Advancing Pasifika Students' Mathematical Thinking

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Five sessions of a year 5 and 6 mathematics class were videotaped and the language used by the teacher and by the students when they were not working with the teacher was analysed. The students in the class were primarily from Pasifika backgrounds. We were interested in the relationship between the teacher's language and the language that students used among themselves. The teacher used the language for advancing children's thinking advocated by Fraivillig et al. (1999). She encouraged students to use similar language, but they did so selectively. They routinely gave their method or asked others for their method rather than for answers, and they reiterated the teacher's request that students be given time to think. However, they used little of the exploratory talk needed to explore mathematical ideas together. The students in this class made excellent progress in relation to national NDP achievement norms. We raise the question of whether or not they would have made more progress or more lasting progress had they engaged in exploratory talk among themselves.

Background

The teacher discussed in this report was observed in 2004 (see Irwin & Woodward, 2005). In that report, we documented her use of enquiry language. In contrast to another teacher who had also received Numeracy Development Project (NDP) training, she asked students more open questions, waited longer for them to think and respond, and was interested in the variety of their responses. In her sessions with students, she did less of the talking and emphasised listening more than did the comparison teacher. A few students also used the language of enquiry with one another. In that study, the students were observed on only one occasion. This study was undertaken in order to study the development of her students' language across a school year. We recorded the teacher's language and that of the students on five occasions.

There has been other research on the nature of students' and teachers' participation in the NDP. Higgins (2003) examined the language that teachers and students used while the teacher was working with a group of students. Young-Loveridge, Taylor, and Hāwera (2005) interviewed students, both in the NDP and not in the NDP, about their views on communicating their strategies for solution to peers. Irwin (2004) interviewed year 9 students on their views of their mathematics programme that was based on the NDP. None of these studies looked at the language that students used among themselves when the teacher was not present. This study extends our brief look at the discourse of student groups in 2004 as well as recording the language used by the teacher.

An analysis of the students' discourse when not with the teacher is important because the general practice in the NDP is to have students working in groups. This means that the majority of students are not with the teacher during group times. The NDP recommends a teaching model for advancing children's thinking (ACT) (Fraivillig, Murphy, & Fuson, 1999), which is based on a study of classes where students were engaged in collaborative problem solving. The examples from that study were of times when groups are working with the teacher, as are those in Higgins (2003) and in other writers such as Bowers, Cobb, & McClain (1999) and Wood (2001). NDP *Book 3: Getting Started* (Ministry of Education, 2005) recommended grouping students for development of strategies. While one group was with the teacher, the other groups would be working independently. Similarly, knowledge sessions usually involved working in groups. Attention was given to managing independent groups in *Getting Started* (2005), but little was said about the expected nature of students' learning in those groups. The general intention appeared to have been that students would practise knowledge skills or strategies when they were not with the teacher.

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that students discuss their mathematical reasoning with one another, and several other studies such as Fraivillig et al. focus on discussion in a group with the teacher present. Wegerif, Mercer, and Dawes (1999) analysed students' talk when the teacher was not present in an experimental programme for advancing students' reasoning. They distinguish among disputative, cumulative, and exploratory talk, all of which involve different degrees of working constructively with each other's ideas. They show that students in the experimental group used exploratory language, including phrases such as "because", "I think", and "agree", and made greater gains on a reasoning task than students in a control group.

Other writers have found that it is beneficial for students to use exploratory talk with one another when not working with the teacher. Is this happening in a NDP class that is successful on the Numeracy Framework? Is this a goal that the NDP should further?

Method

Participants

The class studied was a combined years 5 and 6 (ages 9–10) in a school classified as decile 1, which indicates the lowest socio-economic level. The membership of the class varied as students came or left the area. At the start of the year, there were 26 students in the class, and at the end of the year, there were 28 students, five of whom had been there only for a term.

Table 1
Students in This Class at the End of 2005 N = 28

Females			Males		
Number of students	Year	Ethnicity	Number of students	Year	Ethnicity
1	5	Cook Islands Māori-Niuean	1	5	Tongan
1	5	Sāmoan	1	5	Māori
1	5	Tongan	2	6	Cook Islands Māori
1 (new)	5	Cook Islands Māori	4	6	Sāmoan
4 (1 new)	6	Cook Islands Māori	1	6	Sāmoan-Tongan
3 (1 new)	6	Sāmoan	2	6	Māori
1	6	Sāmoan-Tongan	1	6	European
1	6	Māori-European			
1 (new)	6	Tongan			
2 (1 new)	6	Māori			

Table 1 shows the year level, sex, and ethnicity of students at the end of the year. Those who had been there only for a term are indicated as "new". This table giving each student is included to show the complex ethnic make-up of this class. This class can be summarised as having, at the end of the year, 6 students in year 5 and 22 students in year 6. Of these 28 students, 21 were Pasifika, 5 were Māori, 1 was European, and 1 was Māori-European. Twenty-three of the students had been in the class for more than one term. These 23 students are the ones whose progress is measured below. The only test of these students' skill in English was made by asking the teacher at the start of the year if any of the students were unable to understand her. She reported that two students could not understand her, and she therefore placed them with other students for help.

The teacher of this class was a New Zealand European in her third year of teaching. She reported that she had trained at Dunedin College of Education and that all of her training there was based on the NDP. Her school was involved in NDP training during her first two years there. She had never taught mathematics any other way than that promoted by the NDP. NDP equipment was in the room and in use. The teacher was not seen to refer to any NDP booklets, but her teaching was compatible with furthering students' NDP stages. She grouped her class by stages and changed the groupings for different topics and as she saw students advancing. She chose to assess all students on the full Numeracy Project Assessment diagnostic tool (NumPA).

Method

We videotaped five class sessions with a hand-held videotape recorder. The teacher selected the class topics in relation to the overall plan for the year. Times for videotaping were not related to the topic taught, although it happened that all videotaped sessions covered number and operations. During each lesson, the camera focused on the teacher for the full-class portions of the lessons, with some panning of the students to record what they were doing. When the class broke into groups, we focused on one group recommended by the teacher. The size of the group and the students in the group changed, but some of the same students were filmed in four of the sessions. The group filmed was usually the second-to-top group by assessed stages.

The content of the lessons videotaped is given in Table 2.

Table 2
Characteristics of the Five Lessons Videotaped

Date of class	Topic	Group and activity videotaped
31 March	Showing multiple ways of writing a number (for example, 21 is the same as 3 x 5 + 6); how to write a subtraction problem	3 girls and 3 boys each contributing ideas to a single question
11 May	Multiplication: deriving unknown tables from 2 times, 5 times, and 10 times tables	1 girl and 2 boys finding products from numbers on thrown dice
16 August	Finding fractional parts of whole numbers; translating improper fractions to mixed numbers and vice versa	6 girls and 4 boys with individual work sheets, talking when they needed help from each other
11 October	Review of mental subtraction, 2-digit from 3-digit numbers	5 girls and 3 boys solving problems mentally and sharing their methods
8 November	Multiplication: 2-digit by 1-digit numbers	2 girls and 3 boys finding products from numbers on thrown dice

Findings

Students' Progress on Numeracy Stages in Comparison to National Norms

Table 3 shows the percentage of students in this class at the end of the year in comparison with the 2004 national percentages for year 6 Pasifika students and the 2004 national average. This is an oversimplification of the nature of this class. We did not observe the teacher while she did these assessments, but when observed in 2004, she was flawless in her presentation of items from memory. It is highly likely that she was consistent and accurate in her assessments.

Table 3
Percentage of Students at Each Strategy Stage of the Numeracy Assessment Profile at the End of 2005 in Comparison with National Percentages for Pasifika Students and for the Total National Sample in 2004. N = 23 (not all totals equal 100% because of rounding)

Stage	Add/sub			Mult/div			Proportional		
	This class	Year 6 Pasifika average	Year 6 national average	This class	Year 6 Pasifika average	Year 6 national average	This class	Year 6 Pasifika average	Year 6 national average
Not assessed				4	3	2	4	3	2
< Stage 4	0	1	1	0	5	2			
Stage 4	22	29	16	35	27	15	39 *	39 *	23 *
Stage 5	43	49	46	9	31	26	13	35	33
Stage 6	35	22	37	39	27	36	30	18	25
Stage 7				13	8	18	13	5	14
Stage 8							0	0	2

*Includes all stages up to and including stage 4

This table shows that, at the end of 2005, students in this class in a decile 1 school had a higher proportion of students at the upper stages of the Framework than the national percentages for all Pasifika students in year 6 and was comparable to the overall national average for these strategies. This is true even though this class had five students in year 5 whose stages were usually lower than those of the year 6 students (the national figures for year 5 are markedly lower than year 6 for some stages). We note that, in this class, low percentages of students were judged to be at stage 5, early additive, for multiplicative reasoning or proportional reasoning, although the majority could use additive or part–whole reasoning for addition. This may have been related to the class emphasis on multiplication, which is essential for both of these strategies. Note that in a class of 23, one student is equivalent to 4.3%.

We compared the students' strategy stages at the end of the year with their stages at the beginning of the year or the end of the previous year. In looking at this information, we note that progress between stages is not equally difficult (see Irwin, 2003; Young-Loveridge, 2004). Table 4 shows the number of stages changed for each strategy. This data shows that this is a class in which students make good progress in NDP stages. Previous reports of progress at the higher stages of the Number Framework showed that about 40% of students at these higher stages progressed to further stages (for example, Irwin, 2004; Irwin & Niederer, 2002; Young-Loveridge, 2004). On these three scales, 60%, 53%, and 58% of the students who were assessed and who were not at the ceiling initially moved to a higher stage. This excellent progress provides an important background for examining the teacher's and the students' language in this class across the year.

Table 4
Number of Students Changing Stages on Each Strategy from the Beginning to the End of 2005 (N = 23)

Stage	Add/Sub	Mult/Div	Proportion
At ceiling initially	3	0	0
Not assessed initially	–	4	4
Decreased	–	1	1
Stayed at the same stage	8	8	7
Gained 1 stage	8	6	7
Gained 2 stages	4	4	4

Teacher's language

The teacher's language was compared to that suggested by the model proposed by Fraivillig, Murphy, and Fuson (1999). These authors identify ways in which teachers orchestrate classroom discourse through eliciting children's solution methods and supporting and extending children's understanding. Combining these three components is shown through allowing additional time for student thinking, assisting students with their narrations, probing for better descriptions or solution methods, asking the students to generate alternative solution methods, using challenging follow-up questions, highlighting and discussing errors, providing assisted practice at the top of students' performance levels, assessing students' thinking on an ongoing basis and adjusting instruction accordingly, and continually adapting classroom discussions to accommodate the students' zones of next development (see Fraivillig et al., 1999, Figure 1).

This teacher carried out all of these activities. Some or all of these activities were noted in each of the five lessons recorded. Especially frequent were eliciting of different solution methods, waiting for and listening to student explanations, asking students to elaborate, accepting effort and different answers, listing all solution methods, pushing students beyond original methods, promoting more efficient solution methods, and encouraging a love of challenge.

Common responses by this teacher were:

> Do you want to share your way, B? (lesson 1)
>
> How do you know it's going to be 8? (lesson 2)
>
> Good to see you thinking and really taking time. (lesson 2)
>
> Who can tell me what they did? (lesson 3)
>
> If S comes up with an answer and none of you agree with him, ask him how he did it and get him to work through how he did it. (lesson 4)
>
> So how are you going to work it out? ... F got a different answer. How did you work your one out? (lesson 5)

She had high expectations for her students and repeatedly told them that they were clever and could do it, accompanied by appropriate praise. This was particularly evident in lesson 3 when the worksheet was difficult for the students, and they told her so. She replied:

> Smile because it's hard. It's hard because you're clever and you can handle it.

A time when the teacher routinely challenged students and asked for different methods was in the "Quick 20" that came at the start of four of the five lessons. An example from lesson 4 of these questions and her responses after students gave their answers appears in Table 5.

Table 5
"Quick 20" Questions Asked in Lesson 4 and the Teacher's Response to the Answers Given by Selected Students

Questions	Teacher's response to students' answers
7 x 3	(nod)
2 x 15	Good. How did you work it out?
14 ÷ 2	No, sorry. (To other student.) Good job.
48 – 17	Yes, how did you work it out? Who else did it that way? Did anyone do it a different way?
6 x 6	Sorry. (Asks another student.)
9 x 7	No, I'm sorry, it is not 54. J?
124 – 25	How did you work it out?
18 x 3	How did you work it out?
3 x 4 x 8	Tell me how you did it. ... Who agrees? Who doesn't agree? Why don't you agree? Oh really? How did you do it? Okay, does everyone have 96 apart from those who added?
10 x 100	1 000 (revoiced)
$\frac{1}{4}$ of 80	How did you work it out? Okay, did anyone do it a different way?
25 is what fraction of 50?	(revoiced)
5 x 2	(Asks a lower stage child.) Not quite. (Asks another student.)
7 + 8	(nod)
19 – 6	Very good
12 is $\frac{1}{3}$ of what number?	No, 4 is a third of 12. (To other student.) How did you work it out?
Name a shape with no corners	There will be different answers, so keep your hand up until we have got your shape. Circle, oval, sphere, and cylinder. (A student offered a spiral.) No, a spiral isn't a shape because it doesn't join up.
How many corners on an octagon?	Nice

After seeing who got how many correct, she said:

> Some of you are making really good progress. Well done, guys.

Students' Discourse When Not with the Teacher

Although students worked in groups and the teacher gave guidelines for how they should work together, the groups observed did not appear to be working to solve problems co-operatively except in the first session, which required multiple answers to one question. They were selective in their adoption of the language used by the teacher. On two of the observations, lesson 2 and lesson 4, the discourse involved one student imitating the teacher's role and language and the others playing the role of students. The methods of discussion that the teacher had asked them to use appeared to be taken as class rules rather than models for discussion or sociomathematical norms (Bowers, Cobb, & McClain, 1999). In lesson 2, a boy adopted the teacher's words, for example, saying "Are you sure?" and scaffolding to help a student derive 7 times 4 by asking her what 7 times 5 was, saying "We've got to give her time to think," as well as scolding another student by saying "You're not supposed to tell." Other students played their role as students, for example M asked, "Can I please say it?" when she knew an answer. In lesson 4, a girl took over leadership in a large group, but as an instructor rather than as discussion leader. She said things such as "Turn your cards (number fans) around", "I said, we're doing that one", "No, you've got to work it out with your team", and writing new problems on the board. These episodes have the feel of children "playing school" and not that of co-operative learning. Where students did help each other, they used a very different type of dialogue. In lesson 3, for example: "What is number 3?" "I know you got it, but how did you get it?" "How did you get that?" This was a difficult activity for the students and one in which they genuinely needed each other's help. The fact that a student asked "But how did you get it?" indicates that he realised that "how" was an essential thing to know.

An initial question behind this research was whether or not students improved in using the language of enquiry with one another over the year. This was difficult to determine because of the different groups and tasks. However, lessons 2 and 5 included the same students and similar tasks. Comparison of these two small-group sessions is given in Table 6. See Table 2 for the tasks.

Table 6
Types of Student Discourse in Lesson 2 and Lesson 5

Lesson	Participants	Explaining answers/methods	Barriers to adopting the teacher's language
2	S, N (boys) M (girl)	S: "Are you sure?" x 3 S attempts to guide M to an answer through teacher's steps. When N gives answer, S tells him, "You're not supposed to tell." S and N: "You've got to give her time to think."	Competition between S and N S and N boast about knowing more tables: • S: "I'm going to win, man." • N: "You're going to lose. I'll get revenge with my bare hands." • M taunting: "I know the answer." x 2 • N: "I won."
5	S, N, and L (boys) M and A (girls)	L proves his answer when others show doubt. S asks M for an answer, so she shows her workings on paper. S asks M: "What are you doing?" and she gives her method. L explains his answer to N by giving method.	Competition between boys and girls: A and N argue about who goes first. • S to L: "Don't cheat." • N awards the boys "points". • A: "Beating you." • M: "We got 3 points." • L: "We're winning, you're losing." • L: "Yeah, we won."

The quality of discussion about mathematical methods was better in lesson 5. It was also the case that the intensity of competition was greater on the second occasion. In both of these groups, students were supposed to be deriving answers from known facts. This never happened when the teacher was not there.

In the framework of disputational, cumulative, and exploratory talk, as defined by Wegerif, Mercer, and Dawes (1999), the students' talk was mostly disputational or cumulative, although they used some of the language of exploratory talk to justify their answers. Instead of reasoning collaboratively, on most occasions the students were working as individuals trying to solve problems. They adopted some of the teacher's language, but what was valued was knowledge and "winning". They valued both the knowledge of facts and of procedures. In the second lesson involving multiplication, knowledge of the tables won out every time over attempts to derive them. In lesson 5, on multiplication of two-digit by one-digit numbers, it was skill with the vertical algorithm that they valued. They viewed the competent mathematics student as one who knew the answer and how to get it rather than as a partner in mathematical reasoning.

Influences on Students' Language

Other factors besides a teacher's language influence the discourse of students. Lindfors (1987) wrote about the nature of children's questions in and out of school. She argues that questions out of school are influenced by curiosity, while their questions in school are largely procedural. She gave several examples, including the out-of-school question, "If everybody in the world keeps drinking water, are we going to run out of water some day?" The same child in school asked, "Do you want us to skip every other line?" and "Do we write the date on this paper?" (Lindfors, 1987, p. 287). The dialogue in these five lessons was usually procedural – if "How did you get that?" is considered a procedural question.

It is possible for teachers to foster the language of curiosity in school, but it is not easy in the face of school traditions. In adopting language, Lindfors points out that children use whatever is salient and interesting to them. Similarly, Cazden writes of the classroom containing "the official world of the teacher's agenda, and the unofficial world of the peer culture" (Cazden, 1988, p. 150). These two worlds appeared to intersect in group work in this class when the teacher was not present.

Cobb et al. (2003) have written about the influence of the school structures on instructional practices. That was evident in this case in that the school had decided to focus on writing, and therefore mathematics had been moved to after lunch. The teacher noted that the students did not seem alert at this time. The students may also have been more influenced by their discussions on the playground at that time.

Discussion

On two of the five occasions when small groups were observed, the activities were primarily for practice; on the three other occasions observed, the teacher appeared to expect students to use exploratory language in small-group work. The students used some of the teacher's exploratory language but often used a different style of discourse. While these students were mostly Pasifika, this might happen in any class.

The NDP advocates a set of behaviours for teachers for advancing students' thinking that is based on a model that furthers co-operative learning in classrooms. Teaching in this model usually involves giving one challenging problem to a group to work on throughout their time in the group. This is quite different from group work, which is primarily for practice. In the lessons discussed, competition and the lack of reflection were influenced by the fact that there were several problems to be solved. Again, this is appropriate for practice but less appropriate for co-operative mathematical enquiry. Despite attempts by the teacher to get group co-operation when she was not present, the students did not adopt that pattern. They did show evidence of other characteristics necessary for success. They appeared to be willing to work hard, despite the fact that mathematics was held after lunch when attention wandered. They were interested in getting right answers and in using techniques that would help them do mental mathematics quickly. They decided for themselves what was the easiest way to do a mathematics problem, and using tidy numbers or decomposing multiplication problems were not easy for them. They knew that it is faster to know your tables and carry out the vertical algorithm mentally, usually from left to right. Most of the students observed seemed to have a good sense of number.

Prior to 2006, NDP resources did not suggest co-operative mathematical reasoning by groups working on their own[1]. However, the 2006 version of *Getting Started* (for example, p. 13) does recommend co-operative learning for groups who are not with the teacher. As indicated above (for example, Wegerif, Mercer, & Dawes, 1999), co-operative reasoning has been found to result in better individual reasoning. Theoretical reasons why students should be encouraged to reason mathematically largely relate to the theory that social reasoning precedes individual reasoning and that language is the tool students use for thinking (Mercer, 2002; Resnick, 1987; Vygotsky, 1978, 1986). An example of a NDP class with a large proportion of Pasifika and Māori students that developed a culture of mathematical exploratory talk is described by Hunter (2005). She indicated that in this class, the students' use of enquiry and argument increased their autonomy and deepened their collective responsibility to engage in mathematical practices. This paper would be useful for the NDP to consider.

Watson and Chick (2001) list factors affecting collaboration in groups. These include the nature of the task, disagreement, doubt, and tenacity of ideas as well as social factors of leadership, social conflict, and egocentrism. Among the effects that they found in groups was that students looked for the simplest way of solving a problem. It was evident that these students were not interested in multiple ways, as in lessons 2, 4, and 5. The simplest method was knowledge of tables or of the vertical algorithm done left to right for subtraction (but by using a method that did not require "borrowing") and right to left with carrying for multiplication. Other "simplest" methods seen were looking at a wall chart for tables and using a calculator. In the sessions we observed, the teacher's attempts to get students to use known facts or tidy numbers for finding an answer did not succeed when she was not with them.

Small groups of students can either practise strategies and knowledge or they can explore novel problems. If both types of activities are intended, it would be useful to make the purpose of an activity clear to the students. If groups are to explore novel problems, this is likely to enhance their ability to reason mathematically when not guided by a teacher. If a teacher uses both purposes, it would be useful to be explicit about the expectations for a particular session.

Many classroom styles and discourses can lead to success for students. This classroom demonstrated one of them. It was a style in which the teacher appeared to set rules for thinking for yourself, being able to describe your strategies, listening to others, and respecting different strategies. She repeatedly told the students that they could do difficult work because they were clever. Results showed that they lived up to her expectations. What we do not know is if these students would have been even more successful if they had engaged in co-operative mathematical reasoning. We also do not know if they will continue to reason at high stages on the Number Framework.

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[1] Note that although the process may have been used in some classes, we have found no written recommendation that this should happen.

The use of co-operative mathematical reasoning in groups is an issue for the NDP as a whole. Does the project want to fully implement the programme for advancing children's thinking advocated by Fraivillig et al. (1999) by having classes adopt sociomathematical norms in which students help each other engage in mathematical reasoning even when not with a teacher? The literature suggests that this would provide them with greater success than they now enjoy.

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