanding of a particular mathematical idea. That knowledge guides teachers' choice and use of equipment.

Equipment: Dialogical Orientation

The use of equipment to support mathematical discussion emphasises the use to which the equipment is put. This focus on activity "shifts away from the analysis of symbols [including equipment] as external supports for reasoning and moves towards students' participation in practices that involve symbolising" (Cobb, 2002, p. 187). McClain's (2002) study, investigating the role of tools in both supporting and constraining communication in the classroom, underscored the importance of a teacher's understanding of the student's activity of explaining and justifying their thinking.

> I just feel that the activity-based [approach] works really well with the language type [approach]. … If they don't have that oral language and that ability to talk and explain things … That's the crux of it … It's the talking, it's the explaining, it's the using the words, the language … They'll talk to their partner, even if it's just working out who's putting up how many fingers ... They're still talking with somebody and they're still using that language. (Nancy, interview)

An item of equipment does not have any inherent meaning apart from that arising from the context of use (Roth & McGinn, 1998). In the quote above, the students use their fingers in the context of their discussion of mathematical ideas.

Teacher: Algorithmic Orientation

When there is a focus on procedures, the teacher's role is to give directions to the students about the actions to take with the equipment to complete an algorithm. Emphasis is given to students' "proper" use of the materials. Concern is given to physical dependency on the equipment and so this use of equipment is frequently followed by individual bookwork to practise the procedures learned. In this orientation, bookwork is seen as a more "advanced stage" than using equipment and is one possible reason for minimal use of equipment with older classes.

> That's the thing because in their previous classrooms they were not allowed to share, they had to do it in their books, so they reduced down to an individual. (Vicki, interview)

The emphasis is on following procedures rather than asking students to think about what they are doing and talk about it with others.

Teacher: External Orientation

The teacher demonstrates the elements of the problem with the equipment. In the quote below, the teacher clearly sees her use of equipment as a part of a progression in the teaching of mathematical ideas.

> I like using equipment for the simple fact that I find it easier for children to understand, particularly at the beginning stages … even if they are on … you know … early part–whole … I just find it easier to use it to start with and then they can wean off it again … just the teaching of the initial concept. (Nancy, interview)

The teacher here, in her use of equipment, is focused on giving students a starting point that is concrete; that is, in an external form.

Teacher: Conceptual Orientation

The teacher uses equipment to foster student thinking about mathematical ideas. Equipment choice is based on the teacher's purpose of the lesson. The choice of equipment varies in response to students' developing understanding and is limited by the teacher's own content and pedagogical content knowledge.

> Vicki to the students: "What I'm trying to do is to lead your thinking."

> Vicki to the students: "So why have I given you this set?"

In the above quotes, the teacher, in her use of equipment, has emphasised thinking. Her comments suggest that her choice of equipment is aligned to the overall purpose of encouraging students to think, but that this thinking is shaped by her choice of equipment.

Teacher: Dialogical Orientation

In a dialogical orientation, the teacher's focus is on extending concepts in response to students' actions and explanations. Essentially, this is a mediation role in which the teacher mediates student dialogue in justifying their mathematical thinking using equipment. Vicki, in the quote that follows, mediates by "decoding and rephrasing".

> They don't have the words to explain what they're doing … or put it out in a logical way that I can understand … I mean, you heard all that decoding and rephrasing … paraphrasing … I do that all the time because it's all jumbling out I guess … That's where the equipment fits for these kids. (Vicki, interview)

For this teacher, her use of equipment is to support the students' discussion of mathematical ideas.

Students: Algorithmic Orientation

The students' actions are to imitate exactly the teacher's actions with the equipment that shows the algorithmic operation. They observe the teacher's demonstration using equipment of the sequence to be followed. Typically, the students are expected to individually follow the teacher's procedure rather than do their own thinking as they answer a mathematical problem.

> Follow this procedure and you'll get the right answer at the end. (Nancy, interview)

The teacher here is emphasising that the way to get the right answer is to stick to the procedure that she has shown them.

Students: External Orientation

The students follow the teacher's models as they are introduced to the new ideas. It is important to note this teacher's reference to the students finding different sorts of equipment helpful in the initial stages of introducing a new idea.

> Well, I use the equipment for making the learning of new concepts much easier … If the kids are able to see that different equipment can be used in different ways … I think that maybe helps their thinking strategies as well and gives them more variety in the way they can work things out. (Nancy, interview)

The use of different pieces of equipment is important as it is unlikely that any single piece of equipment illustrates all aspects of a mathematical idea. However, this also assumes that teachers have knowledge on which to base their choice of equipment for students.

Students: Conceptual Orientation

In a conceptual orientation, students use equipment to represent their mathematical thinking.

> It's a visual thing and they can see their processes, they can work their processes through and manipulate through … Particularly those ones who don't think as quickly as the others, they can see and do it as they think it, they can get the … you know … the one and then change it to two and into three and into nine and then add one more for the ten. So they can manipulate the equipment and see their thinking at the same time. … It's like visualising what they're thinking and they can see it happening. (Nancy, interview)

The emphasis in this orientation is to offer the students a way of representing their thinking about a mathematical idea through the use of equipment.

Students: Dialogical Orientation

Students use equipment as a dialogical support for their explanations. The focus in the quote below is on Jerry's participation in the practice of using equipment to symbolise the mathematical ideas in his solution to the problem.

> Some of the time when they're talking … I struggle to understand how they've got there, but they know exactly how they've got there … Jerry got all the pirates off the island, but his system and his way of working it out was totally different to what I would have done … It was different to everyone else's too, but he got the answer at the end, he went though his own processes and explained it, although I didn't quite understand it … but he knew in his own mind and he showed me with the counters. (Nancy, interview)

Student explanations to mathematical problems can become complex and challenging for others to follow. The use of equipment is a way of supporting students as they work through their explanation.

Discussion

Equipment use is complex because it is dynamic in terms of the shifting balance of the teacher and student roles as well as its shifting place in the learning progression for each new idea from an external referent to a tool in the student's practice of symbolising (Cobb, 2002).

Any equipment has been designed with a purpose. Much of the equipment associated with the NDP has been chosen because the purpose of the developer of the equipment was to foster part–whole thinking. There is also the teacher's purpose in using the equipment. The purpose of the equipment designer and the teacher's purpose in using the equipment do not necessarily match. The teacher may adopt or reject or simply not understand the designer's purpose.

The function of a piece of equipment can be clarified in answer to the question "What does the equipment ask us to do?" For instance, a number line tells us to use the sequence of numbers to represent a mathematical idea; a tens frame tells us to use ten as a reference point; and a hundreds chart tells us to use groups of ten (up to ten groups of ten) as the basis for the solution strategies.

The choice of equipment for classroom use depends on a teacher's purpose, as exemplified in this paper through four broad orientations towards mathematics teaching. In the early stages of professional development in the NDP, where teachers are still shifting towards a conceptual and dialogical approach to mathematics teaching, their choice of equipment may be framed by their desire to model mathematical activities. Where teachers work from conceptual and dialogical orientations, their choice of equipment will be based on the number properties they are wanting to highlight to the students as well as the discussion of mathematical ideas they want to foster among students.

Teachers' knowledge of mathematics might also govern the range of equipment used in their teaching. Where teachers' knowledge is still developing, their choice of equipment is restricted and shaped by their understanding of mathematical ideas.

> So the equipment … you can't use one for one purpose only, one form … You need all of it because there's not one piece of equipment that I can think of that will show, particularly for a concept like decimals and fractions, that would show everything about it … yeah … I think that's the limit of something like … procedure type teaching, you don't show … you don't give the kids the versatility, the flexibility of it. (Vicki, interview)

Acknowledgments

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Going Public: Students' Views about the Importance of Communicating Their Mathematical Thinking and Solution Strategies

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This paper reports on data from 183 nine- to eleven-year old students attending six Waikato schools, four of which had participated in the Numeracy Development Project (NDP) and two that had not. Students' responses to questions about communicating their mathematical thinking and strategies to peers were analysed and patterns identified. There was considerable variation from school to school in students' ideas about the value of communicating mathematically with others. Students at the two schools that had not yet been involved in the NDP did not differ markedly from those at participating schools in the interviews about communicating mathematically with others. Overall, the students were more positive about the value of explaining their own thinking to other people than they were about knowing what strategies their peers used to solve mathematics problems.

For more than a decade, mathematics reform documents have been calling for a shift in approach to teaching and learning mathematics (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 2000). A key aspect of the reforms is the idea that mathematics learning should no longer be a private, individual activity but instead involve groups of learners who challenge and support one another as they reason their way through problems (Lampert & Cobb, 2003). Students benefit not just by learning to communicate mathematically; by communicating, they also learn mathematics (NCTM, 2000). According to the current curriculum document for New Zealand schools, communication is essential if students are "to express ideas, and listen and respond to the ideas of others" (Ministry of Education, 1992, p. 9).

As part of the mathematics reforms, much work has been done on the ways that teachers and students approach mathematics teaching and learning in classrooms, the so-called "sociomathematical norms" identified by Yackel and Cobb (1996). Such studies provide exemplars of what is possible when conditions are favourable and teachers are exceptionally capable and committed. Less is known about implementing mathematics reforms in "ordinary" classrooms and schools.

In a separate but parallel development, there has been substantial work on the value of listening to and talking with students themselves (Carr, 2000; Davies, 1982; Devereux, 2001; Paley, 1986; Roberts, 2000; Rudduck & Flutter, 2000; Smith, 1995; Smith, Taylor, & Gollop, 2001). Recent research has focused on the issue of student "voice" and, in particular, on the importance of finding out how students see themselves as learners (Fielding, Fuller, & Loose, 1999; Freeman, McPhail, & Berndt, 2002; Phelan, Davidson, & Cao, 1992; Pollard, Thiessen, & Filer, 1997; Young-Loveridge & Taylor, 2003). According to McCallum, Hargreaves, and Gipps (2000), pupils' voice is important in understanding schools and schooling. The UN Declaration on Human Rights states explicitly that children should be given a voice on matters that have an impact on them (New Zealand Ministry of Foreign Affairs & Trade, 1997).

New Zealand has put in place a major initiative in numeracy that is designed to raise mathematics achievement by improving the professional capability of teachers in teaching mathematics (see *Curriculum Update no. 45 and no. 51.* Wellington: Ministry of Education, 2001). Key components of this initiative include a Number Framework that outlines progressions in students' learning about number, an assessment tool to pinpoint students' learning needs, professional development programmes for teachers to help them become familiar with these tools, and additional resources to support students' learning. As part of the evaluation process, the perspectives of teachers, principals, and facilitators have been researched, but little has been done to find out from students themselves how they see the project and its impact on their mathematics learning.

This paper presents data from part of a larger project that set out to explore students' perceptions and dispositions towards learning mathematics. A range of different issues were explored with students, including their views about the nature of mathematics, mental computation processes, communication of solution strategies with others, and teachers' and family/whānau roles in supporting mathematics learning. This paper focuses on one aspect of the larger study: namely, students' ideas about communicating mathematical thinking and strategies with their peers.

Method

Participants

The participants in this study consisted of 183 year 5 and 6 students (nine- to eleven-year-olds) at six schools. Table 1 shows the composition of the sample. More than half of the students were Māori, over a third were European, a tenth were Pasifika, and the remainder were Asian or another ethnic group. Four of the schools (marked with an asterisk) had participated in the NDP and were from a large urban centre. The two schools that had not yet participated in the numeracy projects (non-NDP) were from a small neighbouring town.

Table 1
Composition of the Sample in Terms of Gender, Ethnicity, and Decile (Eur = European, Ma = Māori, Pas = Pasifika, As = Asian)

School	Decile	Boys	Girls		Eur	Ma	Pas	As	Other	Total
Arch*	low	21	17		9	20	5	2	2	38
Bank*	low	8	15		1	17	5			23
City*	middle	16	14		21	3	1	5		27
Dale*	low	15	15		11	11	3	4	1	30
Edge	low	19	11		2	28				30
Farm	low	17	15		9	22	1			32
Total		96	87		53	101	15	11	3	183

Procedure

Schools were asked to nominate about 30 year 5 and 6 students from across a range of mathematics levels. Students were interviewed individually in a quiet place away from the classroom. Students were told that the interviewer was interested in finding out more about "how kids learn maths and how their teachers can help them" and "what kids themselves think about learning maths". Interviews were transcribed for later analysis. Once the interviews were complete, schools were asked to identify each student's current stage on the Number Framework, or, if at a non-NDP school, whether the student was "average", "above average" or "below average" in mathematics, using their knowledge of the students' achievement that year.

This information was used to categorise students as *high* (above average or at stage 6: Advanced Additive Part–Whole or above), *middle* (average or at stage 5: Early Additive Part–Whole), or *low* (below average or at stage 4: Advanced Counting or below).

During the interview, students were asked to comment on a range of topics, including the importance of working out problems mentally, of getting answers correct, and whether they thought there was only one way or several different ways of working out an answer. They were then asked the following questions and the reasons for their responses:

> Do you think it is important for you to know how other people get their answers? Is it important for you to be able to explain to other people how you worked out your answer?

Results

Table 2 presents a summary of students' responses to the questions explored in the study. This table shows considerable variation from school to school in students' responses to the questions.

Table 2
Percentages of Students Who Responded to the Questions about Knowing and Sharing Solution Strategies with Peers (numbers are shown in brackets)

(DK = Don't know or an ambiguous response)

	Importance of knowing others' strategies				Importance of explaining one's own strategies to others			
School	Yes	No	DK	Total	Yes	No	DK	Total
Arch*	27.8 (10)	52.8 (19)	19.4 (7)	36	48.6 (18)	27.0 (10)	24.3 (9)	37
Bank*	39.1 (9)	43.5 (10)	17.4 (4)	23	56.5 (13)	39.1 (9)	4.3 (1)	23
City*	77.8 (21)	11.1 (3)	11.1 (3)	27	74.1 (20)	7.4 (2)	18.5 (5)	27
Dale*	46.7 (14)	46.7 (14)	6.7 (2)	30	60.0 (18)	23.3 (7)	16.7 (5)	30
Edge	17.9 (5)	71.4 (20)	10.7 (3)	28	48.3 (14)	17.2 (5)	34.5 (10)	29
Farm	31.3 (10)	56.3 (18)	12.5 (4)	32	50.0 (16)	46.9 (15)	3.1 (1)	32
NDP	*46.6 (54)*	*39.7 (46)*	*13.8 (16)*	*116*	*59.0 (69)*	*22.2 (26)*	*18.8 (22)*	*117*
Non-NDP	*25.0 (15)*	*63.3 (38)*	*11.7 (7)*	*60*	*49.2 (30)*	*32.8 (20)*	*18.0 (11)*	*61*

Of the six schools, City had the highest levels of agreement from students about the importance of knowing others' strategies (77.8%) and also about the importance of explaining one's own strategies to others (74.1%). The majority of students at Edge School, on the other hand, felt strongly that it was *not* important to know how other people work out their answers (71.4%). Overall, there were higher levels of agreement about explaining one's thinking to other people than to knowing about other people's strategies and thinking (55.4% vs 38.8%).

As Table 2 shows, there was considerable variation from one school to another in students' views about the importance of communicating about mathematical thinking with their peers. When the responses of students from the four NDP schools were put together and compared with those of the non-NDP students, an interesting pattern emerged. There was a statistically significant difference between the two groups, but only with respect to knowing about how other people had worked out the answers to their problems [$\chi 2$ (2) = 9.47, $p < 0.01$]. Almost half of the students at NDP schools thought that knowing about how other students solved problems was important, compared to only a quarter of the non-NDP group (see Figure 1). The reverse pattern was evident in the responses of students who thought that knowing others' solution strategies was *not* important; almost twice as many non-NDP as NDP students thought that knowing others' solution strategies was *not* important (63.3% vs 39.7%). However, the responses of students at non-NDP schools differed little from those of students at NDP schools on the question about the importance of explaining one's strategies to others (see Figure 1).

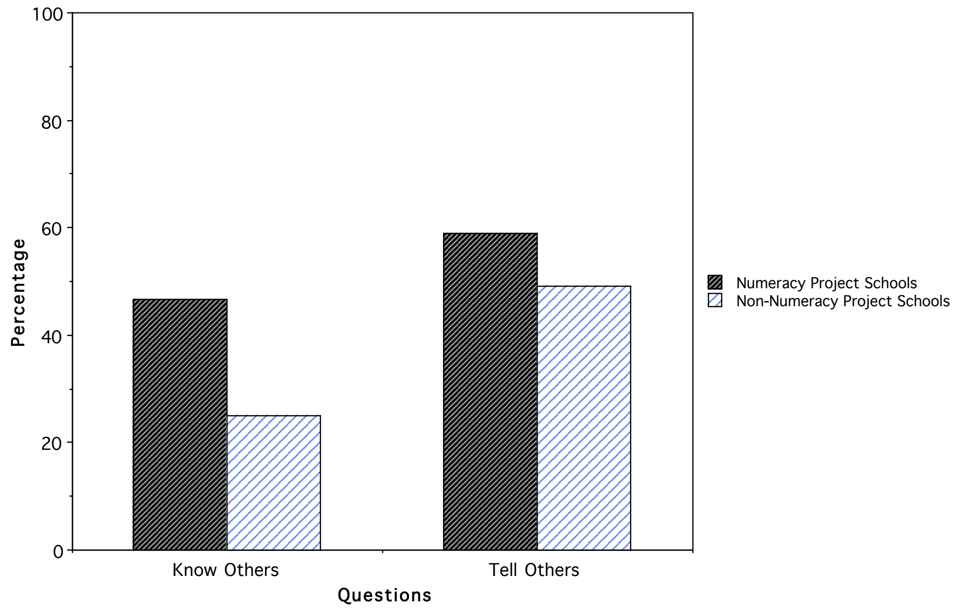

Figure 1. Percentages of year 5–6 students who thought that particular ideas were important as a function of participation in the NDP

Students' responses to the interview questions reflect the extent to which they thought communication was important for their mathematics learning. In the interviews, we endeavoured to find out *why* students held particular views. The following sections present students' responses to the two questions that are the focus of this paper, about *knowing* about other people's solution strategies and about *explaining* one's own solution strategies to others. The responses of those who thought that it was important are presented first, followed by the ideas of those who thought that it was *not* important.

1. Knowing others' strategies is important

Several students referred to the usefulness of having *alternative* ways of solving problems as their reason for saying that knowing others' strategies was important:

> They might have another way. (A11, boy, high)
>
> I could try and do it their way. (A24, girl, middle)
>
> Because other people may have different ways and you can learn off them and you maybe get better and better. (C18, boy, low)

Some students referred explicitly to the value for their own *learning*:

> Because then I can learn from it and other people can too. (B19, boy, high)
>
> So you can learn from them. (F12, girl, high)
>
> Because they're helping you understand. (F8, girl, high)
>
> Yes because then I can learn from it and other people can too. (B8, boy, middle)
>
> It could help your learning by getting better if they can show you how they got to work it out. Sometimes you can get very confused about what other people say, but it's good to hear their learning because sometimes it's most likely to help your learning as well. (C19, girl, low)

It was interesting to note that some of these responses (F12 & F8) came from students attending Farm School, a non-NDP school.

Helping other students learn was given as the reason by some students:

> Because you want to see people succeed with stuff because they do have a bit of trouble, but I just sometimes help them a bit. (F2, girl, high)

A group of students seemed preoccupied with the *correctness* of their answers, even though the NDP had tried to shift the focus away from correct answers onto examining the variety of strategies:

> To get the answer right. (B5, girl, middle)
>
> Because if they get what the answer is, if they get it right … it can be in your head too. (F32, boy, low)

Some students seemed more concerned about classroom *etiquette* and showing respect for their classmates:

> Because when you listen to other people, those other people might listen to you and learn your answer. (A29, girl, middle)

Relationships were an important consideration for some students. One student thought that knowing how other people solve problems was only important among close friends:

> Only if they're really close to you and they're really good friends. (A37, boy middle)

2. Knowing others' strategies is <u>not</u> important

Students who disagreed with the idea that knowing others' strategies is important had a range of different reasons for their view. The most frequent reason given referred to *dishonesty*:

> Because it's just like cheating. (F3, girl, middle)
>
> Because you're using their brains not yours. (A31, girl, low)
>
> Because you don't do that, because that's cheating. (A8, boy, middle)
>
> No, because then you're just like cheating off them and you don't really learn for yourself, you're learning off other people and when you go to do it by yourself you don't actually know because you've cheated off someone else. (A13, girl, high)

Privacy was referred to by some of the students who thought that knowing about other students' strategies was not important:

> Because it's their business and not mine. (F4, girl, middle)
>
> Because it's their work and not yours. (B7, boy, middle)
>
> Because you don't want to know about it, it's only important to them. (F31, boy, high)
>
> Because it's their own business how they do it … It's not important for me because I know my own way to work things out. (C15, boy, low)

> Because it's their way they do it, you don't have to, 'cause it's really none of your business. (C26, girl, low)

Some students thought that *individuality* was important:

> Because everyone's different and everyone has their own way (F5, girl, middle)
>
> Because you should know yourself. (F22, girl, middle)
>
> Because I like doing it by myself. (A2, boy, middle)

The idea of *reciprocity* came through in the responses of some students. They expected to alternate between being the "helper" and the person being helped.

> Because if they get the answer right then they can tell you and help you and then if they don't know a question, if they don't know the answer then we can help them. (A16, boy, high)

3. Explaining One's Strategies to Others is Important

Students tended to be in greater agreement about the importance of explaining their strategies to other students than they had been about knowing others' strategies. Many of these students referred to helping others with their *learning*:

> So they can learn how to do it next time they go to do it. (B15, girl, high)
>
> Because they can learn and they can go up another level. (B6, girl, middle)
>
> It's helping my learning as well so it's good to share what your side of the story is. (C19, girl, low)
>
> Because others may not understand it and you might. (A31, girl, low)
>
> So other people can learn from my answer. (B8, boy, middle)

Sometimes the helping involved withholding some information because of *concern* that too much information might prevent other students from working something out for themselves:

> Just how you solved it up, only a little bit, not too much, because people get out too much information. [Asked: "What would be the problem if you gave them too much information?"] They'll know heaps about it so they won't figure it out for themselves (D1, boy, middle)
>
> Sometimes if they're really stuck, but I won't actually give them my answer … I'll give them clues on how to work it out and that. (A13, girl, high)

Sometimes the reason for helping others with their learning was that the teacher was very busy and it facilitated *classroom organisation* if some of the more proficient mathematicians explained their ways of solving problems to less proficient classmates:

> Because if they wanted to learn something and they asked the teacher and the teacher's busy then they can go to a person that will know the answer and they can explain to them how you can add up to that. (B11, girl, high)

The value of *alternative* strategies was mentioned by some students:

> Because there's lots of different ways, and maybe I have a different way to them. (B17, boy, middle)
>
> Because it's good that I know how I worked out the answer first of all, and it's good 'cause I like sharing my ideas with other people and my point of view of how I can work it out, and so if I say my way and another person tells their way and their way's a bit easier, I can just try it their way, and then you get lots of different ways by telling your one, then other people go "I've got a simpler way of doing that", so it helps you to learn. (C2, boy, high)

The responses of some students hinted at the need to "*prove*" that they had solved the problem themselves before finding out how other students had come up with a solution:

> Because sometimes people ask you how you did it, so you've got to know how you did it, so before you take their answer, you say the answer, you think about how you did it. (C20, boy, high)

> So they know that you don't copy off other people. (B1, boy, middle).

Building and maintaining *relationships* was important for many students and this came through in their responses to the questions:

> So you can help your mates out. (A17, boy, middle).

4. Explaining one's strategies to others is <u>not</u> important

Relatively few students thought that explaining their thinking to others was *not* important. The reasons these students gave for their responses were similar. *Dishonesty* was the most frequent reason given:

> It's just cheating. (A19, girl, low).

> Because that's cheating. (F21, boy, middle)

> Because that's just like cheating. (F28, boy high)

Individuality was given as a reason for not explaining one's strategies to others:

> You should always worry about your work before you go to other people about their work. You never know when they could get a wrong answer and you could as well. (B1, boy, middle)

> Because they could have their own way to work it out. (C15, boy, low)

> Because different people have different ways of doing their answers. (F4, girl, middle)

Privacy was again referred to as a reason for not explaining one's own strategies to others:

> Because it's my work and not theirs. (F5, girl, middle)

> Because if we tell them how you worked out the answer, then they'll go round telling everybody else, then everybody else would get the same answers. (A38, girl, low)

Some students thought that explaining one's strategies to other students might *hinder learning r*ather than help it:

> If you told them the answer they won't be able to learn. (A18, girl, middle).

> Because they've got to learn, they've got to learn … (F10, boy, high)

Discussion

Although the mathematics reforms have called for mathematics to be a more public activity with learners communicating openly about their ways of solving problems, the findings reported in this paper show that it is not easy to achieve this goal. There was considerable variation from school to school in the proportion of students who thought that the communication of mathematical thinking and strategies was important. On average, students at NDP schools were twice as likely as those from non-NDP schools to think that knowing about others' strategies was important. Both groups were similar in their view that explaining their strategies to others was important. However, there was still quite a number of students in NDP schools who, despite having experienced the sharing of mathematics strategies as part of the numeracy projects (in particular, in-class modelling by numeracy facilitators), still seemed to believe that mathematics should be private and was "no one else's business". This finding highlights the difficulties involved in bringing about changes in students' ideas about learning mathematics.

Similar comments have been made about the challenges of mathematics reform for teachers and students in the US (Lampert & Cobb, 2003).

The students' responses seem to reflect confusion about the difference between dishonesty/cheating and being helpful/co-operative. There appears to be a lack of clarity for students about what constitutes "cheating". It would seem that for them, the line between being dishonest and being helpful/co-operative is a very fine one. The fact that substantially more students thought it important to be able to explain their strategies to others (because they saw it as helpful for their peers) than to know about others' strategies (often seen as cheating) is consistent with this idea. Students may be receiving contrary messages in other contexts (for example, assessment situations) in which talking to others is seen as not appropriate.

Some of the students' responses reflected a concern about developing and maintaining relationships with others. This is consistent with the claims of some writers that having friends in a class is so important that sometimes students put being with their friends ahead of academic considerations (Duffield, Allan, Turner, & Morris, 2000; Phelan et al., 1992). Maintaining relationships may also have been an underlying issue for those students in our study who were worried about "cheating" or the risk of seeming too interested in "someone else's business". Relationships between students and their classroom teachers has been identified as a major issue that teachers need to pay more attention to (for example, Bishop et al., 2003; Hawk et al., 2003).

Related to this is the issue of power relations, not just between teachers and their students, but also among students themselves. According to some writers, students identify the feeling of emotional safety as an important feature of classroom climate (Phelan et al., 1992). Duffield and colleagues (2000) discussed the anxiety about social norms that many students experience at school as they become increasingly aware and concerned about what is, and is not, a "cool" topic for discussion with peers at school.

The extent to which students are active participants in helping to determine learning goals seems to be important. In another paper, we have analysed how students perceive the role of their teachers (Taylor, Hawera, & Young-Loveridge, in press). We identified four major roles that teachers seem to adopt, including the roles of mentor, manager, transmitter (of information), and arbiter. In City School, where students seem to value communication with peers very highly and there is a strong emphasis on formative assessment, students are encouraged to collaborate with their teacher in setting their own learning goals and record their learning in individual "logs". We found more students at City tended to see their teacher in the role of mentor. Further, the management team at City had previously been involved in a Ministry contract that focused on assessment and learning.

The findings of this study have some important implications for teachers and other educators. Although there is much rhetoric about the value of communication in mathematics (for example, Ministry of Education, 1992), there is clearly some way to go before students feel comfortable about and appreciate the benefits to their learning of communicating with peers about their thinking. To help students appreciate and value communication processes in mathematics, attention needs to be given to the messages that are conveyed about situations in which communication is to be encouraged and situations in which it is not. One way of supporting/encouraging students that communication in mathematics is valued is by making a point of reporting to parents/caregivers on how well students are doing in terms of their mathematical communication. In our experience, it is more usual to receive information about a child's proficiency on particular strands of the curriculum, such as number, measurement, geometry, algebra, or statistics (often after a test of some sort), than to hear about how well he or she is able to think, reason, and communicate mathematically (part of the mathematical processes strand).

Less obvious, but potentially just as powerful, is the need to keep families/whānau and the community abreast of changes in the approach to mathematics learning. Views of mathematics have shifted considerably over recent decades, and it is important for the community to be kept up-to-date about changes in the way that mathematical processes and thinking are being emphasised. We noticed that most students had family members willing and able to support their mathematics learning at home, but much of this help tended to reflect ways of thinking about mathematics more typical of past generations. The leaflets produced by the Ministry of Education in 2004 and 2005 go some way towards informing parents about the numeracy projects. A key message in the leaflets is that:

> There is usually more than one way to solve a problem. If your child has a strategy that works, praise them. If yours is different, that's quite OK.

The leaflets also list the kinds of things that children are learning on the NDP, including calculating "in their head where possible, rather than using a calculator or pen and paper" (Ministry of Education, 2004a, 2004b). It would be helpful if teachers were to make a point of highlighting the importance of communication in mathematics in their interactions with families/whānau.

The findings of this study, in particular the verbatim quotes from individual students, provide valuable insights into the students' unique perspectives on their mathematics learning and underline the importance of taking children's views into account (Civil & Planas, 2004; Cook-Sather, 2002; Young-Loveridge & Taylor, in press). We believe that teachers should be encouraged to make reciprocal communication in mathematics a major goal in their teaching. Furthermore, students should be encouraged by their teachers to collaborate in setting goals for their learning in mathematics. The findings of this study indicate that much more support is needed for teachers in order to sustain their learning from the NDP professional development over the long term.

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Students' Views about Mathematics Learning: A Case Study of One School Involved in the Great Expectations Project

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This paper reports on the responses of students attending a school that was involved in the study of students' perceptions and dispositions towards learning mathematics and also in the Great Expectations project, a Teaching Learning and Research Initiative designed to raise teachers' expectations of achievement. Twenty-seven year 5–6 students at City School were asked to comment on their views about communicating their mathematical thinking and strategies with peers and teachers. City students regarded the communication of mathematical thinking and strategies as extremely important for their learning and were very articulate in explaining the reasons for their views. Further information gathered from the school helped to explain why City students were so aware of the importance of mathematical communication. For some years, the school had been working towards developing assessment practices that put student learning at the centre of its teaching. As part of this process, students were expected to communicate with their teachers about their learning goals and reflect on how well these were met. The school's emphasis on strengthening teacher–student relationships and encouraging self-responsibility in students seems to have played a major role in helping students appreciate the importance of reciprocal communication in mathematics learning.

There has been increasing concern over recent years about disparities in students' achievement and what can be done to address the learning needs of students more effectively. Analysis of the major sources of variance in students' achievement has identified teachers as the single biggest factor (apart from students themselves) in explaining the variance in achievement among learners (Alton-Lee, 2003; Hattie, 2002). According to the meta-analyses of Hattie, teachers account for about a third of the variance in students' achievement, whereas other factors such as home, school, and peer group explain no more than 5–10 percent each of the differences in achievement among students. The growing recognition of the importance of teachers has led to considerable research on what it is that teachers do that can make the difference for their students. However, a problem with research such as this can be that studies are done in isolation from one another, even though there is often great potential for making links between related studies.

The Teaching Learning and Research Initiative (TLRI), a recent initiative by the government, was designed to fund research projects involving partnerships between teachers and education research experts. The goal of the TLRI was "to find out what works in terms of lifting student achievement, and then to apply those lessons in the real world so students do actually enjoy the benefits" (Mallard, 2003). One of the first projects to be funded through the TLRI was Great Expectations (GE), a project directed by Dr Mary Hill that focused on "enhancing learning and strengthening teaching in primary schools with diverse student populations through action research". The GE project involved teacher researchers from six schools in the Waikato/Auckland region investigating how teaching and learning can be systematically improved, and how expectations are implicated in this (Hill & Robertson, 2004a, 2004b). Each of the six schools chose a different area of particular concern to its teachers, including information and computer technology, literacy, numeracy, teacher professional development, and assessment.

The director of the GE project was approached by me to see whether she would be interested in the possibility of linking the GE project with the Numeracy Development Project (NDP) evaluation research. She was enthusiastic about the way that this could enhance both studies and invited me to present a proposal to teachers in the GE project. City School was one of two local schools invited to participate in the study of perceptions and dispositions towards mathematics learning. The students from City School had been very articulate about their mathematics learning and about the importance of communicating with others about mathematical thinking and strategies. Further information was gathered from the school in order to understand better how City students had come to have such views and insights into their mathematics learning. The research question that guided this particular study was:

> How is one school's involvement in an initiative designed to raise teachers' expectations of achievement related to students' perceptions and dispositions towards learning mathematics?

Method

Participants

The participants in this study consisted of 27 year 5 and 6 students (15 boys and 12 girls) in three classes at City School, an urban school of about 400 pupils serving a medium socio-economic status community (decile 5). Nineteen of the students were Pākehā/European, three were Māori, four were Asian, and one was of Pasifika ancestry. The students were selected by their teachers from a range of stages on the New Zealand Number Framework. The Principal and Deputy Principal were key informants for the second part of the study, which explored the reasons for students' views.

Procedure

Students were interviewed individually in a quiet place away from the classroom. They were told that the interviewer was interested in finding out more about "how kids learn maths and how their teachers can help them" and "what kids themselves think about learning maths". Students were asked to comment on a range of topics, including the importance of working out problems mentally, of getting answers correct, and whether they thought there was only one way or several different ways of working out an answer. They were then asked the following questions and the reasons for their responses:

> Do you think it is important for you to know how other people get their answers?
> Is it important for you to be able to explain to other people how you worked out your answer?

Interviews were transcribed for later analysis. Once the interviews were complete, the school was asked to identify each student's current stage on the Number Framework used as part of the NDP assessment. This information was used to categorise students as *high* (at stage 6: Advanced Additive Part–Whole or above), *middle* (at stage 5: Early Additive Part–Whole), or *low* (at stage 4: Advanced Counting or below).

Results

As Young-Loveridge, Taylor, and Hawera's paper (p. 97 in this compendium) has shown, 21 out of 27 City students thought that it was important to know how other people worked out the answers to their mathematics problems. The remaining six were equally divided between disagreeing with the idea that knowing how others solve problems is important (three), and being unsure about what they thought (three). When the students were asked about the importance of explaining their own solution strategies to others, the level of agreement was also high (20 out of 27). Only two students seemed sure that explaining their thinking to others was not important, and the remaining five were unsure. When the students were asked to explain the reasons for their initial responses, City students gave very detailed answers. Their responses were organised into four major sections, as detailed below.

1. Knowing others' strategies is important

The responses of students from City School were notable, not just because more of them thought that knowing about others' strategies was important, but also because they were very articulate about their reasons for holding particular views. They tended to refer to multiple advantages of knowing about other students' strategies and were able to elaborate about classroom learning processes:

> Because it might help you in working out your ways, because you might be working out a really difficult way but you're not knowing it, and then somebody else shares something with the class, and then it would be really good because you would then find out that you might be able to use that way. (C1, girl, middle)

> Because it can help you grow and develop because if you are always just doing your way and never seeing anybody else's way of doing it … Sometimes when you are stuck, other people's ways can help you, 'cause one time … my way didn't work for it, and then I just sat there and thought about some other people's way and one person's way helped me solve that problem. (C2, boy, high)

> Because other people may have different ways and you can learn off them and you maybe get better and better. (C18, boy, low)

> It could help your learning by getting better, if they can show you how they got to work it out. Sometimes you can get very confused about what other people say, but it's good to hear their learning because sometimes it's most likely to help your learning as well. (C19, girl, low).

2. Knowing others' strategies is <u>not</u> important

Only three students from City School were clearly of the view that knowing about other people's solution strategies was not important. Two of them, assessed by their teachers as being low in mathematics (that is, at Stage 4 Advanced Counting), referred to privacy issues:

> Because it's their own business how they do it … It's not important for me because I know my own way to work things out. (C15, boy, low)

> Because it's their way they do it, you don't have to, 'cause it's really none of your business. (C26, girl, low)

3. Explaining one's strategies to others is important

The majority of students from City School thought that explaining their thinking was important and gave detailed explanations of their reasons for responding positively to the question. For example:

> Because it's good that I know how I worked out the answer first of all, and it's good 'cause I like sharing my ideas with other people and my point of view of how I can work it out, and so if I say my way and another person tells their way and their way's a bit easier, I can just try it their way, and then you get lots of different ways by telling your one, then other people go "I've got a simpler way of doing that", so it helps you to learn. (C2, boy, high)

> It's helping my learning as well, so it's good to share what your side of the story is. (C19, girl, low)

> Because sometimes people ask you how you did it, so you've got to know how you did it, so before you take their answer, you say the answer, you think about how you did it. (C20, boy, high)

4. Explaining one's strategies to others is <u>not</u> important

The two students who did not think it was important to explain their strategies to others talked about the importance of individuality as their reason:

> Because they could have their own way to work it out. (C15, boy, low)

City Students' Perceptions of the Numeracy Project

Although the interviewer had not set out to ask explicitly about students' experiences of the NDP, several students spontaneously referred to the project. A decision was made then to ask students to comment on: "What do you think about this ANP [Advanced Numeracy Project] programme you've been doing in maths?" Unlike same-age peers in a previous study (Young-Loveridge & Taylor, in press), City students were aware that they had been "doing" the NDP and were happy to comment.

> We've done ANP and that was fun because the best part I liked about it was that we could, we learnt more and that we had enough time to do what we wanted … We got to use blocks, we had heaps of maths games about it and we still play them now. (C15, boy, low)

> It has made a big difference … It's shown me easy ways to work things out and it's helped me to remember things and it's shown me some patterns and things. (C16, girl, high)

> I know about this ANP thing … I think it's good in some ways, but sometimes it can be a bit tricky and hard to understand, but in a good way, it's alright to learn. (C19, girl, low)

> It teaches you different strategies to answer a question and easier ways. (Asked if she thought it was better than the way she was doing before?) Yes and no, because there are some parts that sort of, don't exactly, they're just sort of lengthening the questions and yeah, it's quite good. (C21, girl, high)

> I think it's easier to learn things than with the old maths because I think more people are generally happy with that because it's sort of just fun and also it's got lots of good ways to work it out, like doubling up. (C23, boy, high)

> I think it's different and it makes your brain think a bit more about the equation rather than just doing it one way, it makes you think a bit more about it which is good I think, because you should really know a few ways to work it out or something like that. (C24, girl, middle)

> It helps way better. Way more … Well it actually tells you the questions and makes sense and so gives you a little bit of detail and doesn't really tell you much about it, you can really use it to help you. (C26, boy, middle)

> Well, it gets you learning and it shows you different ways of how you can add stuff. (C27, girl, low)

> I think it's cool because we get to use equipment and it makes maths easier. (C28, girl, middle)

The School's Perspective

Further information provided by the school helped us to make sense of the ideas that students shared with us. This information included discussions with the Principal and Deputy Principal about what they have been trying to achieve in their school and writing by the DP on aspects of the school's practices as part of her Masters thesis. Both stressed that they were speaking not as individuals but on behalf of the whole management team, which had spent considerable time brainstorming ideas as part of developing school policy and practice. The management team had previously been involved in a Ministry contract that focused on assessment and learning.

According to City staff, it is the culture and climate of the school that is its distinctive feature. They believe that "teachers feel supported in the school". They commented on the pride staff feel about their relationships with students: "We are keen to communicate with students openly and honestly." … "Students know the school listens to them – they trust us." They talked about both staff and students having an ethos of self-responsibility in the school, with the management team providing a model to all members of the school community. They remarked on the importance of having strong leadership within the school and about the fact that the school is a "non-judgemental environment" where staff can reflect on their practices honestly, with a view to addressing any problems as part of an ongoing process of improvement. They commented on the importance of distinguishing between responding to students' learning needs and delivering the curriculum. They made it clear that students' learning needs have a very high priority at City School. There was also discussion about the focus on the behaviour management of students, and the way that this has an impact on classroom learning. Previously, a more punitive model had been used, with negative consequences for unacceptable behaviour. A conscious decision had been made to take a more positive approach to behaviour, emphasising virtues, values, and overall social development, with the goal of fostering self-regulation of behaviour in students. This was described by City staff as a "solution-oriented focus".

Another important practice at City School was the use of "learning logs" (a book in which comments about a student's learning were recorded) as a way to get students to think about their learning. The students were helped to write about their "learning intentions" in the learning log, and later to select pieces of work to include in the learning log that showed how well they had met their learning intentions. Staff described the way that the students had to "talk about their learning and relate it back to their learning intentions".

A deliberate decision seems to have been made at City School to try to reduce the power imbalance in the relationships between teachers and students. As Cullingford (1995) has pointed out, "the difficulty for children is that schools automatically put all the power into the hands of teachers" (p. 2). Some schools, however, are moving away from having teachers hold an authoritarian role over their students towards building more collaborative relationships in which goals are negotiated with students. The emergence of the learning logs seems to have played an important role in this shift towards more democratic relationships with students. According to City staff:

> What the learning logs have sparked for us really is the importance of the teacher–student relationship and the power that teachers have traditionally held over students, and the ways we've been breaking that down, working on that, anyway. [We] think that's probably why you've had the sort of feedback from our students that you did.

It appeared that City School's involvement in the NDP had come at an opportune time (teachers at years 5 and 6 participated in 2004, and those at years 1–4 the previous year). The teachers had already established a climate within the school in which students were expected to converse with their teachers and with other students about many aspects of their learning, and this was happening across the whole curriculum. The idea of discussing ways of thinking and strategies for solving problems in mathematics was not new to teachers at City School, in contrast to many other teachers participating in the NDP. These kinds of conversations with students were already part of accepted practice in the school across all areas of the school curriculum. City staff spoke about the way that the philosophy behind the NDP fitted very well with what was already happening at City School in curriculum areas apart from mathematics.

> ANP and ENP have had a huge influence, and our staff love it, they just love it … It fits in with the way we work with kids.

An Independent Perspective on the School

A report from the Education Review Office (ERO), based on a review conducted just after we interviewed the students, was made available to the school earlier this year. The ERO report is useful here because it provides an independent view of the school that corroborates the perspectives of City staff as well as the students' views on what is happening in their school. Because it is a written report, it provides a permanent record of the practices evident at City School at the time of the review. The following excerpts from the ERO report helps to capture the character of City School. The ERO report commented on the learner-centred approach to student assessment, including the role of self-responsibility by students:

> A comprehensive range of assessment practices is well used to identify needs, monitor progress and report on achievement of individuals and groups of students. These practices involve teachers using a variety of standardised and diagnostic testing tools to accurately determine individual learning needs and inform planning; teachers making wide use of aspects of formative assessment to monitor learning and provide feedback to students; learning logs for all students which highlight learning intentions and reflect the use of self, peer and teacher and parent feedback; analysis of achievement information at syndicate level that is effectively used to develop annual action plans to target learning needs; and procedures through which parents are well informed about learning progress and achievement levels of their children.
>
> A feature of the school is the emphasis on student achievement and strategies to improve learning for all students. Teachers are setting challenging benchmarks for student achievement in literacy and numeracy … Students are achieving well and school benchmarks have been raised to recognise this achievement and progress.
>
> A central focus of school operations is strengthening students' ability to learn and achieve. An emphasis on the 4R's of resourcefulness, resilience, reflection and relationships encourages students to be actively involved in the learning process. Students take responsibility for their own learning and supporting the learning of others.

Quality teaching was also referred to in the report:

> High quality teaching is evident school wide with several examples of outstanding practice. Teachers are hard working, committed and plan and organise programme information by an analysis of relevant student achievement data and current best practice. Teachers are encouraged to be reflective practitioners, and to focus on teaching to the learning needs of individual students. Effective professional learning opportunities and a rigorous performance management system support teachers to continually improve teaching practice.
>
> Planning, classroom organisation and the effective use of a variety of teaching strategies are informed by relevant student achievement data and current best practice and research. Appropriate and effective grouping based on the identified needs of students is evident particularly in literacy and numeracy.

The report also commented on the overall climate within the school and the importance of relationships between students and their teachers.

> A positive climate prevails across the school, with warm, constructive interactions between and among teachers and students. An effective behaviour management system based on courtesy, consideration, co-operation and common sense underpins relationships throughout the school. Students are well taught in stimulating, well-resourced classrooms and demonstrate high levels of on-task behaviour.
>
> The school's focus on the 4 C's of courtesy, consideration, co-operation and common sense underpin positive relationships between teachers and students. Students are actively engaged in learning.

Discussion

The comments of the Principal and Deputy Principal, as well as statements made in the report by ERO, all help to put the responses of City students into a wider context. It quickly became clear from discussions that staff at City School had been working over many years to shape its philosophy and practice and to ensure that student learning was its central focus. Threading through the comments of City students and their teachers was a clear commitment to democratic decision-making processes within the school. City students were expected to show initiative and responsibility in relation to their own learning. Students tended to see their teachers in a "mentor" role, as sources of help and assistance who could be consulted whenever the need arose, rather than as authorities who disseminate knowledge (Blumenfeld et al., 1997; Nuthall, 1997). There was a strong sense of agency evident for both teachers and students, and this was accompanied by feelings of ownership about the learning, particularly for students. Being members of several professional learning communities committed to improving students' learning and raising achievement (for example, school, GE team) has probably further enhanced the professional learning of City staff (Camburn, Rowan, & Taylor, 2003; Copland, 2003; Little et al., 2003).

Ideally, it would have been good to make links between what happened in classrooms and improvements on the Number Framework, as assessed by the diagnostic interview. Unfortunately, changes in staffing meant that a decision was made not to gather final assessment data from two of the year 5–6 classes. The result was that there was complete data from only half of the initial cohort, and it did not make sense to proceed with the quantitative analysis.

Further insight into the way City School "does things" could have been gained by doing classroom observations as well as having conversations with the students. This would have allowed the interviewer to talk to the students about specific activities and events that had happened within the classroom mathematics programme and to explore the students' perspectives on those activities and events, as well as asking about the "general". It might also have been valuable to talk to younger students (that is, years 3–4) about their views.

Some might argue that City School is atypical, and hence the findings cannot be generalised to other New Zealand schools. However, City School provides a powerful and telling example of just what can be achieved in a school when the conditions are favourable and staff are committed to changing their practices in order to improve learning. What is clear from this account of City School is that bringing about change in how schools teach mathematics is a hugely complex and challenging issue. Consideration needs to be given not just to what happens in the classroom during a mathematics session, but also to the wider perspective of the school as a whole, including the overall climate and practices of the school (Hiebert et al., 1997).

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Appendix A (Patterns of Performance and Progress)

Percentages of students in each year group as a function of gender, ethnicity, school decile band, and framework stage for each domain on the Diagnostic Interview (NumPA) in 2004

Year	**1**	**2**	**3**	**ENP**	**4**	**5**	**6**	**ANP**	**7**	**8**	**INP**
(Number of students)	*(7793)*	*(8197)*	*(8516)*	***(24507)***	*(10013)*	*(9868)*	*(9959)*	***(29840)***	*(8374)*	*(7306)*	***(15680)***
Gender											
Girls	48.3	48.5	48.6	**48.5**	49.1	48.2	48.7	**48.7**	49.9	50.7	**50.3**
Boys	51.7	51.5	51.4	**51.5**	50.9	51.8	51.3	**51.3**	50.1	49.3	**49.7**
Ethnicity											
European	64.7	62.5	61.9	**63.0**	59.7	59.1	59.2	**59.3**	58.9	58.2	**58.6**
Māori	17.3	17.6	18.4	**17.8**	18.7	19.5	18.2	**18.8**	24.5	24.6	**24.6**
Pasifika	8.5	9.1	10.2	**9.3**	11.3	10.9	11.2	**11.2**	9.5	10.0	**9.7**
Asian	5.8	6.3	5.6	**5.9**	5.9	5.9	6.4	**6.1**	3.6	3.4	**3.5**
Other	3.8	4.5	4.0	**4.1**	4.4	4.7	4.9	**4.7**	3.6	3.8	**3.7**
School Decile Band											
Low (1–3)	20.1	22.1	25.8	**22.8**	26.6	29.4	29.7	**28.5**	28.4	31.5	**29.8**
Medium (4–7)	38.3	36.4	35.7	**36.8**	37.8	38.9	37.4	**38.1**	49.3	48.5	**48.9**
High (8–10)	41.6	41.5	38.4	**40.5**	35.6	31.8	32.8	**33.4**	22.2	20.0	**21.2**
Addition/Subtraction											
Initial Stage											
0: Emergent	15.7	3.2	1.7	**6.6**	1.7	0.9	0.9	**1.2**	0.9	0.7	**0.8**
1: One-to-One Counting	29.8	14.6	4.9	**16.1**	1.6	0.7	0.4	**0.9**	0.3	0.2	**0.2**
2 Count All with materials	43.7	39.5	17.8	**33.3**	5.3	2.2	1.1	**2.9**	1.0	0.4	**0.7**
3 Count All with imaging	8.5	20.2	14.2	**14.4**	5.5	2.9	1.3	**3.3**	1.2	0.8	**1.0**
4: Advanced Counting	2.2	19.4	46.3	**23.3**	48.9	40.7	34.2	**41.3**	27.4	21.6	**24.7**
5: Early Additive P–W	0.2	3.0	14.5	**6.1**	32.6	43.6	46.6	**40.9**	46.2	44.4	**45.4**
6: Adv. Additive P–W	0.0	0.2	0.6	**0.3**	4.3	9.0	15.5	**9.6**	23.0	31.9	**27.1**
Final Stage											
0: Emergent	2.0	1.0	0.1	**1.0**	0.2	0.2	0.1	**0.1**	0.3	0.3	**0.3**
1: One-to-One Counting	9.6	2.7	0.9	**4.3**	0.3	0.1	0.1	**0.2**	0.1	0.0	**0.1**
2 Count All with materials	42.6	16.8	4.9	**20.9**	1.6	0.8	0.3	**0.9**	0.4	0.1	**0.3**
3 Count All with imaging	25.9	20.2	7.2	**17.5**	2.3	1.1	0.5	**1.3**	0.6	0.4	**0.5**
4: Advanced Counting	17.7	44.0	47.2	**36.8**	32.5	22.1	16.2	**23.6**	14.0	8.3	**11.4**
5: Early Additive P–W	2.1	14.4	35.4	**17.8**	49.7	52.3	46.3	**49.4**	43.4	35.6	**39.8**
6: Adv. Additive P–W	0.1	0.8	4.3	**1.8**	13.3	23.4	36.5	**24.4**	41.2	55.1	**47.7**
Multiplication/Division											
Initial Stage											
Not given	98.6	84.0	46.7	**75.7**	17.8	7.9	4.7	**10.2**	3.9	1.9	**3.0**
2–3: Count All	1.1	9.2	21.9	**11.0**	17.3	11.7	7.9	**12.3**	4.9	3.5	**4.2**
4: Advanced Counting	0.3	5.8	24.9	**10.7**	43.3	39.8	31.2	**38.1**	24.1	17.0	**20.8**
5: Early Additive P–W		0.9	5.4	**2.2**	15.4	25.0	29.5	**23.2**	30.6	30.2	**30.4**
6: Adv. Additive P–W		0.1	1.1	**0.4**	5.5	13.3	21.5	**13.4**	28.3	32.8	**30.4**
7: Adv. Mult. P–W		0.0	0.0	**0.0**	0.8	2.2	5.2	**2.8**	8.3	14.5	**11.2**

Year	1	2	3	ENP	4	5	6	ANP	7	8	INP
Muliplication/Division cont.	*(7793)*	*(8197)*	*(8516)*	***(24507)***	*(10013)*	*(9868)*	*(9959)*	***(29840)***	*(8374)*	*(7306)*	***(15680)***
Final Stage											
Not given	87.5	53.6	19.8	**52.7**	6.8	3.7	2.3	**4.2**	1.5	0.8	**1.2**
2–3: Count All	5.8	12.1	12.0	**10.1**	6.1	3.2	2.0	**3.8**	1.9	1.1	**1.5**
4: Advanced Counting	6.0	27.4	43.1	**26.1**	37.3	24.8	15.3	**25.8**	12.1	7.3	**9.9**
5: Early Additive P–W	0.6	5.9	17.5	**8.2**	28.7	30.0	26.2	**28.3**	24.1	19.7	**22.0**
6: Adv. Additive P–W	0.1	1.0	6.8	**2.7**	17.4	29.6	36.1	**27.7**	38.0	37.3	**37.7**
7: Adv. Mult. P–W		0.1	0.6	**0.3**	3.7	8.7	18.2	**10.2**	22.4	33.8	**27.7**
Proportion/Ratio											
Initial Stage											
Not given	98.7	84.2	46.8	**75.8**	18.5	8.1	4.9	**10.5**	4.5	2.5	**3.6**
1: Unequal Sharing	0.8	6.7	17.0	**8.4**	15.1	11.1	8.1	**11.4**	5.4	3.5	**4.5**
2–4: Equal Sharing	0.5	8.6	32.9	**14.5**	50.5	49.8	41.1	**47.1**	32.7	26.3	**29.7**
5: Early Additive P–W	0.0	0.4	3.1	**1.2**	12.7	22.4	27.7	**20.9**	29.2	28.2	**28.7**
6 Adv. Additive P–W		0.0	0.2	**0.1**	2.5	6.8	13.1	**7.5**	19.1	24.0	**21.4**
7: Adv. Mult. P–W		0.0	0.0	**0.0**	0.6	1.8	4.6	**2.4**	8.0	12.6	**10.2**
8: Adv. Proportional P–W					0.1	0.1	0.5	**0.2**	1.0	2.8	**1.8**
Final Stage											
Not given	87.6	53.6	20.6	**53.0**	7.1	4.2	2.4	**4.5**	1.9	1.0	**1.4**
1: Unequal Sharing	2.9	6.3	6.6	**5.3**	3.8	1.7	1.2	**2.2**	1.1	0.6	**0.9**
2–4: Equal Sharing	9.1	35.7	52.4	**33.0**	46.1	32.5	22.0	**33.6**	19.1	13.1	**16.3**
5: Early Additive P–W	0.4	3.8	16.3	**7.0**	29.1	34.8	32.7	**32.2**	28.8	23.5	**26.3**
6 Adv. Additive P–W	0.1	0.5	3.5	**1.4**	10.4	18.9	25.2	**18.2**	26.8	29.0	**27.9**
7: Adv. Mult. P–W		0.1	0.5	**0.2**	3.2	7.4	14.4	**8.3**	17.9	23.4	**20.5**
8: Adv. Proportional P–W			0.0	**0.0**	0.3	0.5	2.2	**1.0**	4.3	9.3	**6.7**
FNWS											
Initial Stage											
0 Emergent FNWS	11.2	2.4	1.4	**4.9**	1.9	0.7	0.6	**1.1**	1.3	1.2	**1.2**
1 Initial FNWS to 10	27.8	7.7	2.1	**12.2**	0.6	0.2	0.1	**0.3**	0.1	0.0	**0.1**
2 up to 10	33.1	20.5	6.1	**19.5**	1.6	0.5	0.3	**0.8**	0.3	0.2	**0.2**
3 up to 20	19.5	27.7	14.1	**20.4**	4.5	2.4	1.2	**2.7**	1.1	0.5	**0.8**
4 up to 100	7.6	34.8	48.4	**30.9**	36.6	23.4	14.0	**24.7**	10.2	6.5	**8.5**
5 up to 1000	0.8	6.6	26.6	**11.7**	50.3	61.8	63.0	**58.4**	58.0	51.2	**54.8**
6 up to 1,000,000	0.0	0.2	1.2	**0.5**	4.5	11.0	20.7	**12.1**	29.1	40.4	**34.3**
Final Stage											
0 Emergent FNWS	1.9	1.9	0.9	**1.5**	1.4	0.6	0.7	**0.9**	0.8	0.7	**0.7**
1 Initial FNWS to 10	5.4	0.8	0.3	**2.1**	0.1	0.1	0.0	**0.1**	0.0	0.0	**0.0**
2 up to 10	16.5	4.0	1.0	**6.9**	0.3	0.2	0.1	**0.2**	0.1	0.0	**0.1**
3 up to 20	33.9	15.7	4.5	**17.6**	1.3	0.7	0.4	**0.8**	0.4	0.2	**0.3**
4 up to 100	35.9	48.5	34.4	**39.6**	17.6	8.7	5.0	**10.5**	4.1	2.0	**3.1**
5 up to 1000	6.2	27.4	51.5	**29.0**	62.5	58.8	47.6	**56.3**	40.7	29.5	**35.5**
6 up to 1 000 000	0.2	1.7	7.4	**3.2**	16.8	30.9	46.4	**31.4**	53.8	67.6	**60.2**

Year	1	2	3	ENP	4	5	6	ANP	7	8	INP
	(7793)	*(8197)*	*(8516)*	***(24507)***	*(10013)*	*(9868)*	*(9959)*	***(29840)***	*(8374)*	*(7306)*	***(15680)***
Initial Stage											
0 Emergent BNWS	38.3	8.2	2.6	**15.8**	2.3	1.0	0.8	**1.4**	1.6	1.4	**1.5**
1 Initial BNWS from 10	21.3	13.1	4.7	**12.8**	1.2	0.4	0.1	**0.6**	0.2	0.1	**0.1**
2 back from 10	29.7	33.6	15.2	**26.0**	4.6	1.7	1.0	**2.4**	0.4	0.4	**0.4**
3 back from 20	6.7	15.7	12.1	**11.6**	5.5	2.7	1.2	**3.1**	1.4	0.5	**1.0**
4 back from 100	3.5	22.9	38.4	**22.1**	32.3	24.2	16.4	**24.3**	13.0	8.8	**11.0**
5 back from 1000	0.5	6.3	25.9	**11.3**	49.7	59.6	61.1	**56.8**	54.9	49.5	**52.4**
6 back from 1 000 000	0.0	0.2	1.2	**0.5**	4.3	10.4	19.5	**11.4**	28.4	39.3	**33.5**
Final Stage											
0 Emergent BNWS	6.1	2.5	1.2	**3.2**	1.5	0.8	0.8	**1.1**	1.3	1.4	**1.4**
1 Initial BNWS from 10	10.1	2.7	0.9	**4.4**	0.3	0.2	0.1	**0.2**	0.1	0.0	**0.1**
2 back from 10	31.7	12.3	3.5	**15.4**	0.9	0.4	0.2	**0.5**	0.1	0.1	**0.1**
3 back from 20	22.4	14.8	6.0	**14.1**	2.1	0.8	0.5	**1.1**	0.5	0.2	**0.4**
4 back from 100	23.6	39.7	32.3	**32.0**	18.5	10.7	6.0	**11.8**	5.2	3.0	**4.2**
5 back from 1000	5.9	26.2	49.0	**27.7**	60.2	57.7	47.6	**55.2**	39.8	29.2	**34.8**
6 back from 1 000 000	0.2	1.7	7.2	**3.1**	16.5	29.4	44.8	**30.2**	52.9	66.1	**59.1**
Numeral ID											
Initial Stage											
N/A	2.0	15.0	54.1	**24.5**	81.6	91.8	94.8	**89.3**	94.1	94.5	**94.3**
0 Emergent	27.5	3.5	0.4	**10.1**	0.5	0.4	0.4	**0.4**	0.2	0.3	**0.2**
1 Numerals to 10	36.0	14.8	2.9	**17.4**	0.5	0.2	0.1	**0.3**	0.1	0.0	**0.1**
2 Numerals to 20	17.0	15.0	3.9	**11.8**	1.2	0.4	0.2	**0.6**	0.2	0.2	**0.2**
3 Numerals to 100	15.1	37.8	21.3	**24.8**	5.6	1.9	0.8	**2.8**	0.9	0.6	**0.8**
4 Numerals to 1000	2.3	13.8	17.4	**11.4**	10.7	5.3	3.8	**6.6**	4.5	4.4	**4.5**
Final Stage											
N/A	8.7	35.7	73.6	**40.3**	89.5	94.8	96.3	**93.5**	95.2	95.3	**95.2**
0 Emergent	4.0	0.4	0.1	**1.4**	0.4	0.3	0.2	**0.3**	0.3	0.2	**0.2**
1 Numerals to 10	13.7	2.7	0.5	**5.4**	0.2	0.1	0.0	**0.1**	0.0		**0.0**
2 Numerals to 20	17.0	4.9	1.0	**7.4**	0.3	0.2	0.1	**0.2**	0.1	0.0	**0.0**
3 Numerals to 100	41.2	25.8	7.4	**24.3**	1.7	0.7	0.3	**0.9**	0.4	0.2	**0.3**
4 Numerals to 1000	15.3	30.6	17.4	**21.1**	7.9	4.0	3.1	**5.0**	4.0	4.3	**4.2**
Fractions											
Initial Stage											
Not given	98.7	84.3	45.6	**75.4**	17.3	7.1	3.6	**9.4**	4.0	2.2	**3.2**
2–3 Unit fracts not recog	1.2	14.9	46.5	**21.5**	47.4	37.9	28.0	**37.8**	16.9	11.8	**14.5**
4 Unit fractions recog.	0.1	0.7	5.9	**2.3**	21.1	27.9	30.2	**26.4**	27.4	25.1	**26.3**
5 Order unit fractions	0.0	0.1	1.9	**0.7**	12.8	23.4	29.6	**21.9**	34.5	33.8	**34.2**
6 Coord. num'r/denom'r		0.0	0.1	**0.0**	0.9	2.5	5.4	**2.9**	10.9	15.4	**13.0**
7 Equivalent fractions			0.0	**0.0**	0.4	0.9	2.3	**1.2**	4.4	7.6	**5.9**
8 Order fractions					0.1	0.3	0.8	**0.4**	1.9	4.0	**2.9**

Year	**1**	**2**	**3**	**ENP**	**4**	**5**	**6**	**ANP**	**7**	**8**	**INP**
Fractions cont.	*(7793)*	*(8197)*	*(8516)*	***(24507)***	*(10013)*	*(9868)*	*(9959)*	***(29840)***	*(8374)*	*(7306)*	***(15680)***
Final Stage											
Not given	87.1	54.2	21.0	**53.3**	7.9	3.9	1.9	**4.6**	2.1	1.3	**1.7**
2–3 Unit fracts not recog.	8.8	23.0	25.6	**19.4**	15.4	9.0	6.2	**10.2**	4.7	2.7	**3.8**
4 Unit fractions recog.	2.8	15.4	27.8	**15.7**	27.6	22.6	17.1	**22.5**	17.0	12.0	**14.7**
5 Order unit fractions	0.6	7.1	23.6	**10.8**	40.5	45.8	42.9	**43.0**	36.9	31.2	**34.3**
6 Coord. num'r/denom'r	0.1	0.3	1.8	**0.7**	6.5	13.0	19.1	**12.9**	20.0	22.5	**21.2**
7 Equivalent fractions			0.2	**0.1**	1.6	4.1	8.3	**4.7**	12.0	17.5	**14.6**
8 Order fractions		0.0	0.1	**0.0**	0.5	1.6	4.5	**2.2**	7.2	12.7	**9.8**
Place Value											
Initial Stage											
0–1 Emergent	50.8	22.0	8.5	**26.5**	4.6	1.8	1.2	**2.5**	5.0	6.3	**5.6**
2–3 One as a unit	47.3	61.0	45.0	**51.1**	21.9	11.1	6.0	**13.0**	5.5	2.8	**4.2**
4 Ten as counting unit	1.9	15.7	38.5	**19.3**	47.4	42.5	32.7	**40.9**	23.7	17.4	**20.8**
5 Tens in nos. to 1000	0.0	1.1	7.2	**2.9**	21.7	34.2	39.1	**31.6**	34.7	32.9	**33.9**
6 Ts, Hs, Th whole nos.	0.0	0.1	0.8	**0.3**	4.1	9.7	17.9	**10.5**	24.2	29.6	**26.7**
7 10ths in decimals/order			0.0	**0.0**	0.3	0.7	2.8	**1.3**	5.7	8.0	**6.8**
8 Decimal conversion		0.0		**0.0**	0.1	0.1	0.3	**0.2**	1.2	2.9	**2.0**
Final Stage											
0–1 Emergent	16.1	6.7	2.0	**8.1**	2.1	0.8	0.6	**1.2**	2.0	2.6	**2.3**
2–3 One as a unit	62.2	40.5	21.1	**40.7**	8.6	3.8	2.2	**4.9**	2.0	0.9	**1.5**
4 Ten as counting unit	20.7	44.0	47.1	**37.7**	34.3	22.9	14.4	**23.9**	11.2	6.7	**9.1**
5 Tens in nos. to 1000	0.8	7.5	24.4	**11.3**	40.1	43.9	38.2	**40.7**	31.0	24.5	**28.0**
6 Ts, Hs, Th whole nos.	0.1	1.2	5.2	**2.2**	13.3	23.8	32.1	**23.1**	34.5	36.4	**35.4**
7 10ths in decimals/order	0.0	0.0	0.2	**0.1**	1.2	4.1	10.2	**5.2**	14.0	18.8	**16.2**
8 Decimal conversion			0.0	**0.0**	0.3	0.6	2.3	**1.1**	5.3	10.0	**7.5**
Basic Facts											
Initial Stage											
0–1 Non-grouping w 5	94.8	79.5	46.9	**73.0**	22.3	10.3	6.5	**13.1**	12.2	11.5	**11.8**
2–3 Within/w. 5, w'in 10	4.5	14.8	25.6	**15.3**	23.1	14.7	9.3	**15.7**	6.6	4.3	**5.5**
4 Add'n w. 10s/doubles	0.7	5.4	23.5	**10.2**	35.3	30.7	23.2	**29.7**	16.0	11.9	**14.1**
5 Addition facts	0.0	0.2	3.3	**1.2**	15.6	29.2	30.4	**25.1**	25.3	24.2	**24.8**
6 Subtr'n & mult'n facts		0.0	0.6	**0.2**	3.0	12.8	24.1	**13.3**	29.0	31.8	**30.3**
7 Division facts		0.0	0.0	**0.0**	0.6	2.1	5.6	**2.8**	9.9	12.7	**11.2**
8 Common factors/multiple					0.0	0.2	0.9	**0.4**	1.0	3.7	**2.2**
Final Stage											
0–1 Non-grouping w 5	61.7	33.5	13.6	**35.5**	6.9	2.6	1.9	**3.8**	5.8	5.5	**5.7**
2–3 Within/w. 5, w'in 10	28.1	29.5	18.4	**25.2**	9.3	5.5	3.1	**5.9**	2.3	1.3	**1.9**
4 Add'n w. 10s/doubles	9.7	31.3	43.0	**28.5**	32.4	18.9	10.9	**20.8**	8.2	5.2	**6.8**
5 Addition facts	0.4	5.0	20.0	**8.8**	34.0	35.5	27.9	**32.5**	22.0	15.9	**19.2**
6 Subtr'n & mult'n facts	0.1	0.6	4.6	**1.8**	14.0	27.3	34.2	**25.1**	32.3	32.0	**32.1**
7 Division facts	0.0	0.0	0.4	**0.2**	2.9	8.9	17.7	**9.8**	23.3	26.6	**24.9**
8 Common factors/multiple			0.0	**0.0**	0.5	1.3	4.4	**2.1**	6.1	13.4	**9.5**

Appendix B (Patterns of Performance and Progress)

Table B1

Percentages of Year 0–8 Students as a Function of Gender, Ethnicity, and School Decile Band in 2004 and 2003

	Gender		Ethnicity					Decile Band		
Year	Boys	Girls	European	Māori	Pasifika	Asian	Other	Low	Mid	High
2004	51.0	49.0	60.4	19.7	10.2	5.4	4.3	26.8	40.0	33.2
2003	51.0	49.0	57.8	23.6	9.7	4.7	4.1	35.6	37.9	26.4

Total number of students in 2004 = 70 027
Total number of students in 2003 = 138 829

Table B2

Percentages of Year 0–8 Students at Each Framework Stage for Addition/Subtraction as a Function of Gender, Ethnicity, and School Decile Band in 2004 and 2003

	Gender		Ethnicity				Decile Band		
Year	Boys	Girls	European	Māori	Pasifika	Asian	Low	Mid	High
2004									
	(35740)	(34286)	(42331)	(13801)	(7120)	(3794)	(18132)	(27064)	(22488)
Initial Stage									
0–3	29.1	28.4	28.0	29.7	32.3	26.6	30.2	27.0	30.0
4 AC	28.4	34.3	29.3	34.9	39.4	25.5	35.9	30.6	29.0
5 EA	30.1	29.4	31.2	27.6	23.6	33.0	26.5	31.3	30.4
6 AA	12.5	7.9	11.5	7.8	4.7	14.9	7.5	11.1	10.6
Final Stage									
0–3	16.8	16.5	16.0	18.0	19.7	13.2	17.3	16.5	16.7
4 AC	23.3	27.7	23.5	28.5	33.9	21.3	30.3	23.9	24.2
5 EA	35.1	37.3	36.6	36.1	34.3	34.7	35.3	36.9	36.1
6 AA	24.8	18.5	23.8	17.4	12.1	30.7	17.0	22.7	23.0
2003									
	(70823)	(68004)	(80249)	(32784)	(13523)	(6566)	(48063)	(51187)	((35648)
Initial Stage									
0–3	32.8	32.9	30.6	35.0	42.8	28.5	36.3	30.9	31.7
4 AC	29.5	35.1	30.8	35.6	36.1	26.5	35.0	32.0	29.0
5 EA	26.7	25.0	28.1	23.6	17.3	28.0	22.3	27.4	28.1
6 AA	11.0	7.0	10.6	5.8	3.8	16.9	6.4	9.7	11.2
Final Stage									
0–3	20.6	20.9	18.3	23.5	31.0	16.3	24.8	19.3	17.7
4 AC	23.9	28.3	24.3	28.3	32.3	22.0	29.0	24.9	23.9
5 EA	34.3	35.1	36.2	34.4	27.7	33.1	32.2	36.0	36.3
6 AA	21.2	15.7	21.2	13.8	8.9	28.6	14.0	19.8	22.1

Table B3

Percentages of Year 0–8 Students at Each Framework Stage for Multiplication/Division as a Function of Gender, Ethnicity, and School Decile Band in 2004

	Gender		Ethnicity				Decile Band		
Year	Boys	Girls	European	Māori	Pasifika	Asian	Low	Mid	High
Initial Stage									
Not Given	31.8	31.2	31.1	31.6	34.9	29.1	32.5	30.0	33.0
2–3	9.5	10.6	9.0	11.4	14.3	9.2	12.0	9.4	9.6
4 AC	23.2	26.2	23.5	27.6	27.8	20.8	27.2	24.4	23.1
5 EA	17.0	17.9	18.1	16.9	14.6	18.3	16.2	18.5	17.2
6 AA	13.8	11.5	14.0	10.2	7.4	15.8	9.7	13.7	13.2
7 AM	4.7	2.6	4.3	2.3	1.1	6.7	2.4	4.1	3.9
Final Stage									
Not Given	20.5	20.5	20.2	21.0	23.4	16.7	20.4	20.7	20.8
2–3	5.2	5.8	4.7	6.5	8.0	5.4	6.8	4.7	5.3
4 AC	21.1	23.6	20.8	25.1	28.0	18.5	25.7	21.0	21.8
5 EA	18.9	20.9	19.5	20.9	21.1	18.6	21.1	19.7	19.2
6 AA	21.7	20.6	22.6	19.2	15.5	23.8	18.9	22.5	21.0
7 AM	12.6	8.6	12.2	7.5	4.0	17.0	7.0	11.4	11.9

Table B4

Percentages of Year 0–8 Students at Each Framework Stage for Proportion/Ratio as a Function of Gender, Ethnicity, and School Decile Band in 2004

	Gender		Ethnicity				Decile Band		
Year	Boys	Girls	European	Māori	Pasifika	Asian	Low	Mid	High
Initial Stage									
Not Given	32.1	31.5	31.4	32.2	35.3	29.3	33.1	30.2	33.2
1 Unequal	9.3	8.4	7.9	9.9	11.2	10.0	10.1	8.7	8.4
2–4 Equal	30.0	33.7	30.6	34.2	36.8	28.4	34.8	31.2	30.3
5 EA	15.7	15.9	16.4	15.8	12.1	15.0	14.6	16.7	15.3
6 AA	8.3	7.7	9.0	6.0	3.8	11.1	5.6	9.0	8.3
7 AM	3.9	2.6	4.1	1.7	0.7	4.8	1.5	3.7	3.9
8 AP	0.7	0.3	0.6	0.2	0.1	1.4	0.3	0.5	0.6
Final Stage									
Not Given	20.8	20.8	20.5	21.2	23.9	17.1	20.6	20.8	21.1
1 Unequal	3.3	2.7	2.5	4.0	3.6	3.7	3.7	2.7	2.9
2–4 Equal	28.3	30.8	27.6	32.4	37.0	26.6	33.9	28.2	28.4
5 EA	21.4	22.8	21.8	23.6	22.3	19.8	23.3	22.2	20.9
6 AA	14.6	14.4	15.5	12.7	10.1	17.3	12.8	15.2	14.7
7 AM	9.2	7.1	9.8	5.1	2.8	11.6	4.7	8.7	10.0
8 AP	2.5	1.4	2.3	0.9	0.3	3.9	1.0	2.1	2.1

Table B5

Percentages of Students at Framework Stages for Addition/Subtraction at the End of the Project as a Function of Ethnicity and Gender in 2004 and 2003

	European		Māori		Pasifika		Asian	
Final Stage	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
				2004				
No. of students	*(21391)*	*(20940)*	*(7212)*	*(6589)*	*(3605)*	*(3515)*	*(1969)*	*(1825)*
0–3	16.1	16.0	18.5	17.5	20.7	18.6	13.0	13.4
4 AC	21.1	26.0	26.9	30.1	32.6	35.2	18.1	24.8
5 EA	35.3	38.0	35.4	36.8	34.1	34.5	33.0	36.6
6 AA	27.6	20.0	19.1	15.6	12.6	11.6	35.9	25.2
				2003				
No. of students	*(42518)*	*(39556)*	*(17043)*	*(15741)*	*(6782)*	*(6741)*	*(3382)*	*(3184)*
0–3	17.8	18.7	23.7	23.2	31.7	30.3	15.9	16.6
4 AC	21.6	27.1	26.8	29.9	31.3	33.4	20.0	24.1
5 EA	35.6	36.8	34.4	34.5	27.8	27.6	32.4	33.8
6 AA	24.9	17.4	15.1	12.4	9.1	8.7	31.6	25.4

Table B6

Percentages of Students at Framework Stages for Multiplication/Division at the End of the Project as a Function of Ethnicity and Gender in 2004

	European		Māori		Pasifika		Asian	
Final Stage	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Not Given	20.2	20.3	21.2	20.7	24.4	22.5	16.3	17.2
2–3	4.3	5.1	6.5	6.5	7.4	8.6	5.1	5.8
4 AC	19.1	22.6	24.7	25.4	28.4	27.7	16.5	20.7
5 EA	18.5	20.5	19.7	22.1	20.6	21.5	16.7	20.6
6 AA	23.5	21.7	19.2	19.1	15.0	16.0	23.9	23.7
7 AM	14.4	9.9	8.6	6.2	4.3	3.8	21.6	12.1

Table B7

Percentages of Students at Framework Stages for Proportion/Ratio at the End of the Project as a Function of Ethnicity and Gender in 2004

	European		Māori		Pasifika		Asian	
Final Stage	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Not Given	20.4	20.6	21.4	21.0	25.1	22.8	16.5	17.7
1 Unequal	2.7	2.2	4.3	3.8	3.8	3.4	3.8	3.6
2–4 Equal	26.0	29.3	32.5	32.3	36.1	38.0	24.8	28.5
5 EA	21.1	22.5	22.4	25.0	22.2	22.4	18.7	20.9
6 AA	15.8	15.1	12.5	12.9	9.3	10.8	17.5	17.1
7 AM	11.0	8.5	5.8	4.3	3.2	2.4	13.3	9.8
8 AP	3.0	1.7	1.2	0.7	0.4	0.2	5.4	2.3

Table B8

Percentages of Students Who Progressed to a Higher Stage for Addition/Subtraction as a Function of Initial Stage, Gender, Ethnicity, and School Decile Band (2004 and 2003)

	Gender		Ethnicity				Decile Band		
Initial Stage	Boys	Girls	European	Māori	Pasifika	Asian	Low	Medium	High
				2004					
Stages 0–3	(10397)	(9742)	(11869)	(4094)	(2300)	(1008)	(5476)	(7313)	(6741)
To stage 4	34.3	36.2	35.3	34.6	34.4	37.0	35.9	33.1	37.2
To stage 5	8.2	6.2	7.6	5.5	5.5	12.5	6.9	6.6	7.2
To stage 6	0.8	0.5	0.7	0.6	0.1	1.4	1.4	0.2	0.3
Total	*43.3*	*42.9*	*43.6*	*40.7*	*40.0*	*50.9*	*44.2*	*39.9*	*44.7*
Stage 4	(10143)	(11760)	(12397)	(4813)	(2806)	(966)	(6505)	(8286)	(6520)
To stage 5	48.0	46.1	49.3	44.3	40.5	48.0	42.6	47.7	50.0
To stage 6	5.6	4.2	5.0	4.6	3.3	7.5	4.4	4.5	5.6
Total	*53.6*	*50.3*	*54.3*	*48.9*	*43.8*	*53.5*	*47.0*	*52.2*	*55.6*
Stage 5	(10747)	(10066)	(13198)	(3815)	(1682)	(1253)	(4796)	(8460)	(6839)
To stage 6	35.7	31.3	34.8	29.4	27.3	41.6	30.4	33.1	35.5
				2003					
Stages 0-3	(23215)	(22364)	(24572)	(11471)	(5785)	(1870)	(17424)	(15801)	(11283)
To stage 4	32.0	33.8	34.5	30.4	28.9	35.9	30.8	32.5	37.1
To stage 5	6.9	4.8	6.5	4.9	4.2	7.7	4.8	6.2	7.1
To stage 6	0.6	0.5	0.7	0.3	0.2	1.9	0.2	0.4	0.9
Total	*39.5*	*39.1*	*41.7*	*35.6*	*33.3*	*45.5*	*35.8*	*39.1*	*45.1*
Stage 4	(20907)	(23902)	(24685)	(11679)	(4876)	(1743)	(16830)	(16404)	(10340)
To stage 5	50.3	47.2	51.5	46.5	39.1	50.3	44.4	50.1	54.1
To stage 6	4.8	4.0	4.6	4.2	3.2	6.0	4.2	4.4	4.7
Total	*55.1*	*51.2*	*56.1*	*50.7*	*42.3*	*56.3*	*48.6*	*54.5*	*58.8*
Stage 5	(18895)	(16995)	(22516)	(7734)	(2346)	(1841)	(10732)	(14020)	(10032)
To stage 6	33.9	29.9	33.7	28.7	24.1	35.7	29.3	32.8	34.0

Table B9

Percentages of Students Who Progressed to a Higher Stage for Addition/Subtraction as a Function of Initial Stage, Ethnicity, and Gender in 2004 and 2003

	European		Māori		Pasifika		Asian	
Initial Stage	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
				2004				
Stages 0–3	(6081)	(5788)	(2178)	(1916)	(1205)	(1095)	(509)	(499)
To stage 4	34.4	36.2	34.1	35.2	33.4	35.6	34.6	39.5
To stage 5	9.0	6.2	5.9	5.0	5.4	5.7	14.3	10.6
To stage 6	1.0	0.4	0.6	0.6	0.1	0.2	1.4	1.4
Total	*44.4*	*42.8*	*40.6*	*40.8*	*38.9*	*41.5*	*50.3*	*51.5*
Stages 4	(5563)	(6834)	(2352)	(2461)	(1340)	(1466)	(433)	(533)
To stage 5	51.0	47.9	44.6	44.1	41.2	39.9	49.0	47.3
To stage 6	6.1	4.1	5.1	4.1	2.8	3.9	9.5	5.8
Total	*57.1*	*52.0*	*49.7*	*48.2*	*44.0*	*43.8*	*58.5*	*53.1*
Stages 5	(6712)	(6486)	(2018)	(1797)	(882)	(800)	(668)	(585)
To stage 6	37.4	32.1	30.1	28.7	28.1	26.4	45.4	37.3
				2003				
Stages 0-3	(12274)	(12298)	(5997)	(5474)	(3003)	(2782)	(981)	(889)
To stage 4	33.7	35.4	29.4	31.5	28.5	29.3	34.7	37.2
To stage 5	7.7	5.3	5.5	4.3	5.1	3.2	10.1	5.1
To stage 6	0.7	0.6	0.3	0.2	0.2	0.3	2.4	1.3
Total	*42.1*	*41.3*	*35.2*	*36.0*	*33.8*	*32.8*	*47.2*	*43.6*
Stage 4	(11189)	(13496)	(5765)	(5914)	(2306)	(2570)	(785)	(958)
To stage 5	54.0	49.4	47.3	45.8	39.8	38.4	50.8	49.8
To stage 6	5.3	4.1	4.4	4.1	2.9	3.4	6.6	5.5
Total	*59.3*	*53.5*	*51.7*	*49.9*	*42.7*	*41.8*	*57.4*	*55.3*
Stage 5	(11815)	(10701)	(4157)	(3577)	(1196)	(1150)	(957)	(884)
To stage 6	35.9	31.2	30.1	27.1	24.0	24.2	36.8	34.5

Appendix C (Patterns of Performance and Progress)

Table C1

Comparison of Average Framework Stages on <u>*Addition/Subtraction*</u> *(Standard Deviations shown in brackets) for Younger Children* <u>*Before*</u> *the Project with Older Children* <u>*Before*</u> *the Project for Adjacent Year Groups in 2004*

Year groups	Younger students	Older students	Younger students **before** project	Older students **before** project	Diff	t value	df	Prob.	Effect size
1 & 2	7793	8197	1.52 (0.94)	2.48 (1.15)	–0.96	–57.72	15653	0.000	–0.83
2 & 3	8197	8516	2.48 (1.15)	3.44 (1.20)	–0.96	–53.14	16711	0.000	–0.76
3 & 4	8516	10013	3.44 (1.20)	4.13 (1.08)	–0.69	–40.99	17344	0.000	–0.58
4 & 5	10013	9868	4.13 (1.08)	4.48 (0.94)	–0.35	–24.40	19599	0.000	–0.34
5 & 6	9868	9959	4.48 (0.94)	4.69 (0.92)	–0.21	–15.76	19799	0.000	–0.22
6 & 7	9959	8374	4.69 (0.92)	4.85 (0.94)	–0.15	–11.08	17625	0.000	–0.16
7 & 8	8374	7306	4.85 (0.94)	5.03 (0.90)	–0.19	–12.72	15564	0.000	–0.20

Table C2

Comparison of Average Framework Stages (SDs in brackets) for Younger Students <u>*After*</u> *the Project with Older Students* <u>*Before*</u> *the Project for Adjacent Year Groups*

Addition/Subtraction

Year groups	Younger students	Older students	Younger students **after** project	Older students **before** project	Diff	t value	df	Prob	Effect size
1 & 2	7794	8197	2.54 (1.02)	2.48 (1.15)	0.07	3.81	15924	0.000	0.06
2 & 3	8197	8516	3.50 (1.10)	3.44 (1.19)	0.06	3.21	16678	0.001	0.05
3 & 4	8516	10013	4.24 (0.91)	4.13 (1.08)	0.10	7.11	18527	0.000	0.10
4 & 5	10013	9868	4.69 (0.84)	4.48 (0.94)	0.21	16.40	19523	0.000	0.23
5 & 6	9868	9959	4.95 (0.80)	4.69 (0.92)	0.26	21.36	19476	0.000	0.30
6 & 7	9959	8374	5.17 (0.76)	4.85 (0.94)	0.33	25.63	16041	0.000	0.38
7 & 8	8374	7306	5.23 (0.81)	5.03 (0.90)	0.20	14.22	14844	0.000	0.23

Multiplication/Division

Year groups	Younger students	Older students	Younger students **after** project	Older students **before** project	Diff	t value	df	Prob	Effect size
2 & 3	1287	4524	4.18 (0.77)	3.73 (0.73)	0.45	18.59	1975	0.000	0.59
3 & 4	4524	8217	4.45 (0.85)	4.14 (0.86)	0.31	19.67	9332	0.000	0.36
4 & 5	8217	9068	4.83 (0.96)	4.51 (0.97)	0.32	21.97	17158	0.000	0.33
5 & 6	9068	9485	5.21 (1.00)	4.84 (1.04)	0.37	24.39	18550	0.000	0.35
6 & 7	9485	8027	5.57 (1.01)	5.12 (1.04)	0.45	29.07	16871	0.000	0.43
7 & 8	8027	7145	5.71 (1.00)	5.39 (1.05)	0.32	19.34	14787	0.000	0.31

Proportion/Ratio

Year groups	Younger students	Older students	Younger students **after** project	Older students **before** project	Diff	t value	df	Prob	Effect size
2 & 3	1274	4517	4.17 (0.59)	3.75 (0.58)	0.42	22.89	2026	0.000	0.70
3 & 4	4517	8130	4.37 (0.72)	4.06 (0.74)	0.31	23.20	9590	0.000	0.42
4 & 5	8130	9016	4.69 (0.89)	4.34 (0.86)	0.36	26.50	16820	0.000	0.40
5 & 6	9016	9454	5.03 (0.98)	4.65 (1.01)	0.38	25.98	18466	0.000	0.38

6 & 7	9454	7950	5.40 (1.08)	4.95 (1.10)	0.45	27.11	16827	0.000	0.40
7 & 8	7950	7094	5.58 (1.15)	5.25 (1.17)	0.33	17.43	14781	0.000	0.28

Table C3

Comparison of Final Framework Stages on Addition/Subtraction (SDs in brackets) for Particular Sub-groups at Each Initial Framework Stage in 2004

European vs Māori

Initial stage	No. of European	No. of Māori	Final stage European	Final stage Māori	Diff	t value	df	Prob	Effect size
0	1119	495	2.68 (1.74)	2.16 (1.51)	0.53	6.17	1075	0.000	0.31
1	2544	885	2.46 (0.99)	2.36 (0.99)	0.10	2.58	1532	0.010	0.10
2	5478	1794	3.14 (0.94)	3.11 (0.95)	0.03	1.22	3040	0.223	0.03
3	2728	920	3.91 (0.71)	3.85 (0.67)	0.07	2.59	1677	0.010	0.10
4	12397	4813	4.58 (0.64)	4.52 (0.65)	0.06	5.82	8586	0.000	0.09
5	13198	3815	5.33 (0.54)	5.25 (0.60)	0.08	7.31	5737	0.000	0.14
Average									***0.13***

European vs Pasifika

Initial stage	No. of European	No. of Pasifika	Final stage European	Final stage Pasifika	Diff	t value	df	Prob	Effect size
0	1119	292	2.68 (1.74)	1.98 (1.29)	0.71	7.72	596	0.000	0.42
1	2544	478	2.46 (0.99)	2.47 (1.02)	-0.01	-0.22	656	0.829	–0.01
2	5478	963	3.14 (0.94)	3.07 (0.95)	0.07	2.23	1314	0.026	0.07
3	2728	567	3.91 (0.71)	3.82 (0.73)	0.09	2.67	809	0.008	0.13
4	12397	2806	4.58 (0.64)	4.46 (0.62)	0.12	9.59	4251	0.000	0.19
5	13198	1682	5.33 (0.54)	5.23 (0.54)	0.10	7.10	2136	0.000	0.19
Average									***0.17***

Asian vs Pasifika

Initial stage	No. of Asian	No. of Pasifika	Final stage Asian	Final stage Pasifika	Diff	t value	df	Prob	Effect size
0	106	292	3.29 (1.76)	1.98 (1.29)	1.32	7.06	148	0.000	0.86
1	175	478	2.69 (1.04)	2.47 (1.02)	0.22	2.39	303	0.017	0.21
2	475	963	3.37 (0.94)	3.07 (0.95)	0.30	5.66	953	0.000	0.31
3	252	567	3.96 (0.68)	3.82 (0.73)	0.14	2.67	510	0.008	0.20
4	966	2806	4.62 (0.66)	4.46 (0.62)	0.16	6.60	1579	0.000	0.25
5	1253	1682	5.41 (0.51)	5.23 (0.54)	0.18	9.10	2767	0.000	0.34
Average									***0.36***

High Decile vs Low Decile

Initial stage	No. of high decile	No. of low decile	Final stage high decile	Final stage low decile	Diff	t value	df	Prob	Effect size
0	453	791	2.03 (1.27)	2.77 (1.81)	-0.74	–8.44	1191		-00.44
1	1532	1137	20.55 (00.97)	20.45 (10.01)	00.11	20.76	2387	0.006	00.11
2	3182	2249	30.23 (00.93)	30.13 (00.93)	00.11	40.24	4828	0.000	00.12
3	1574	1299	30.94 (00.67)	30.85 (00.68)	00.09	30.63	2757	0.000	00.13
4	6520	6505	40.61 (00.61)	40.49 (00.65)	00.11	100.04	12974	0.000	00.17
5	6839	4796	50.34 (00.51)	50.26 (00.58)	00.08	70.93	9395	0.000	00.15
Average									***00.10***

Table C4

Comparison of Final Framework Stages on <u>Addition/Subtraction</u> (SDs in brackets) for Particular Sub-groups at Each Initial Framework Stage in 2003

European vs Māori

Initial stage	No. of European	No. of Māori	Final stage European	Final stage Māori	Diff	t value	df	Prob	Effect size
0	2416	1629	2.03 (1.56)	1.81 (1.33)	0.22	4.81	3835	0.000	0.15
1	4933	2278	2.40 (0.97)	2.31 (0.97)	0.08	3.43	4431	0.001	0.08
2	12749	5618	3.16 (0.97)	3.03 (0.97)	0.13	8.14	10787	0.000	0.13
3	4474	1946	3.90 (0.73)	3.80 (0.79)	0.10	5.00	3452	0.000	0.13
4	24685	11679	4.59 (0.63)	4.51 (0.72)	0.08	10.67	20445	0.000	0.12
5	22516	7734	5.30 (0.58)	5.21 (0.72)	0.09	10.51	11481	0.000	0.14
Average									***0.13***

European vs Pasifika

Initial stage	No. of European	No. of Pasifika	Final stage European	Final stage Pasifika	Diff	t value	df	Prob	Effect size
0	2416	874	2.03 (1.56)	1.90 (1.21)	0.13	2.57	1980	0.010	0.09
1	4933	1153	2.40 (0.97)	2.29 (0.95)	0.10	3.32	1756	0.001	0.10
2	12749	2728	3.16 (0.97)	2.99 (0.98)	0.17	8.06	3955	0.000	0.17
3	4474	1030	3.90 (0.73)	3.71 (0.81)	0.19	6.97	1433	0.000	0.25
4	24685	4876	4.59 (0.63)	4.32 (0.96)	0.27	19.01	5732	0.000	0.38
5	22516	2346	5.30 (0.58)	4.97 (1.20)	0.33	13.33	2463	0.000	0.49
Average									***0.25***

Asian vs Pasifika

Initial stage	No. of Asian	No. of Pasifika	Final stage Asian	Final stage Pasifika	Diff	t value	df	Prob	Effect size
0	250	874	2.64 (1.80)	1.90 (1.21)	0.74	6.11	316	0.000	0.53
1	301	1153	2.55 (1.05)	2.29 (0.95)	0.25	3.81	439	0.000	0.26
2	928	2728	3.20 (0.97)	2.99 (0.98)	0.21	5.56	1616	0.000	0.21
3	391	1030	3.91 (0.75)	3.71 (0.81)	0.20	4.30	762	0.000	0.25
4	1743	4876	4.60 (0.68)	4.32 (0.96)	0.28	13.23	4371	0.000	0.31
5	1841	2346	5.29 (0.71)	4.97 (1.20)	0.32	10.89	3925	0.000	0.31
Average									***0.31***

High Decile vs Low Decile

Initial stage	No. of high decile	No. of low decile	Final stage high Decile	Final stage Low Decile	Diff	t value	df	Prob	Effect size
0	1068	2466	2.29 (1.70)	1.90 (1.33)	0.40	6.79	1663	0.000	0.27
1	2182	3466	2.51 (0.99)	2.35 (0.99)	0.17	6.12	4621	0.000	0.17
2	5960	8458	3.21 (0.97)	3.03 (0.97)	0.18	11.11	12846	0.000	0.18
3	2073	3034	3.96 (0.71)	3.76 (0.81)	0.20	9.51	4801	0.000	0.26
4	10340	16830	4.62 (0.62)	4.45 (0.81)	0.17	19.16	25992	0.000	0.23
5	10032	10732	5.31 (0.55)	5.16 (0.90)	0.16	15.29	18016	0.000	0.21
Average									***0.22***

Table C5

Comparison of Final Framework Stages on Addition/Subtraction (SDs in brackets) for Students in Low-decile Schools Involved in the Manurewa Enhancement Initiative Versus Students at all Other Low-decile Schools at Each Initial Framework Stage (2004)

Low-decile Manurewa Enhancement Initiative vs Low-decile non-MEI

Initial stage	No. of MEI	No. of non-MEI	Final stage MEI	Final stage non-MEI	Diff	t value	df	Prob	Effect size
0	20	771	1.85 (0.93)	2.79 (1.82)	–0.94	–4.30	23	0.000	–0.52
1	55	1082	3.00 (1.17)	2.42 (1.00)	0.58	3.62	58	0.001	0.57
2	123	2126	3.07 (0.89)	3.13 (0.93)	–0.06	–0.68	138	0.498	–0.06
3	59	1240	4.00 (0.56)	3.84 (0.69)	0.16	2.08	67	0.041	0.24
4	385	6120	4.61 (0.63)	4.49 (0.65)	0.12	3.64	437	0.000	0.18
5	251	4545	5.19 (0.46)	5.26 (0.59)	–0.08	–2.56	298	0.011	–0.14
Average									***0.05***

Table C6

Comparison of Final Framework Stages on Each Operational Domain (SDs in brackets) for Each Initial Framework Stage as a Function of Gender in 2004

Addition/Subtraction

Initial stage	No. of boys	No. of girls	Final stage boys	Final stage girls	Diff	t value	df	Prob	Effect size
0	1153	959	2.43 (1.66)	2.57 (1.65)	–0.14	–2.00	2046	0.046	-0.08
1	2191	2056	2.44 (1.02)	2.45 (0.98)	0.00	0.06	4243	0.949	0.00
2	4613	4501	3.15 (0.97)	3.13 (0.92)	0.02	1.07	9108	0.285	0.02
3	2440	2226	3.93 (0.73)	3.86 (0.67)	0.07	3.28	4664	0.001	0.10
4	10143	11760	4.57 (0.66)	4.53 (0.62)	0.04	4.52	20887	0.000	0.06
5	10747	10066	5.33 (0.55)	5.28 (0.55)	0.05	6.35	20754	0.000	0.09
Average									***0.05***

Multiplication/Division

Initial stage	No. of boys	No. of girls	Final stage boys	Final stage girls	Diff	t value	df	Prob	Effect size
2-3	3370	3634	4.13 (0.81)	4.06 (0.78)	0.08	3.98	6906	0.000	0.10
4	8245	8950	4.81 (0.80)	4.74 (0.77)	0.07	5.98	16917	0.000	0.09
5	6085	6132	5.70 (0.69)	5.63 (0.68)	0.06	5.21	12209	0.000	0.09
6	4924	3935	6.38 (0.56)	6.33 (0.55)	0.05	4.57	8501	0.000	0.09
Average									***0.09***

Proportion/Ratio

Initial stage	No. of boys	No. of girls	Final stage boys	Final stage girls	Diff	t value	df	Prob	Effect size
1	3293	2846	4.22 (0.75)	4.17 (0.70)	0.04	2.33	6111	0.020	0.06
2-4	10639	11506	4.71 (0.81)	4.65 (0.77)	0.06	5.62	21799	0.000	0.08
5	5584	5426	5.64 (0.78)	5.55 (0.75)	0.09	6.10	11005	0.000	0.12
6	2964	2621	6.49 (0.69)	6.42 (0.66)	0.07	4.03	5546	0.000	0.10
7	1398	901	7.24 (0.51)	7.21 (0.51)	0.03	1.15	1916	0.249	0.06
Average									***0.08***

Appendix D: Stages of the Number Framework

Stage Zero: Emergent

Students at this stage are unable to consistently count a given number of objects because they lack knowledge of counting sequences and/or the ability to match things in one-to-one correspondence.

Stage One: One-to-one Counting

This stage is characterised by students who can count and form a set of objects up to ten but cannot solve simple problems that involve joining and separating sets, such as 4 + 3.

Stage Two: Counting from One on Materials

Given a joining or separating of sets problem, students at this stage rely on counting physical materials, such as their fingers. They count all the objects in both sets to find an answer, as in "Five lollies and three more lollies. How many lollies is that altogether?"

Stage Three: Counting from One by Imaging

This stage is also characterised by students counting all of the objects in simple joining and separating problems. Students at this stage are able to image visual patterns of the objects in their mind and count them.

Stage Four: Advanced Counting (Counting On)

Students at this stage understand that the end number in a counting sequence measures the whole set and can relate the addition or subtraction of objects to the forward and backward number sequences by ones, tens, and so on. For example, instead of counting all objects to solve 6 + 5, the student recognises that "6" represents all six objects and counts on from there: "7, 8, 9, 10, 11."
Students at this stage also have the ability to co-ordinate equivalent counts, such as "10, 20, 30, 40, 50," to get $50 in $10 notes. This is the beginning of grouping to solve multiplication and division problems.

Stage Five: Early Additive Part–Whole

At this stage, students have begun to recognise that numbers are abstract units that can be treated simultaneously as wholes or can be partitioned and combined. This is called *part–whole thinking*. A characteristic of this stage is the derivation of results from related known facts, such as finding addition answers by using doubles or teen numbers.

Stage Six: Advanced Additive Part–Whole

Students at the advanced additive stage are learning to choose appropriately from a repertoire of part–whole strategies to estimate answers and solve addition and subtraction problems. They see numbers as whole units in themselves but also understand that "nested" within these units is a range of possibilities for subdivision and recombining. Simultaneously, the efficiency of these students in addition and subtraction is reflected in their ability to derive multiplication answers from known facts. These students can also solve fraction problems using a combination of multiplication and addition-based reasoning. For example, 6 x 6 as (5 x 6) + 6.

Stage Seven: Advanced Multiplicative Part–Whole

Students at the advanced multiplicative stage are learning to choose appropriately from a range of part–whole strategies to estimate answers and solve problems involving multiplication and division. Some writers describe this stage as "operating on the operator". This means than one or more of the numbers involved in a multiplication or division is partitioned and then recombined.

For example, to solve 27 x 6, 27 might be split into 20 + 7 and these parts multiplied then recombined, as in 20 x 6 = 120, 7 x 6 = 42, 120 + 42 = 162. This strategy uses the distributive property.

A critical development at this stage is the use of reversibility, in particular, solving division problems using multiplication. Advanced multiplicative part–whole students are also able to estimate answers and solve problems with fractions using multiplication and division.

Stage Eight: Advanced Proportional Part–Whole

Students at the advanced proportional stage are learning to select from a repertoire of part–whole strategies to estimate answers and solve problems involving fractions, proportions, and ratios. This includes strategies for the multiplication of decimals and the calculation of percentages.

These students are able to find the multiplicative relationship between quantities of two different measures. This can be thought of as a mapping. For example, consider this problem: "You can make 21 glasses of lemonade from 28 lemons. How many glasses can you make using 8 lemons?"

To solve the problem, students need to find a relationship between the number of lemons and the number of glasses. This involves the creation of a new measure, glasses per lemon. The relationship is that the number of glasses is three-quarters the number of lemons. This could be recorded as: 21:28, □:8, 21 is ¾ of 28, or ¾ of 8 is 6.

Appendix E (Te Poutama Tau: A Case Study of Two Schools)

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Te Rārangi Patapatai mō te Hunga Tumuaki

Te Tumuaki

1. E hia tau e mahi ana koe hei kaiako?
2. E hia tau e mahi ana koe hei tumuaki?
3. He aha ngā tohu whakaako kei a koe?
4. Kua uru atu koe i tētahi/ētahi wānanga pāngarau? Whakamāramatia mai.
5. Kei te kaingākaunui koe ki te pāngarau?

Ngā āhuatatega –ā-iwi, ā-whānau

6. He pēhea āhua o te whanaungatanga i waenganui i te kura me te iwi kāinga, ngā iwi/hapū?
7. He aha ngā kapapori pāpori, kapapori ohaoha (socio-economic background) o ngā tamariki?

He teitei	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau
Kei waenganui	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau
He hakahaka	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau

8. Te āhua o te whānau. He pēhea te āhua o ngā whānau?

Kotahi anake te matua	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau
Tokorua ngā mātua	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau
He whānau whānui	te katoa/te nuinga/ētahi/ruarua noa iho/ kare kau

9. Kōrero ai te whānau i te reo Māori i te kāinga?

I ngā wā katoa	te katoa o ngā whānau/ ētahi/ruarua noa iho/ kare kau
I te nuinga o te wā	te katoa o ngā whānau/ ētahi/ruarua noa iho/ kare kau
I ētahi wā	te katoa o ngā whānau/ ētahi/ruarua noa iho/ kare kau

10. Te takiwā o te kura. He aha te āhua o te takiwā o te kura?
    He Rural   Minor urban (small town)   Major urban (big town/city)

Te tataunga o te kura

11. What is the school decile?
    He teitei (8–10) Kei waenganui (4–7) He hakahaka (1–3)
12. E hia ngā tamariki kei roto i te kura?
13. Ngā Whakaritenga o te pāngarau?
13. I pēhea koe i whakarite a te kura mō te marau pāngarau? Whakamāramatia mai.

Te Poutama Tau

14. I pēhea koe i tautoko ai ngā pouako hei mahi i Te Poutama Tau?
15. Ki ōu whakaaro, pēhea te neketanga whakamua o tō kura kei roto i Te Poutama Tau?
    He tino neke He āhua neke He iti noa Kāore i neke
16. Ki ōu whakaaro e tautoko ana Te Poutama Tau i te piki whakarunga o ngā tamariki kei roto i te pāngarau?
    He tino tautoko He āhua tautoko He iti noa Kāore i tautoko
17. Ki ōu whakaaro he aha ngā āhuatanga me ngā whakaritenga o te kura i tino tautoko mai i te piki whakarunga o tōu kura i roto i Te Poutama Tau?
    Hei tauira: te tautoko mai o ngā pouako?
    te tautoko mai o te whānau?
    te matatau o ngā pouako ki te pāngarau?
    Te kaingakaunui o ngā tamariki ki te pāngarau?
18. He korero anō āu mo Te Poutama Tau, mo te whakaako rānei i te pāngarau?

Te Rārangi Patapatai mō te Hunga Tumuaki
(Principals' Questionnaire)

(These are indicative questions only)

Principal

1. How many years have you been teaching?
2. How many years have you been principal?
3. What academic qualifications do you have?
4. Have you done any courses or professional development?
5. What are your own interests in pāngarau?

Demographic characteristics

6. Iwi identification. Is the school closely connected to iwi/hapū?
   One iwi/hapū   mixture of iwi/hapū
7. What is the socio-economic background of the tamariki?
   High   All/Most/Some/Few/None
   Middle   All/Most/Some/Few/None
   Low   All/Most/Some/Few/None
8. Family Type. What are the characteristics of the whānau?
   Single Parent   All/Most/Some/Few/None
   Nuclear family   All/Most/Some/Few/None
   Extended family   All/Most/Some/Few/None
9. Do the whānau speak te reo Māori?
   All the time   All/Most/Some/Few/None
   Most of the time   All/Most/Some/Few/None
   Sometimes   All/Most/Some/Few/None
10. School locality
    What are the characteristics of the local area?
    Rural   Minor urban (small town)   Major urban (big town/city)

School Characteristics

11. What is the school decile?
    High (8–10)   Medium (4–7)   Low (1–3)
12. What is the school roll?
13. How do you organize the school for pāngarau?

Te Poutama Tau

14. What kinds of support do you provide to teachers for the implementation of Te Poutama Tau?
15. How well do you rate your school's progress in Te Poutama Tau?
16. Do you think Te Poutama Tau has raised general pāngarau achievement?
17. What do you think are the factors that have lead to your school's success in Te Poutama Tau?
    For example, teacher support/attitudes, whānau involvement/support, peer (teachers' and students') support, resource quality, facilitator support, and so on.
18. Do you have anything else to add about Te Poutama Tau or mathematics?

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Te Rārangi Patapatai mō te Hunga Pouako

Ngā mahi whakaako

1. E hia tau koe e mahi ana hei kaiako?
2. E hia tō roa i tēnei kura?
3. He aha te/nga tohu whakaako kei a koe?
4. Kua uru atu koe ki tētahi atu wānanga pāngarau i kō atu i Te Poutama Tau?
5. He aha te/nga marautanga e tino kaingākautia ana e koe?

Te āhua o tō akomanga

6. He aha te āhua o tō kura/akomanga i te tau 2003:
    - He kura kaupapa Māori?
    - He kura rumaki?
    - He kura-ā-iwi?
    - He akomanga rumaki i te kura auraki?
    - He akomanga reo rua?
    - He momo kura kē atu?
7. Tokohia nga tamariki i tō akomanga?
    5–10 11–15 15–20 20–30 30+
8. He aha te/nga tau kura o nga tamariki i tōu akomanga?
    Tau 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
9. E hia tau koe i whakaako i tēnei reanga/karaehe?
10. He pēhea te matatau o tōu karaehe ki te reo Māori?
    - He tino matatau te katoa
    - He matatau te nuinga
    - He āhua matatau te nuinga
    - Kāore i te matatau te nuinga
11. He pēhea tō rātau ngakaunui ki te pāngarau i mua mai i Te Poutama Tau?
    a. He tino ngakaunui te katoa
    b. He ngakaunui te nuinga
    c. He āhua ngakaunui te nuinga
    d. He iti nei ō rātau ngakaunui
12. He pēhea te whakaaro o nga tamariki ki te pāngarau i naianei?
    He ōrite tonu He āhua rerekē He tino rerekē

Te Whakaako Poutama Tau

13. E hia tau koe e whai atu ana i Te Poutama Tau?
14. I whakahaeretia Te Poutama Tau i te whānuitanga o te kura?
    Ae Kao
15. Mehemea ko koe te kaiwhakahaere o Te Poutama Tau ki tō kura, he aha ētahi o nga wharitenga matua mō tēnei kaupapa?
16. He pēhea tō whakaako i te pāngarau i naianei? He rite tonu, he rerekē? Whakamāramatia mai.
17. He aha nga rautaki whakaako o Te Poutama Tau e tino pai ki a koe? Whakamāramatia mai.
18. He aha nga wāhanga tino pai o Te Poutama Tau ki a koe? Whakamāramatia mai.
19. He aha nga wāhanga tino pai o Te Poutama Tau ki ō tamariki?
20. I pēhea koe i whakamahi ai nga rauemi o Te Poutama Tau? Whakamāramatia mai.
21. He aha nga rauemi matua ki a koe? Whakamāramatia mai.

Te Tautoko o te Kura

22. He pēhea nei te tautoko mai o tōu kura i a koe e whai atu ana i Te Poutama Tau;
    - ka tino tautoko
    - ka āhua tautoko mai
    - kāore e tino tautoko mai i ētahi wā
    - kāore i te tino tautoko.
23. He korero anō āu mo Te Poutama Tau, mo te whakaako rānei i te pāngarau?

Te Rārangi Patapatai mō te Hunga Pouako
(Teacher's Questionnaire)

(These are indicative questions only)

Teaching Experience

1. How many years have you been teaching?
2. How many years have you been teaching in this school?
3. What academic qualifications do you have?
4. Have you done any courses or professional development in pāngarau outside of Te Poutama Tau?
5. What are your main curriculum areas?

Characteristics of class

6. Is/was your class:
   - kura kaupapa Māori?
   - total immersion school?
   - total immersion class in an English-medium school?
   - bilingual class?
   - another type of class?
7. How many children did you have in your class?
   5–10 11–15 15–20 20–30 30+
8. What year group were they?
   Y1, 2, 3, 4, 5, 6, 7, 8, 9, 10
9. How many years have you been teaching this age group?
10. How would you rate te reo Māori fluency of your class?
11. What is/was the attitude of the children to pāngarau?
12. Has their attitude to pāngarau changed?

Teaching Te Poutama Tau

13. How many years have you been involved in the Te Poutama Tau project?
14. Do you have school-wide responsibilities for Te Poutama Tau?
15. If you are the lead teacher, what are some of the main factors to consider?
16. Has your own teaching style been affected by Te Poutama Tau?
17. What are some of the effective strategies of Te Poutama Tau?
18. What do you find most effective about Te Poutama Tau? Explain.
19. What aspects of Te Poutama Tau do your children enjoy most?
20. How have you used the equipment?
21. What has been the key equipment?

School support

22. How has the school supported you in the Te Poutama Tau project?
23. Do you have anything else to add about Te Poutama Tau or mathematics?