# Performance on the Number Framework 

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## Introduction

In 2005, the Ministry of Education offered New Zealand secondary schools an opportunity to improve the teaching and learning of number concepts and skills through the Secondary Numeracy Pilot Project (SNP), a professional development programme for teachers.

The overall aim of the SNP is to develop teachers' knowledge of number concepts, student strategies for operating with number, and instructional practice in order to improve student achievement in year 9 .

For the SNP, the Numeracy Development Project (NDP) diagnostic interview used to assess students against the Framework in both strategy and knowledge was modified to better suit the needs of secondary schools. SNP teachers have only one set of strategy questions to use in their individual interviews with students at the beginning of the project (term 1) and at the end (term 4). The knowledge domains are assessed as a pen-and-paper whole-class assessment.

A key aspect of the evaluation of the SNP is to quantify any improvement. This chapter aims to address the following research questions:

1. Do SNP students make progress on the Number Framework?
2. How does this progress compare to that of year 8 students in NDP schools?
3. What demographic factors impact on the progress and performance of SNP students?
4. Is there a relationship between students' number knowledge profiles and their progress on the multiplicative strategy domain?

The results in this chapter are divided into three sections:

- The performance of students on the strategy domains. This describes students' abilities to operate with number.
- The performance of students on the knowledge domains. This describes the key items of number knowledge.
- The relationship between students' use of strategies and their number knowledge.

Where overall differences are described between groups, a T-test has been carried out to verify a difference to at least the $95 \%$ confidence level. In addition, differences in percentages of students at particular levels of each domain are not reported unless they are greater than $5 \%$. It needs to be noted that, in some instances, the figures show some significantly different mean gains that may be smaller than other gains in the same figures that are not statistically significant due to differences in sample size. In all tables, percentages are rounded. Percentages less than 0.5\% are therefore shown as $0 \%$, and where there are no students represented, the cell is left blank.

## Sample

The results reported in this chapter were obtained by downloading data from the online Numeracy Database on 23 January 2006. Results from all students in SNP schools were included, providing these results included an initial and final entry for each of the seven domains assessed.

Of the 43 schools that participated in the SNP in 2005, results were available at this time for 3975 students from 31 schools. Table 1 comprises a breakdown of these students by ethnicity. Twothirds ( $66 \%$ ) of the students were of New Zealand European origin, $20 \%$ identified as Māori, and approximately $5 \%$ identified as each of Pasifika, Asian, or other. There were more male students than female ( $57 \%$ compared with $43 \%$ ). (This gender imbalance in the results was caused by the data from several girls' schools not being available at the time that data for this evaluation was downloaded.)

Table 1
Profile of SNP Students by Ethnicity

| Ethnicity | Percentage |
| :--- | :---: |
| NZ European | 66 |
| Màori | 20 |
| Pasifika | 5 |
| Asian | 4 |
| Other | 5 |
| Total | 3975 |

## Performance on the Strategy Domains

## Additive Domain

Tables 2-5 and figures 1-3 present the results of SNP students on the additive domain, which describes students' strategies for solving addition and subtraction problems.

Table 2, comparing initial and final additive stages, shows that the percentage of students at the top two stages ( 6 and 7 ) of the additive domain increased from $45 \%$ at the start of the project to $69 \%$ by the final assessment. These students are able to use a range of mental part-whole strategies to solve addition and subtraction problems. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from 15\% to 5\% over the course of the project. There were still $5 \%$ of students unable to partition numbers mentally, and this is a cause for concern.

Table 2
Initial and Final Additive Stages

|  | \% initial additive | \% final additive |
| :--- | :---: | :---: |
| 0: Emergent | 0 | 0 |
| 1: One-to-one counting | 0 |  |
| 2: Counting from one on materials | 0 | 0 |
| 3: Counting from one by imaging | 1 | 0 |
| 4: Advanced counting | 14 | 5 |
| 5: Early additive part-whole | 39 | 26 |
| 6: Advanced additive part-whole | 37 | 46 |
| 7: Advanced multiplicative part-whole | 8 | 23 |
| Number of students | 3975 | 3975 |

Note: Percentages may not total to 100 due to rounding.

Table 3 shows the initial and final additive stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were more likely than Māori or Pasifika students to be at the higher stages of the additive domain, with $72 \%, 53 \%$, and $57 \%$ respectively reaching the top two stages by the end of project.

Table 3
Initial and Final Additive Stage by Ethnicity

|  | NZ European |  |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Additive stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |  |
| 0: Emergent | 0 |  | 0 | 0 |  |  |  |
| 1: One-to-one counting | 0 |  |  |  |  |  |  |
| 2: Counting from one on materials | 0 | 0 | 1 | 0 | 1 |  |  |
| 3: Counting from one by imaging | 1 | 0 | 2 | 1 | 2 | 1 |  |
| 4: Advanced counting | 12 | 5 | 22 | 9 | 29 | 6 |  |
| 5: Early additive part-whole | 38 | 23 | 45 | 37 | 42 | 36 |  |
| 6: Advanced additive part-whole | 40 | 47 | 27 | 42 | 24 | 45 |  |
| 7: Advanced multiplicative part-whole | 9 | 25 | 4 | 11 | 2 | 12 |  |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |  |

Note: Percentages may not total to 100 due to rounding.
Table 4 shows that male students ( $26 \%$ ) were more likely than female students ( $18 \%$ ) to reach the top stage of the additive domain. Similar percentages ( $5 \%$ and $6 \%$ respectively) remained at the advanced counting stage or lower.

Table 4
Initial and Final Additive Stage by Gender

|  | Male |  | Female |  |
| :--- | :---: | :---: | :---: | :---: |
| Additive stage | \% initial | \% final | \% initial | \% final |
| 0: Emergent | 0 |  | 0 | 0 |
| 1: One-to-one counting |  |  | 0 |  |
| 2: Counting from one on materials | 0 | 0 | 0 | 0 |
| 3: Counting from one by imaging | 1 | 0 | 1 | 0 |
| 4: Advanced counting | 12 | 5 | 18 | 6 |
| 5: Early additive part-whole | 35 | 22 | 44 | 31 |
| 6: Advanced additive part-whole | 42 | 46 | 30 | 45 |
| 7: Advanced multiplicative part-whole | 10 | 26 | 6 | 18 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.

Table 5, a breakdown of additive performance by school decile band, shows that students in the high-decile bands performed better than students in either medium- or low-decile bands, with $30 \%$ reaching stage 7 , compared with $19 \%$ of students in medium-decile and $14 \%$ of students in low-decile bands.

Table 5
Initial and Final Additive Stage by Decile

|  | Low |  | Medium |  | High |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Additive stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0: Emergent | 1 |  | 0 |  | 0 | 0 |
| 1: One-to-one counting |  |  | 0 |  |  |  |
| 2: Counting from one on materials | 0 |  | 0 | 0 | 0 | 0 |
| 3: Counting from one by imaging | 1 | 1 | 1 | 0 | 1 | 0 |
| 4: Advanced counting | 20 | 8 | 17 | 7 | 9 | 3 |
| 5: Early additive part-whole | 47 | 33 | 39 | 28 | 37 | 22 |
| 6: Advanced additive part-whole | 26 | 45 | 36 | 46 | 41 | 45 |
| 7: Advanced multiplicative part-whole | 5 | 14 | 6 | 19 | 12 | 30 |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |

Note: Percentages may not total to 100 due to rounding.
Figure 1 compares the numbers of stages that SNP students gained on the additive domain with the numbers of stages gained from the same starting stage by year 8 students participating in the NDP. For both samples, the numbers of students initially rated at stages $0-3$ were very small, so these results have been excluded from the following figures. The pattern of students' performance initially rated at stages 4-6 is similar for the two samples, with a higher proportion of SNP students initially at stage 5 progressing to at least stage 6 ( $p<0.01$ ) and a higher proportion of year 8 students initially at stage 6 progressing to stage 7 ( $p<0.001$ ).


Figure 1. Number of stages gained by initial additive stage for SNP and year 8 students

Figure 2 shows the progress made on the additive domain by students' initial stage and ethnicity. A significantly higher proportion of New Zealand European students than Māori students progressed from stage 5 ( $p<0.001$ ) and from stage 6 ( $p<0.001$ ).


Figure 2. Number of stages gained by initial additive stage and ethnicity

Figure 3 shows that students from high-decile schools are significantly more likely than mediumdecile ( $\mathrm{p}<0.05$ ) or low-decile ( $\mathrm{p}<0.01$ ) students to make progress from stage 5 on the additive domain. The same pattern applies to students initially at stage 6 ( $\mathrm{p}<0.001$ and $\mathrm{p}<0.05$ respectively). The proportions of students from medium- and low-decile schools who made progress were similar.


Figure 3. Number of stages gained by initial additive stage and decile band

Tables 6-9 and figures 4-6 present the results of SNP students on the multiplicative domain, which describes students' strategies for solving multiplication and division problems.

Table 6 compares the initial and final multiplicative stages and shows that the percentage of students at the top two stages (7 and 8) of the multiplicative domain increased from $25 \%$ at the start of the project to $46 \%$ by the final assessment. These students are able to use a range of advanced mental strategies to solve multiplication and division problems. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from $14 \%$ to $6 \%$ over the course of the project. There were still $6 \%$ of students who used skipcounting as their most advanced multiplicative strategy, and this is a cause for concern.
Table 6
Initial and Final Multiplicative Stage

|  | \% initial | multiplicative |
| :--- | :---: | :---: | \% final multiplicative.

Table 7 shows the initial and final multiplicative stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were considerably more likely than Māori or Pasifika students to be at the higher stages of the multiplicative domain, with $51 \%, 30 \%$, and $27 \%$ respectively reaching the top two stages by the end of the project.

Table 7
Initial and Final Multiplicative Stage by Ethnicity

|  | NZ European |  |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Multiplicative stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |  |
| 2-3: Counting from one | 1 | 0 | 3 | 1 | 5 | 1 |  |
| 4: Advanced counting | 9 | 5 | 20 | 10 | 24 | 11 |  |
| 5: Early additive part-whole | 25 | 14 | 33 | 23 | 36 | 20 |  |
| 6: Advanced additive part-whole | 35 | 31 | 31 | 37 | 23 | 41 |  |
| 7: Advanced multiplicative part-whole | 23 | 33 | 11 | 23 | 11 | 22 |  |
| 8: Advanced proportional part-whole | 6 | 18 | 2 | 7 | 2 | 5 |  |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |  |

Note: Percentages may not total to 100 due to rounding.

Table 8 shows that male students (53\%) were more likely than female students (39\%) to reach the top two stages of the multiplicative domain. Similar percentages ( $5 \%$ and $6 \%$ respectively) remained at the advanced counting stage or lower.

Table 8
Initial and Final Multiplicative Stage by Gender

|  | Male |  | Female |  |
| :--- | ---: | ---: | ---: | ---: |
| Multiplicative stage | \% initial | \% final | \% initial | \% final |
| 2-3: Counting from one | 2 | 0 | 2 | 0 |
| 4: Advanced counting | 10 | 5 | 14 | 6 |
| 5: Early additive part-whole | 23 | 13 | 33 | 19 |
| 6: Advanced additive part-whole | 35 | 29 | 32 | 36 |
| 7: Advanced multiplicative part-whole | 23 | 33 | 15 | 28 |
| 8: Advanced proportional part-whole | 7 | 20 | 3 | 11 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.
Table 9 presents a breakdown of additive performance by decile band. Students from highdecile schools performed better than students from either medium- or low-decile schools, with $58 \%$ reaching stage 7 or higher compared with $41 \%$ of students from medium-decile schools and $32 \%$ from low-decile schools. At the lower stages of the domain, only $3 \%$ of students from highdecile schools and $6 \%$ of students from medium-decile schools remained at the advanced counting stage or lower, compared with $13 \%$ from low-decile schools.

Table 9
Initial and Final Multiplicative Stage by Decile

|  | Low |  | Medium |  | High |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Multiplicative stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 2-3: Counting from one | 4 | 1 | 2 | 0 | 1 | 0 |
| 4: Advanced counting | 21 | 12 | 14 | 6 | 6 | 3 |
| 5: Early additive part-whole | 32 | 20 | 29 | 18 | 23 | 11 |
| 6: Advanced additive part-whole | 27 | 35 | 34 | 34 | 36 | 28 |
| 7: Advanced multiplicative part-whole | 14 | 24 | 18 | 30 | 25 | 34 |
| 8: Advanced proportional part-whole | 2 | 8 | 4 | 11 | 9 | 24 |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |

Note: Percentages may not total to 100 due to rounding.

Figure 4 compares the numbers of stages that SNP students gained on the multiplicative domain with the numbers of stages gained from the same starting stage by year 8 students from schools participating in the NDP (many year 8 students were in their second year of the Intermediate Numeracy Project). Again, the numbers of students initially rated below stage 4 were very low and have been excluded from the following figures. The pattern of performance of students initially rated at stages $4-7$ was very similar for the two samples, with SNP students initially at stage 6 making significantly greater gains ( $p<0.01$ ) than year 8 students.


Figure 4. Number of stages gained by initial multiplicative stage for SNP and year 8 students

Figure 5 compares progress made on the multiplicative domain by students' initial stage and ethnicity. A significantly higher proportion of New Zealand European students than Māori students made progress from stage 6 ( $\mathrm{p}<0.001$ ) and from stage 7 ( $\mathrm{p}<0.01$ ).


Figure 5. Number of stages gained by initial multiplicative stage and ethnicity

Figure 6 shows that a significantly higher proportion of students from high-decile schools made progress from stage 6 on the multiplicative domain than medium-decile ( $\mathrm{p}<0.001$ ) or lowdecile ( $\mathrm{p}<0.001$ ) students. A significantly higher proportion of high-decile than medium-decile students also made progress from stage 7 ( $p<0.001$ ).


Figure 6. Number of stages gained by initial multiplicative stage and decile band

## Proportional Strategy Domain

Tables 10-13 and figures 7-9 show the results of SNP students on the proportional domain, which describes students' ability to solve problems involving ratios and proportions.

Table 10, comparing initial and final proportional stages, shows that the percentage of students at the top two stages ( 7 and 8 ) of the proportional domain increased from $36 \%$ in the initial assessment to $53 \%$ by the end of the year. These students use multiplication and division to find fractions of numbers. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from $17 \%$ to $6 \%$ over the course of the project. There were still $6 \%$ of students who needed to share out objects to find fractions of a set, and this is a cause for concern.

Table 10
Initial and Final Proportional Stage

|  | \% initial proportional | \% final proportional |
| :--- | :---: | :---: |
| 1: Unequal sharing | 1 | 0 |
| 2-4: Equal sharing | 16 | 6 |
| 5: Early additive part-whole | 29 | 23 |
| 6: Advanced additive part-whole | 17 | 17 |
| 7: Advanced multiplicative part-whole | 31 | 41 |
| 8: Advanced proportional part-whole | 5 | 12 |
| Number of students | 3975 | 3975 |

Note: Percentages may not total to 100 due to rounding.
Table 11 shows the initial and final proportional stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were more likely than Māori or Pasifika students to be at the higher stages of the proportional domain, with $60 \%, 34 \%$, and $26 \%$ respectively reaching the top two stages.

Table 11
Initial and Final Proportional Stage by Ethnicity

|  | NZ European |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Proportional stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 1: Unequal sharing | 1 | 0 | 2 | 1 | 2 |  |
| 2-4: Equal sharing | 12 | 5 | 27 | 12 | 33 | 12 |
| 5: Early additive part-whole | 26 | 19 | 35 | 35 | 41 | 34 |
| 6: Advanced additive part-whole | 18 | 15 | 15 | 17 | 11 | 27 |
| 7: Advanced multiplicative part-whole | 36 | 46 | 19 | 30 | 11 | 22 |
| 8: Advanced proportional part-whole | 5 | 14 | 2 | 4 | 1 | 4 |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |

Note: Percentages may not total to 100 due to rounding.
Table 12 shows that male students ( $59 \%$ ) are more likely than female students ( $47 \%$ ) to reach the top two stages of the proportional domain. Similar percentages ( $6 \%$ and $7 \%$ respectively) remained at the advanced counting stage or lower (stages 1-4).

Table 12
Initial and Final Proportional Stage by Gender

|  | Male |  | Female |  |
| :--- | :---: | :---: | :---: | :---: |
| Proportional stage | \% initial | \% final | \% initial | \% final |
| 1: Unequal sharing | 1 | 0 | 2 | 0 |
| 2-4: Equal sharing | 15 | 6 | 18 | 7 |
| 5: Early additive part-whole | 27 | 20 | 32 | 26 |
| 6: Advanced additive part-whole | 16 | 15 | 19 | 19 |
| 7: Advanced multiplicative part-whole | 34 | 43 | 27 | 39 |
| 8: Advanced proportional part-whole | 7 | 16 | 2 | 8 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.
Table 13 presents a breakdown of performance on the proportional domain by decile band, showing that students from high-decile schools performed better than students from either medium- or low-decile schools, with $65 \%$ reaching stage 7 or higher compared with $48 \%$ of students from medium-decile schools and $39 \%$ from low-decile schools. At the lower stages of the domain, only $4 \%$ of students from high-decile schools remained at the equal sharing stage or
lower, compared with $7 \%$ of students from medium-decile schools and $16 \%$ of students from low-decile schools.

Table 13
Initial and Final Proportional Stage by Decile

|  | Low |  | Medium |  | High |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Proportional stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 1: Unequal sharing | 5 | 1 | 2 | 0 | 1 | 0 |
| 2-4: Equal sharing | 21 | 15 | 20 | 7 | 10 | 4 |
| 5: Early additive part-whole | 37 | 29 | 30 | 26 | 25 | 16 |
| 6: Advanced additive part-whole | 12 | 16 | 19 | 18 | 17 | 15 |
| 7: Advanced multiplicative part-whole | 23 | 33 | 26 | 39 | 40 | 47 |
| 8: Advanced proportional part-whole | 2 | 6 | 4 | 9 | 7 | 18 |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |

Note: Percentages may not total to 100 due to rounding.

Figure 7 compares the numbers of stages gained on the proportional domain by SNP students with the numbers of stages gained from the same starting stage by year 8 students from schools participating in the NDP. The small numbers of students initially rated below stages $2-4$ have been excluded from the following figures. The pattern of performance of students initially rated at stages $2-4$ and higher is similar for the two samples. Because of the higher proportions of students gaining more than one stage, the mean gains of SNP students initially at stage 5 are significantly greater than those of year 8 students ( $p<0.05$ ).


Figure 8 shows the progress made on the proportional domain by students' initial stage and ethnicity. A significantly higher proportion of NZ European students than Māori students made progress from stage $5(p<0.001)$ and from stage $7(p<0.001)$. A significantly higher proportion of Pasifika students than Māori students made progress from stage 5 ( $\mathrm{p}<0.05$ ).


Figure 8. Number of stages gained by initial proportional stage and ethnicity

Figure 9 shows that a significantly higher proportion of students from high-decile schools than low-decile schools made progress from stages 2 to 4 ( $\mathrm{p}<0.05$ ), 5 ( $\mathrm{p}<0.01$ ), and 6 ( $\mathrm{p}<0.05$ ). A significantly higher proportion of students from medium-decile schools than from low-decile schools made progress from stages $5(\mathrm{p}<0.01)$ and $6(\mathrm{p}<0.05)$. A significantly higher proportion of students from high-decile schools than from medium-decile schools made progress from stage 7 ( $\mathrm{p}<0.05$ ).


Figure 9. Number of stages gained by initial proportional stage and decile band

## Comparison of Strategy Domains

Table 14 shows the percentages of students at each stage at the final assessment for all three strategy domains. It would be expected that students would be more likely to be at the higher stages of the additive domain than of the multiplicative domain and more likely to be at the higher stages of the multiplicative domain than of the proportional domain. While there were similar proportions of students still at the counting stages of each domain, the proportions at the higher stages were mixed, with only $23 \%$ of students rated at stage 7 of the additive domain, compared with $46 \%$ and $53 \%$ at either stage 7 or 8 of the multiplicative and proportional domains respectively.

Table 14
Final Strategy Stages - All Domains

|  | Domain |  |  |
| :--- | :---: | :---: | :---: |
| Stage | \% additive | \% multiplicative | \% proportional |
| < 4: Counting from one | 1 | 0 | 1 |
| 4: Advanced counting | 5 | 6 | 6 |
| 5: Early additive part-whole | 26 | 16 | 23 |
| 6: Advanced additive part-whole | 46 | 32 | 17 |
| 7: Advanced multiplicative part-whole | 23 | 30 | 41 |
| 8: Advanced proportional part-whole | $\mathrm{N} / \mathrm{A}$ | 16 | 12 |

Note: Percentages may not total to 100 due to rounding.

## Performance on the Knowledge Domains

## Forward Number Word Sequence

Tables 15-18 present the results of SNP students on the Forward Number Word Sequence (FNWS) domain, which describes students' ability to identify the number after a given number.

Over the duration of the project, the proportion of students able to identify the number after a given number in the range 1 to 1000000 (stage 6) increased from $54 \%$ to $71 \%$ (see Table 15).

Table 15
Initial and Final FNWS Stage

|  | \% initial FNWS | \% final FNWS |
| :---: | :---: | :---: |
| 0: Emergent | 0 | 0 |
| 1: Initial to 10 | 0 |  |
| 2: To 10 | 1 | 0 |
| 3: To 20 | 1 | 0 |
| 4: To 100 | 3 | 2 |
| 5: To 1000 | 41 | 27 |
| 6: To 1000000 | 54 | 71 |
| Number of students | 3975 | 3975 |

Table 16 shows that a higher proportion of New Zealand European students ( $74 \%$ ) than either Māori (57\%) or Pasifika (65\%) students reached the top stage of the FNWS domain.

Table 16
Initial and Final FNWS Stage by Ethnicity

|  | NZ European |  | Māori |  | Pasifika |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| FNWS stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0: Emergent | 0 |  | 0 |  |  |  |
| 1: Initial to 10 | 0 |  |  |  |  |  |
| 2: To 10 | 0 | 0 | 1 | 1 | 4 | 3 |
| 3: To 20 | 1 | 0 | 3 | 1 |  | 1 |
| 4: To 100 | 2 | 1 | 5 | 3 | 6 | 2 |
| 5: To 1000 | 39 | 25 | 51 | 37 | 47 | 31 |
| 6: To 1000 000 | 58 | 74 | 40 | 57 | 43 | 65 |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |

Note: Percentages may not total to 100 due to rounding.
Table 17 shows that a higher percentage of male ( $73 \%$ ) than female ( $68 \%$ ) students reached the top stage of the FNWS domain.

Table 17
Initial and Final FNWS Stage by Gender

|  | Male |  | Female |  |
| :--- | :---: | :---: | :---: | :---: |
| FNWS stage | \% initial | \% final | \% initial | \% final |
| 0: Emergent | 0 |  |  | 0 |
| 1: Initial to 10 |  |  | 0 |  |
| 2: To 10 | 1 | 1 | 1 | 0 |
| 3: To 20 | 1 | 0 | 1 | 1 |
| 4: To 100 | 3 | 2 | 3 | 1 |
| 5: To 1000 | 38 | 24 | 45 | 30 |
| 6: To 1 000 000 | 57 | 73 | 51 | 68 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.

Table 18 shows that a higher proportion of students from high-decile schools ( $74 \%$ ) than students from medium-decile ( $69 \%$ ) or low-decile ( $64 \%$ ) schools reached the top stage of the FNWS domain.

Table 18
Initial and Final FNWS Stage by Decile

|  | Low |  | Medium |  | High |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| FNWS stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0: Emergent |  |  | 0 |  |  | 0 |
| 1: Initial to 10 |  |  | 0 |  |  |  |
| 2: To 10 | 2 | 2 | 1 | 0 | 0 | 0 |
| 3: To 20 | 0 |  | 1 | 1 | 1 | 0 |
| 4: To 100 | 6 | 3 | 3 | 2 | 1 | 0 |
| 5: To 1000 | 46 | 31 | 40 | 27 | 40 | 24 |
| 6: To 1 000 000 | 46 | 64 | 53 | 69 | 57 | 75 |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |

Note: Percentages may not total to 100 due to rounding.

## Fractions Domain

Tables 19-22 illustrate the performance of SNP students on the fractions domain, which describes students' ability to identify and order fractions.

Table 19 shows that the percentage of students able to identify equivalent fractions (stage 7 or 8) increased from $22 \%$ at the initial assessment to $40 \%$ by the final assessment. The percentage of students still unable to order unit fractions (stage 4 or lower) decreased from $18 \%$ to $9 \%$.

Table 19
Initial and Final Fractions Stage

|  | \% initial fractions | \% final fractions |
| :--- | :---: | :---: |
| 2-3: Non-fractions | 5 | 2 |
| 4: Assigns unit fractions | 13 | 7 |
| 5: Orders unit fractions | 39 | 29 |
| 6: Co-ordinates numerators/denominators | 21 | 23 |
| 7: Equivalent fractions | 15 | 25 |
| 8: Orders fractions | 7 | 15 |
| Number of students | 3975 | 3975 |

[^0]Table 20 shows that New Zealand European students performed better than Māori and Pasifika students on the fractions domain, with $46 \%$ reaching the top two stages of the domain, compared with $22 \%$ and $23 \%$ for Māori and Pasifika students respectively. Fewer Pasifika ( $11 \%$ ) than Māori $(17 \%)$ students remained unable to order unit fractions by the end of the project.

Table 20
Initial and Final Fractions Stage by Ethnicity

|  | NZ European |  |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Fractions stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |  |
| 2-3: Non-fractions | 4 | 1 | 11 | 3 | 8 | 4 |  |
| 4: Assigns unit fractions | 11 | 5 | 19 | 14 | 15 | 7 |  |
| 5: Orders unit fractions | 40 | 28 | 43 | 37 | 49 | 33 |  |
| 6: Co-ordinates numerators/denominators | 22 | 21 | 17 | 24 | 20 | 34 |  |
| 7: Equivalent fractions | 16 | 28 | 8 | 14 | 5 | 16 |  |
| 8: Orders fractions | 8 | 18 | 2 | 8 | 2 | 7 |  |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |  |

Note: Percentages may not total to 100 due to rounding.
Table 21 shows that while a higher percentage of male ( $18 \%$ ) than female ( $12 \%$ ) students reached the top stage of the fractions domain, similar percentages ( $9 \%$ and $8 \%$ respectively) remained unable to order unit fractions (stage 4 or lower).

Table 21
Initial and Final Fractions Stage by Gender

|  | Male |  | Female |  |
| :--- | ---: | :---: | ---: | :---: |
| Fractions stage | $\%$ initial | \% final | \% initial | \% final |
| 2-3: Non-fractions | 7 | 2 | 4 | 1 |
| 4: Assigns unit fractions | 12 | 7 | 13 | 7 |
| 5: Orders unit fractions | 37 | 27 | 42 | 30 |
| 6: Co-ordinates numerators/denominators | 19 | 21 | 23 | 25 |
| 7: Equivalent fractions | 17 | 25 | 12 | 25 |
| 8: Orders fractions | 8 | 18 | 5 | 12 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.

As shown in Table 22, students from high-decile schools performed better than students from medium-decile schools on the fractions domain and both performed better than students from low-decile schools, with $71 \%, 61 \%$, and $46 \%$ respectively reaching at least stage 6 by the end of the project.

Table 22
Initial and Final Fractions Stage by Decile

|  | Low |  |  | Medium |  | High |  |
| :--- | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| Fractions stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |  |
| 2-3: Non-fractions | 11 | 3 | 6 | 2 | 3 | 1 |  |
| 4: Assigns unit fractions | 20 | 12 | 14 | 7 | 9 | 4 |  |
| 5: Orders unit fractions | 40 | 38 | 43 | 30 | 35 | 24 |  |
| 6: Co-ordinates numerators/denominators | 19 | 17 | 19 | 25 | 24 | 21 |  |
| 7: Equivalent fractions | 7 | 17 | 12 | 23 | 20 | 30 |  |
| 8: Orders fractions | 4 | 12 | 6 | 13 | 9 | 20 |  |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |  |

Note: Percentages may not total to 100 due to rounding.

## Place Value Domain

Tables $23-26$ present the results of SNP students on the place value domain, which describes students' ability to partition whole numbers and decimals using their place value.

Table 23 shows that the percentage of students at least able to identify the number of tenths in numbers and order decimals (stage 7 or 8) increased from 18\% at the initial assessment to 35\% by the end of the project.

Table 23
Initial and Final Place Value Stage

|  | \% initial place value | \% final place value |
| :--- | :---: | :---: |
| 0-1: Non-counting | 0 |  |
| 2-3: Counts by ones | 2 | 1 |
| 4: 10s to 100, order to 1000 | 11 | 5 |
| 5: 10s to 1000, order to 10 000 | 49 | 35 |
| 6: 10s, 100s, 1000s, orders whole numbers | 19 | 25 |
| 7: Tenths in and orders decimals | 10 | 18 |
| 8: Tenths hundredths and thousandths | 8 | 17 |
| Number of students | 3975 | 3975 |

[^1]Table 24 shows that a higher proportion of New Zealand European students (39\%) than Pasifika ( $27 \%$ ) or Māori ( $17 \%$ ) students reached the top stages of the place value domain.

Table 24
Initial and Final Place Value Stage by Ethnicity

|  | NZ European |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Place value stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0-1: Non-counting | 0 |  |  |  | 1 |  |
| 2-3: Counts by ones | 2 | 0 | 4 | 1 | 4 | 3 |
| 4: 10s to 100, order to 1000 | 9 | 4 | 16 | 9 | 17 | 4 |
| 5: 10s to 1000, order to 10 000 | 48 | 32 | 56 | 46 | 58 | 41 |
| 6: 10s, 100s, 1000s, orders whole numbers | 20 | 24 | 16 | 28 | 14 | 25 |
| 7: Tenths in and orders decimals | 11 | 20 | 5 | 11 | 5 | 22 |
| 8: Tenths hundredths and thousandths | 10 | 19 | 2 | 6 | 2 | 5 |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |

Note: Percentages may not total to 100 due to rounding.
Table 25 shows that while a higher percentage of male (37\%) than female ( $32 \%$ ) students reached at least stage 7 of the place value domain, a similar percentage of male ( $6 \%$ ) to that of female (4\%) students remained at stage 4 or lower.

Table 25
Initial and Final Place Value Stage by Gender

|  | Male |  | Female |  |
| :--- | :---: | :---: | :---: | :---: |
| Place value stage | \% initial | \% final | \% initial | \% final |
| 0-1: Non-counting | 0 |  | 0 |  |
| 2-3: Counts by ones | 3 | 1 | 2 | 0 |
| 4: 10s to 100, order to 1000 | 11 | 5 | 11 | 4 |
| 5: 10s to 1000, order to 10 000 | 47 | 34 | 52 | 36 |
| 6: 10s, 100s, 1000s, orders whole numbers | 18 | 23 | 20 | 27 |
| 7: Tenths in and orders decimals | 10 | 17 | 10 | 19 |
| 8: Tenths hundredths and thousandths | 11 | 20 | 5 | 13 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.

Table 26 shows that students from high-decile schools were more likely to reach the higher stages of the place value domain than students from medium-decile schools and that both were more likely than students from low-decile schools.

Table 26
Initial and Final Place Value Stage by Decile

|  | Low |  |  |  |  |  |  | Medium |  | High |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Place value stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |  |  |  |  |  |
| 0-1: Non-counting | 0 |  | 0 |  |  |  |  |  |  |  |  |
| 2-3: Counts by ones | 4 | 1 | 2 | 1 | 2 | 1 |  |  |  |  |  |
| 4: 10s to 100, order to 1000 | 21 | 8 | 10 | 5 | 9 | 4 |  |  |  |  |  |
| 5: 10s to 1000, order to 10 000 | 51 | 43 | 51 | 36 | 46 | 31 |  |  |  |  |  |
| 6: 10s, 100s, 1000s, orders whole numbers | 17 | 24 | 20 | 27 | 17 | 21 |  |  |  |  |  |
| 7: Tenths in and orders decimals | 6 | 17 | 9 | 17 | 12 | 20 |  |  |  |  |  |
| 8: Tenths hundredths and thousandths | 2 | 7 | 6 | 14 | 13 | 24 |  |  |  |  |  |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |  |  |  |  |  |

Note: Percentages may not total to 100 due to rounding.

## Basic Facts Domain

Tables 27 to 30 illustrate the performance of SNP students on the basic facts domain, which rates students' ability to quickly recall basic number facts.

Table 27 shows that the percentage of students who knew at least their subtraction and multiplication basic facts (stage 6 or higher) increased from $71 \%$ at the initial assessment to $82 \%$ by the final assessment. At the end of the project, $4 \%$ of students were still unable to recall their multiplication facts for 2,5 , and 10 (that is, were still below stage 5 ).

Table 27
Initial and Final Basic Facts Stage

|  | \% initial basic facts | \% final basic facts |
| :--- | :---: | :---: |
| 0-1: Non-grouping | 1 | 0 |
| 2-3: Within and with five | 2 | 1 |
| 4: Within 10, doubles, and teens | 6 | 3 |
| 5: Addition, multiplication for 2, 5, 10 | 21 | 14 |
| 6: Subtraction and multiplication | 47 | 44 |
| 7: Division | 24 | 38 |
| Number of students | 3975 | 3975 |

[^2]Table 28 shows that New Zealand European students performed better than either Māori or Pasifika students on the basic facts domain. While a slightly higher proportion of Pasifika (78\%) than Māori ( $73 \%$ ) students knew their subtraction and multiplication facts (stage 6 or higher) by the end of the project, a similar proportion ( $7 \%$ compared with $5 \%$ ) remained below stage 5 .
Table 28
Initial and Final Basic Facts Stage by Ethnicity

|  | NZ European |  | Māori |  | Pasifika |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Basic facts stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0-1: Non-grouping | 1 | 0 | 1 | 0 | 1 |  |
| 2-3: Within and with five | 1 | 0 | 3 | 1 | 2 | 2 |
| 4: Within 10, doubles, and teens | 5 | 3 | 8 | 4 | 9 | 5 |
| 5: Addition, multiplication for 2, 5, 10 | 21 | 13 | 27 | 20 | 23 | 16 |
| 6: Subtraction and multiplication | 48 | 45 | 46 | 45 | 45 | 44 |
| 7: Division | 24 | 39 | 16 | 28 | 21 | 34 |
| Number of students | 2642 | 2642 | 783 | 783 | 193 | 193 |

Note: Percentages may not total to 100 due to rounding.
Table 29 shows that female students performed similarly to male students on the basic facts domain, with $84 \%$ knowing their subtraction and multiplication basic facts (stage 6 or higher) by the end of the project and only $2 \%$ still unable to recall their multiplication facts for 2,5 , and 10 (stage 4 or lower), compared with $80 \%$ and $5 \%$ respectively of male students.

Table 29
Initial and Final Basic Facts Stage by Gender

|  | Male |  | Female |  |
| :--- | :---: | :---: | :---: | :---: |
| Basic facts stage | $\%$ initial | \% final | \% initial | \% final |
| 0-1: Non-grouping | 1 | 0 | 0 | 0 |
| 2-3: Within and with five | 2 | 1 | 1 | 0 |
| 4: Within 10, doubles, and teens | 7 | 4 | 4 | 2 |
| 5: Addition, multiplication for 2, 5, 10 | 22 | 15 | 21 | 12 |
| 6: Subtraction and multiplication | 45 | 41 | 50 | 46 |
| 7: Division | 24 | 39 | 23 | 38 |
| Number of students | 2248 | 2248 | 1727 | 1727 |

Note: Percentages may not total to 100 due to rounding.

Table 30 shows that students from high-decile schools performed better than students from medium-decile schools ( $86 \%$ compared with $81 \%$ at stage 6 or higher by the end of the project) on the basic facts domain and that both performed better than students from low-decile schools.

Table 30
Initial and Final Basic Facts Stage by Decile

|  | Low |  | Medium |  | High |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Place value stage | \% initial | \% final | \% initial | \% final | \% initial | \% final |
| 0-1: Non-grouping | 2 | 1 | 0 | 0 | 1 | 0 |
| 2-3: Within and with five | 3 | 1 | 1 | 1 | 1 | 0 |
| 4: Within 10, doubles, and teens | 8 | 5 | 6 | 3 | 5 | 2 |
| 5: Addition, multiplication for 2, 5, 10 | 30 | 16 | 22 | 15 | 17 | 12 |
| 6: Subtraction and multiplication | 42 | 47 | 47 | 45 | 48 | 40 |
| 7: Division | 15 | 29 | 22 | 36 | 28 | 46 |
| Number of students | 435 | 435 | 2059 | 2059 | 1481 | 1481 |

Note: Percentages may not total to 100 due to rounding.

## Comparison of Knowledge Domains

Table 31 presents the percentages of students at each stage of the four knowledge domains at the end of the project. That a small proportion of students remain at stage 4 or lower on each domain should be a cause for concern for teachers in SNP schools. At the higher stages, it can be seen that a similar percentage of students reached stage 7 and 8 on the fractions ( $40 \%$ ), place value ( $35 \%$ ), and basic facts ( $38 \%$ ) domains.

Table 31
Final Knowledge Stages - All Domains

|  | Domain |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Stage | \% FNWS | \% Fractions | \% Place value | \% Basic facts |
| $<4$ | 1 | 2 | 1 | 1 |
| 4 | 2 | 7 | 5 | 3 |
| 5 | 27 | 29 | 35 | 14 |
| 6 | 71 | 23 | 25 | 44 |
| 7 | N/A | 25 | 18 | 38 |
| 8 | N/A | 15 | 17 | N/A |

[^3]
## The Relationship between Strategy and Knowledge

Tables 32 and 33 explore the relationship between students' gains in strategy stage and their number knowledge. The tables present the knowledge results on the fractions, place value, and basic facts domains of students who were initially rated as advanced additive (stage 6) on the multiplicative domain. This subgroup of the students was chosen for two reasons. Firstly, because this is the stage at which the highest proportion (34) of SNP were initially assessed. Secondly, there was a relatively even split of students between those who made progress from stage 7 and those who did not. The students are separated into two groups: those who progressed to at least stage 7 (advanced multiplicative), and those who did not.

Table 32 compares the end-of-project number knowledge profile of the two groups. In each knowledge domain, over half ( 57,51 , and 52 respectively) of the students who progressed were rated as at least stage 7 . Contrastingly, less than a third ( 30,25 , and 32 respectively) of the students who did not progress were rated at stage 7 or 8 on the knowledge domains.

Table 32
Comparing the Number Knowledge Profile of Initially Advanced Additive Students Who Progressed to Advanced Multiplicative with Those Who Did Not

| Number of students | 680 | 661 |
| :---: | :---: | :---: |
|  | \% remained advanced additive | \% became multiplicative |
| Fractions |  |  |
| 2-3: Non-fractions | 1 | 0 |
| 4: Assigns unit fractions | 6 | 2 |
| 5: Orders unit fractions | 33 | 19 |
| 6: Co-ordinates numerators/denominators | 29 | 22 |
| 7: Equivalent fractions | 21 | 33 |
| 8: Orders fractions | 9 | 24 |
| Place value |  |  |
| 2-3: Counts by ones | 1 | 0 |
| 4: 10 s to 100, order to 1000 | 4 | 1 |
| 5: 10s to 1000, order to 10000 | 40 | 23 |
| 6: 10s, 100s, 1000s, orders whole numbers | s 31 | 25 |
| 7: Tenths in and orders decimals | 18 | 24 |
| 8: Tenths, hundredths, and thousandths | 7 | 27 |
| Basic facts |  |  |
| 0-1: Non-grouping | 0 | 0 |
| 2-3: Within and with five | 0 | 0 |
| 4: Within 10, doubles, and teens | 3 | 2 |
| 5: Addition, multiplication for 2, 5, 10 | 15 | 5 |
| 6: Subtraction and multiplication | 49 | 40 |
| 7: Division | 32 | 52 |

[^4]Table 33 presents the same results, but in this case, the percentages represent the proportions of students finishing the project at each knowledge stage who did or did not progress on the multiplicative domain. Over half of those students who were rated at stage 7 or 8 on each knowledge domain progressed on the multiplicative domain, while over half of those who were rated below stage 7 on each knowledge domain remained advanced additive.

Table 33
Comparing the Proportions of Initially Advanced Additive Students at Each Knowledge Stage Who Progressed to Advanced Multiplicative with Those Who Did Not

|  | Number of <br> students |  | \% remained <br> advanced additive |
| :--- | :---: | :---: | :---: |
| Fractions | \% became <br> multiplicative |  |  |
| 2-3: Non-fractions | 10 | 80 |  |
| 4: Assigns unit fractions | 50 | 78 | 20 |
| 5: Orders unit fractions | 351 | 64 | 22 |
| 6: Co-ordinates numerators/denominators | 345 | 58 | 36 |
| 7: Equivalent fractions | 364 | 40 | 42 |
| 8: Orders fractions | 221 | 29 | 60 |
| Place value |  |  | 71 |
| 2-3: Counts by ones | 7 | 86 |  |
| 4: 10s to 100, order to 1000 | 29 | 86 | 14 |
| 5: 10s to 1000, order to 10 000 | 423 | 64 | 14 |
| 6: 10s, 100s, 1000s, orders whole numbers | 373 | 57 | 36 |
| 7: Tenths in and orders decimals | 279 | 43 | 43 |
| 8: Tenths, hundredths, and thousandths | 230 | 22 | 57 |
| Basic facts |  |  | 78 |
| 0-1: Non-grouping | 2 | 100 |  |
| 2-3: Within and with five | 3 | 33 | 0 |
| 4: Within 10, doubles, and teens | 34 | 59 | 67 |
| 5: Addition, multiplication for 2, 5, 10 | 134 | 75 | 41 |
| 6: Subtraction and multiplication | 603 | 56 | 25 |
| 7: Division | 565 | 39 | 44 |

## Concluding Comment and Key Findings

Students in schools participating in the SNP made progress on all three strategy domains. More specifically, the findings were as follows:

- The percentages of students rated in the top two stages of the additive, multiplicative, and proportional domains increased from 45 to 69,25 to 46 , and 36 to 53 respectively.
- The percentages of students still rated stage 4 or lower on the additive, multiplicative, and proportional domains decreased from 15 to 5,14 to 6 , and 17 to 6 respectively.
- Greater percentages of New Zealand European students than Māori and Pasifika students were at higher initial and final stages across all three domains.
- A higher proportion of male students than female students reached the top stages of each strategy domain; similar proportions remained at the bottom stages.
- Students from high-decile schools performed better than students from medium-decile schools, who in turn performed better than students from low-decile schools.
Students also made progress on the four knowledge domains as shown below.
- New Zealand European students performed better than Māori or Pasifika students; the performance of Pasifika students was overall slightly better than that of Māori students, though this varied between domains.
- Male students generally performed slightly better than female students, the exception being in the basic facts domain, where they performed similarly.
- Students from high-decile schools performed better than students from medium-decile schools, who in turn performed better than students from low-decile schools.

A comparison of knowledge profiles of initially advanced additive students showed that those students who made progress on the multiplicative domain were more likely to be at the higher stages on the knowledge domains than those who did not.

## References

Ministry of Education (2005). Book 1: The Number Framework. Wellington: Ministry of Education.
Ministry of Education (2005-). "The Diagnostic Assessment." Unpublished (SNP strategy diagnostic test and whole-class knowledge test)


[^0]:    Note: Percentages may not total to 100 due to rounding.

[^1]:    Note: Percentages may not total to 100 due to rounding.

[^2]:    Note: Percentages may not total to 100 due to rounding

[^3]:    Note: Percentages may not total to 100 due to rounding.

[^4]:    Note: Percentages may not total to 100 due to rounding.

