

## Modelling Books and Student Discussion in Mathematics

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*Ehara taku toa i te toa takitahi,  
engari he toa takitini.*

*My strength is not that of a single warrior but that of many.*

Manipulation of materials, commonly referred to as “hands-on”, as a strategy for learning mathematics is widely applied in New Zealand primary classrooms. A modelling book can enhance a hands-on approach through linking modelling and discussion of mathematical ideas as promoted through the Numeracy Development Projects (NDP). This approach has been interpreted as “kinaesthetic”. One of the potentially most damaging applications of kinaesthetic learning has been to Māori students, among others. Modelling books link modelling and discussion of mathematical ideas, as promoted through the NDP. Modelling books may help teachers to reconceptualise hands-on learning to include discussion of mathematical ideas and provide a means of developing conceptual understanding through the introduction of mathematical abstractions.

### Background

Historically, classroom practices in New Zealand elementary mathematics have emphasised the manipulation of objects. The emphasis on “hands-on” can be traced back to the introduction of a child-centred approach around the beginning of the 1970s. Teachers’ interpretations of the child-centred approach favoured manipulation of materials (Higgins, 1998). However, discussion of the mathematical ideas represented by the materials has not necessarily been included in their practices.

A matter of greater concern is that, guided by popular literature on learning styles, hands-on has been interpreted as “kinaesthetic”. This interpretation does not distinguish between students’ pleasure in handling the materials and their understanding of the mathematical principles.

One of the potentially most damaging applications of kinaesthetic learning has been to Māori and Pasifika students (Higgins, 2001). We have been challenged by Bishop, Berryman, Tiakiwai, and Richardson (2003) to examine dominant teacher-centred “monocultural pedagogies developed in New Zealand on the basis of unchallenged metaphors” (p. 23). Furthermore, the treatment of Māori as a homogeneous rather than as a diverse category has led teachers to consider simplistic pedagogical strategies such as “Māori learning styles” (McKinley, Stewart, & Richards, 2004) as kinaesthetic.

An important pedagogical shift promoted through the Numeracy Development Project (NDP) has been to think of learning as participation in a mathematical conversation. Such discussions are frequently situated in a small teacher-led group of about six to ten students. Part of the reason for the emphasis on such a setting is to enable students to individually and collectively manipulate mathematical representations such as a number line or an abacus and afford all members of the group the opportunity to participate in discussion.

The NDP strategy teaching model (Ministry of Education, 2006) emphasises both the manipulation of materials and the explanation by students of their action in terms of the underlying mathematical ideas. If interpretations of kinaesthetic learners equate in practice to restricting students to “modelling with materials” without an emphasis on discussion of mathematical ideas, students may have little opportunity to move beyond modelling to the imaging and abstraction through the number properties phases of the teaching model.

## The Modelling Book

This investigation is based on the assumption that manipulation and discussion need to be linked. A pedagogical strategy in common use that links manipulation and discussion is to record the thinking of the teacher-led group. Some teachers use a large scrapbook or hand-made book to record the collective thinking. This book has various labels, such as a “recording” or “modelling” book. For the purposes of this discussion, we will call it a modelling book.

Typical practice is for the modelling book to be either placed in the middle of the floor with the group sitting around it in a semi-circle or on an easel with the students in front. In both cases, as the main recorder, the teacher is adjacent to the book.

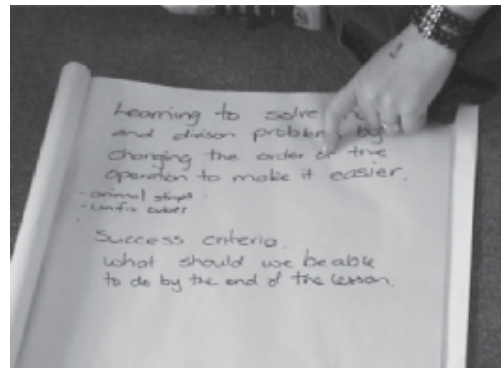


Figure 1. Modelling book

The present research investigated the ways in which a modelling book supported interaction and student learning in teacher-led small group work in mathematics through conversations among Māori students and between them and the teacher. Of particular interest was the extent to which the modelling book helped the teacher to bridge hands-on manipulation of materials with mathematical abstractions; and secondly, the usefulness of the modelling book in generating collective responsibility for the learning of the group. The first part of the paper presents theoretical frames for the analysis of the use of the modelling book as a classroom practice. The latter part of the paper examines evidence gathered from classrooms in the light of the extent to which modelling books were helpful learning tools for Māori students in the classroom investigated.

## Theoretical frames

### *Conceptualising Cognition as Physically and Socially Situated*

For the purposes of the discussion, the modelling book has been conceptualised as an inscription that facilitates interaction (Roth, Woszczyzna, & Smith, 1996). In their study of the affordances and constraints of computers in science education, Roth et al. noted that “the physical presence of the object of talk provides students with a means for coordinating just what they are talking about” (p. 1009). They argue that visual support is more important than manipulation of an object. In this sense, drawing on the earlier work of Erickson (1982), “students coordinate their conversation directly over and about the display which provides an anchor for the conversational topic” (p. 1009). Furthermore, they noted the importance of gestures as an important element of the co-ordination and sense-making and that “participation in discourse is dependent on access to the representational devices” (p. 1008).

This paper is based on the notion of cognition as physically and socially situated. The joining of the physical or material object (in this case, the modelling book) and the social setting (in this case, the small teacher-led group) is an important shift from the traditional notion of cognition as being solely in the heads of learners. While much has been recently written about socially situated cognition, the place of inscriptions in cognition has been less frequently examined. Notable exceptions to this are the work of Cobb (2002), Roth and McGinn (1998), and Sfard (2000), who use inscriptions in the sense that material objects are inscribed with messages.

## Findings

The claim is that modelling books can be used as a strategy for engaging students in mathematical discussion by supporting student interaction in a teacher-led small group. This support is given through co-ordinating the conversation and contributing to students' sense-making activities. Furthermore, modelling books may help teachers to reconceptualise hands-on learning to include discussion of mathematical ideas and as a means of developing conceptual understanding through the introduction of mathematical abstractions. Modelling books are a way of addressing Alton-Lee's (2003) challenge for students to "have the relevance of their learning activities made transparent" (p. 90). The rest of this paper will examine the evidence of the affordances and constraints of using modelling books as a pedagogical strategy to support discussion and manipulative use when solving mathematical problems in the teacher-led group. This will be discussed under the dimensions of physical, social, and conceptual aspects of modelling-book use. The illustrations are excerpts from interviews informed by classroom observations in eight English-medium year 7 and 8 classrooms across two regions in New Zealand as part of the evaluation of the NDP in 2005.

### *Physical Dimensions of Using Modelling Books*

The simple physical existence of a modelling book may afford students and teachers a reference point in teacher-led groups. It may act as a memory jog in tracking previous aspects of the discussion. Where records are illegible, their "scrawly/messy" form serves as a constraint by inhibiting the communication of ideas and distracting focus from the mathematical topic at hand. A visual record of the oral discussion may be particularly helpful for those with impaired hearing or with English as another language.

Writing it down is really important because it's the visual aspect to tune in. (Arthur, teacher)<sup>1</sup>

Cos it [the modelling book] is something to see instead of like doing it in your head and you forget when you go to do another part of it and you can see it on the paper. (Student)

Teachers commented on the usefulness of a modelling book for reminding them not only where a particular group is at but also where they need to go next to develop students' understanding of mathematics. Teachers using the Number Framework (Ministry of Education, 2006) to guide pedagogical decisions typically refer to stages of mathematical thinking (for instance, early additive or advanced multiplicative). The process of applying the Framework to the teaching of specific students is often a complex process involving ongoing analysis of student responses. A modelling book can support this process as a tool that is immediately to hand in the teaching setting (Roth & Tobin, 2001).

Having those [modelling books] open gives me the focus for that particular group and I look at things in the past. I've never grounded myself [before using modelling books] in terms of keeping to what the kids know and where I could take them. (Trisha, teacher)

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<sup>1</sup> Teachers' names have been changed.

Students commented that they found the modelling book helpful as a reminder of material covered, as well as a way of charting the teacher's examples.

Cos then we know what to do and she gives us good examples, and so next time we do it correctly, and she gives us harder ones, and then we do other things, and then she reminds us later on when we do it again. (Student)

She showed us the pattern that we did, just to remind us. (Student)

The placement of the modelling book, in terms of proximity and ease of access to the teacher and to students, is critical to the extent to which it can act as a reference point in discussion. The book may constrain interactions when it is physically accessible to the teacher but not to the students, such as when it is upside down or out of the students' reach. By contrast, the book affords or supports interaction when both students and teachers are able to physically access it, such as when it is placed on the floor in the centre of the group facing the right way up for students to see.

The physical space on a page, or parts of a page, of a modelling book may be regarded by group participants as "teacher", "student", or shared "teacher/student" space. This can be demonstrated by the ways in which participants physically (for example, through gestures) incorporate the inscriptions in the modelling book into their verbal responses. In the observed teacher-led sessions, teachers generally regarded the book as shared teacher/student space.

It's where I write, it's where they write. (Esther, teacher)

Students had a similar understanding that the book was a shared tool.

We can all sit around the table and look at the big maths [modelling] book. Sometimes [when people explain], they have a different way of figuring out from you, so you think, oh yeah, I could have done it like that too. (Student)

Students and teachers also appeared to have shared understanding that a teacher may use parts of a page as "teacher" space. This may be indicated to students through gesture, as in Figure 2,



Figure 2. Teacher space in modelling book indicated through teacher gesture

or as space that is specific to particular students:

Cos when she does it she does different [examples] for kind of like each person, she does a hard one for like Nancy, cos she knows quite a lot of maths and all that, and she does, like, quite an easy one for us lower kind of average kind of maths people. (Student)

### *Social Dimensions of Using Modelling Books*

Modelling books can fulfil a social function through supporting group interactions. The extent to which a book co-ordinates discussion is evident through reference to it in conversations. As a common focal point, it appears to be useful to both the teacher and to students in providing an anchor for the conversation that can contribute to the development of students' understanding.

If someone did jump ahead ... that's the same as a quarter or that's the same as two-quarters ... you could see where they were basically at in terms of just looking at what the [student] picture [in the modelling book] looks like. (Trisha, teacher)

If someone else has a different opinion, she just writes down on the side and then we all figure it out. (Student)

A book may become a shared recorded history of previous learning that affords both the teacher and students a means of informing discussion through linking back to previous mathematics sessions.

The following day, you go back and you've got it right there in front of you, you haven't got everybody's workbook out ... There's times in my classroom when I've had to hunt those books down. (Trisha, teacher)

The kids can go back to it. (Esther, teacher)

Such opportunities may be constrained when the book is merely the teacher's recorded history of previous learning. In this case, it becomes the teacher's choice of whose ideas are recorded and how this is done.

The teacher may specifically use the book as a means of focusing each group as they take their turn with the teacher. The visual record displayed affords both the teacher and students a quick reference point.

And the main thing I use [the modelling book] for is to be focused ... I'll do three groups today, I'll do two groups tomorrow ... then they come back to me in two days' time ... they'll come back, they'll refocus, we'll talk about it and they can see visually and they connect them, that's how I use it. (Arthur, teacher)

... one of the kids said, "Whaea, I don't remember how to play that game." I said, "Gee, I wonder if that will be in our workbook – shall I go and find out?" She goes, "Oh yeah," and away she went, she opened it up and then had a look at the samples that we did and they got it. (Trisha, teacher)

Students find a modelling book useful as a reference point for gauging their progress as a group.

So we know what we have improved on and what we need to improve on. (Student)

She keeps old work in the big scrapbook so we can see if we've improved. Like we started off and we've gone to the end of our maths and seen if we've improved ... sometimes I don't think I've improved. (Student)

[if you didn't have the scrapbook in the middle] we'd miss what we improved and we won't know if we've learnt anything or not. (Student)

Students may become motivated to participate in discussion by seeing their ideas valued through being recorded. From the students' perspective, it becomes a recorded shared history of previous learning while making students' ideas accessible to all other students.

It's got all our pieces of work that we've finished, and she writes down some ideas from the lessons that we've been taught in maths. (Student)

Furthermore, a modelling book has potential as a collective mechanism for supporting the enactment of groups as an instructional tool such as described by a teacher in a previous study who thought of her class as a waka, or groups within groups (Higgins et al., 2005).

### *Conceptual Dimensions of Using Modelling Books*

Having a way of recording immediately to hand allows teachers to introduce the symbolic form through providing students with a bridge between physical models and mathematical abstractions. The modelling book provides a means of making the links between models and abstractions explicit. However, teachers may be unable to make links between recording (inscriptions) and mathematical abstractions because of limited subject and/or pedagogical content knowledge.

Yeah, it's important to have the written recording alongside actual hands-on stuff. You can do lots of hands-on stuff, but then you give them a Figure It Out book and perhaps like three times two and they go what does that mean ... So that recording is there. So anything I do or any equation we talk about we write it down. ... So quite often I give them the pen [and ask] "How would you write that down?" (Esther, teacher)

As part of this process, the teacher and students are using literacy to support the language (vocabulary and syntax/structure) of mathematics.

They say three groups of two ... you write down three groups of two ... and I'll actually put down three times two. So ... they're picking up the language ... (Trisha, teacher)

The act of recording gives students thinking time by slowing down the pace of the lesson through the extra time taken to record the ideas raised in discussion.

She makes us think instead of her just saying it. (Student)

Students saw the modelling book as a tool for the teacher to track their conceptual understanding.

She keeps a [modelling] book so she knows, she can find out the answers that we like to say and how we can improve on doing what we've done, making the same answer but making it sound better. (Student)

She ... tries to make you improve on it so you can find different answers for [the problem] ... She finds other ways for you to answer it so it's easier. (Student)

Sometimes we don't know that we have the answers and we try and ... we end up finding the answer ... we say it [and] find out that we did know. ... It was helping us learn for something we already knew but didn't know we did know it. ... If we didn't have the book, we'd pretty much never know all the things we did know. (Student)

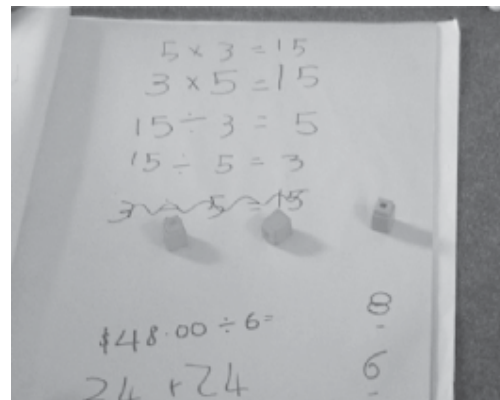


Figure 3. Tracking students' conceptual understanding



## Conclusion

This paper began by discussing the problems of using the notion of kinaesthetic learners as a strategy for working with Māori students. As has been argued elsewhere (Higgins, 2001), the manipulation of materials by itself will not necessarily lead to an understanding of mathematical principles. Nor can we assume that students understand the purpose of a pedagogical tool (McDonald et al., 2005).

The modelling book reinforces the complexities of hands-on learning in ways that the notion of kinaesthetic learning tends to gloss over. The aspects of learning made visible through the use of modelling books in mathematics learning may provide important support for diverse learners.

The modelling book gives the students additional information beyond the manipulation of materials and participation in discussion that they can use in building their mathematical understanding. It also supports the development of collective enterprise in solving mathematical problems. In the settings in which the use of modelling books was investigated, Māori students responded well and there is no reason why this approach should not suit all students.

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