Evaluations of the 2005 Secondary Numeracy Pilot Project and the CAS Pilot Project

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2005 Secondary Mathematics Pilots: Secondary Numeracy Project and CAS

Foreword

In 2005, two pilot projects in secondary school mathematics were implemented by the Ministry of Education. The 2005 Secondary Numeracy Pilot Project (SNP) emphasised the use of mental computational strategies to solve numeric problems, while the 2005 Computer Algebraic Systems (CAS) Pilot Project integrated calculators into classrooms to assist in the teaching of mathematics. The theme that is common to these projects is that they both aim to help students develop a deeper understanding of mathematics. This report is a collection of the research undertaken alongside the introduction of these projects.

The Numeracy Development Projects (NDP) were first implemented by the Ministry of Education in 2001, following a pilot project in 2000. This was developed as a response to the relatively poor performance of New Zealand students in the 1995 Third International Mathematics and Science Study (TIMSS). The early years of the NDP focused on the Early Numeracy Project (ENP) for students in years 1–3 and the Advanced Numeracy Project (ANP) for students in years 4–6. In 2001 and 2002, a smaller number of teachers took part in NDP exploratory studies aimed at students in years 7–10. As a result of this exploratory work and the findings of the evaluations, the Ministry of Education expanded the intermediate and secondary components of the project. In 2005, 43 secondary schools chose to participate in the SNP pilot.

The SNP shares features with the other parts of the NDP. Teachers are introduced to a Number Framework, which describes strategies and knowledge that students have been observed to use to solve numerical problems. The Number Framework contains broad stages of increasingly sophisticated strategies, with progress through the earlier stages tending to occur more readily than with later stages. The teachers conduct a diagnostic interview to assess students' performance against the Framework and use this information as a starting point for teaching. Facilitators introduce and model a teaching approach for developing mathematical understanding that progresses, for most topics, through physical representations, imaging, and on to abstract mathematical concepts and algebraic thinking.

The goal of the SNP is to develop students' capacity to work efficiently with numbers by developing their computational strategies. This structural thinking can then be exploited to develop their understanding of algebra.

However, it all takes time. Time was identified as a pressure by teachers and facilitators both at the start and the end of the SNP pilot. Teaching is an intensely busy profession, and the act of significant teaching is a mentally draining occupation. Patience is needed, and the pace of change needs to be realistic if the project is to successfully build on the significant gains that have already been achieved.

Three chapters in this report examine aspects of the implementation of the SNP: the content of the number knowledge assessment; student performance and progress; and the impact of the SNP on teachers and facilitators. The final chapter reports on the CAS Pilot Project.

Number Knowledge Assessment

The diagnostic interview is an integral part of the NDP. In the SNP, this interview differs in two aspects from the interview that is used in primary and intermediate schools. Both modifications have been introduced in response to trialling in secondary schools in an effort to reduce the time needed to assess students against the Framework.

While primary and intermediate teachers can choose one of three increasingly difficult forms of the strategy interview to use with students, only one set of strategy questions is used in the SNP. This composite of appropriate questions for secondary students is extracted from the two more advanced forms of the interview used in primary and intermediate schools.

The second aspect that differs is the form of the knowledge assessment. This is no longer part of the individual interview with students, as occurs in primary and intermediate schools; rather, it is a pen and paper whole-class assessment. Again, the questions are based on those used in the primary knowledge interview. This knowledge assessment was initially adapted by Peter Hughes and Carol Mayers as part of the Manurewa Enhancement Initiative in 2004; a secondary working party further adapted this when introducing it to SNP schools in 2005.

Given such a change to the form of the knowledge assessment, it seemed sensible to determine whether in fact students found the items within each domain increasingly difficult and whether items across domains but identified as being at the same stage of the Framework were of similar difficulty. In other words, did the student data match the expected progressions of difficulty described in the Number Knowledge Framework? Tagg and Thomas ("Secondary Numeracy Project Knowledge Test Analysis", p. 5) undertook this piece of research, using 1000 scripts from a random sample of schools stratified by decile grouping. The research indicated that the items were generally ordered from easiest to most difficult within each domain. Six changes were subsequently made to the 2006 version to fine-tune this assessment.

Student Performance and Progress

Tagg and Thomas ("Performance on the Number Framework", p. 12) also provide an analysis of the progress of students in the SNP, measured against the Number Framework. Each student was assessed through a diagnostic interview against the Framework at the start of year 9 and reassessed towards the end of the year to ascertain the progress they had made. Tagg and Thomas looked for evidence of progress by the students, compared this with the progress of year 8 students in NDP schools, analysed demographic factors, and investigated the relationship between number knowledge and progress in multiplicative thinking.

Their analysis used data from 3975 students across 31 schools and showed that progress was made in all strategy and knowledge domains. Significant shifts occurred in the proportion of the student population that could perform in the top two stages of the additive (45% to 69%), multiplicative (25% to 46%), and proportional (36% to 53%) domains. Although the final performance of New Zealand European students was better than that of Māori and Pasifika students and the performance and progress of Pasifika students was overall slightly better than that of Māori, all ethnic groups progressed to higher stages in the Framework. Male students generally performed better than female students. Students from high-decile schools as a group performed better than students from medium-decile schools, who in turn performed better as a group than students from low-decile schools. Having said that, there still exist in all decile bands students who experience significant difficulties making sense of their mathematics. There just aren't as many of them in higher decile schools.

While the project appears to have done much to shift students beyond counting strategies, it is a matter for concern that there were still 5% to 6% of students who, at the end of that year, still used counting strategies as their best available strategy for solving additive, multiplicative, and proportional problems. Their ability to understand more sophisticated mathematics in the future is very limited.

Impact of the SNP on Teachers and Facilitators

While Tagg and Thomas researched the impact that the SNP was having on student achievement, Harvey and Higgins, with Jackson, (p. 36) delved into the impact of the SNP on teachers and facilitators. Surveys seeking feedback from SNP teachers and facilitators were conducted at the start and towards the end of the year. In addition, a small sample of in-school facilitators and regional facilitators were interviewed.

The SNP model differs significantly from the primary version, in which facilitators are not based in a school and tend to work full time in a number of schools. In the SNP, most facilitators are practising teachers who are released from some of their teaching to facilitate the project with approximately 12 teachers. Generally, in 2005, these teachers were all at the same school, although there were several instances of facilitators working with up to three smaller schools.

The SNP facilitation model was received positively by schools. It allowed for substantive discussion to occur with the facilitator within mathematics departments throughout the duration of the project on a very regular basis, encouraging professional dialogue about mathematics teaching. Adapting the intentions of the project to local circumstances was more readily achieved. Additionally, teachers reported more willingness to take risks in adopting new practices when the support of the facilitator was readily available.

The diagnostic assessment was significant in influencing teachers' understanding of students' knowledge and thinking. While there was little reported change to teachers' personal mathematical knowledge, both teachers and facilitators reported growth in knowledge of how students learn mathematics.

There has been a need identified by teachers and facilitators for continuing development of secondary specific resources, and the Ministry has taken action on this for 2006.

CAS Pilot Project

The final chapter in this report contains an evaluation of the Computer Algebraic Systems (CAS) Pilot Project by Alex Neill and Teresa Maguire (p. 69). Twelve teachers in six schools undertook professional development that aimed to improve the teaching and learning of mathematics through the use of CAS technology. The pilot encouraged teachers, assisted by the technology, to adopt an exploratory approach with students in order to improve mathematical understanding. The strong modelling of an appropriate pedagogy to accompany the technology was seen as a key ingredient of the pilot. Most of the teachers believed the pilot led to students having a deeper understanding of mathematical concepts. Students were split on this, with a significant group of them believing they had less understanding than before while a similar number claimed more mathematical understanding.

This pilot also explored issues related to the impact of technology on summative assessments and the need for continuing professional development of teachers to support the pedagogical approach. As with the SNP pilot, teachers saw a need for more teaching resources.

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Secondary Numeracy Project Knowledge Test Analysis

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Purpose

While the knowledge domains have been included in the diagnostic interview for the Early Numeracy Project (ENP), Advanced Numeracy Project (ANP), and Intermediate Numeracy Project (INP), a decision was made to use a written test to assess the four knowledge domains for the Secondary Numeracy Project (SNP). This decision was made in an effort to reduce the amount of time required to assess students, which has been identified as a barrier to the sustainability of the numeracy project (Thomas, Tagg, and Ward, 2006). While the strategy domains require students to explain how they work out answers to items, the knowledge domains assess the number knowledge that students can recall without needing to carry out calculations, so a written test taken simultaneously by the whole class is seen to be an appropriate method of assessment. The four knowledge domains assessed are number word sequence, fractions, place value, and basic facts. This chapter reports on the findings of a research project that investigated the robustness of the knowledge test with the goal of informing its future development. These research questions are addressed:

- Do the items within each domain of the SNP knowledge test get progressively more difficult?
- Are the items within each stage of each domain of the test of similar difficulty?

Sample

A random sample of six schools, stratified by decile group, was selected from the 43 schools participating in the SNP in 2005. These schools were invited to submit all of their initial student knowledge record sheets for independent analysis. These tests were completed at the beginning of the project, in Term One 2005. All of the low-decile (282) tests were marked, and similar numbers of the high-decile (350) and medium-decile (368) tests were added to take the sample to a total of 1000. The responses of each student to each item were analysed.

Rasch Analysis

Rasch item analysis allows logit scores to be calculated for both item difficulty and student ability. These scores are calculated using an iterative process that assumes that for any given item there is a probability that any given student will answer correctly; this probability can be expressed in terms of the item's difficulty and the student's ability. Both values are expressed in logits, with a negative score representing lower difficulty and a positive score representing higher difficulty. This method of calculating item difficulty takes into account which other items were answered correctly by each respondent, rather than just the proportion of correct responses. A Rasch analysis of the 1000 tests was carried out using the RASCAL program (Assessment Systems Corporation, 1996).

Figure 1 shows a scatterplot of the difficulty score for each item, with vertical lines separating the four domains. The figure clearly illustrates the increase in difficulty of items within each domain and also shows several points that appear to lie outside the trend for each domain. These will be discussed in the following section.

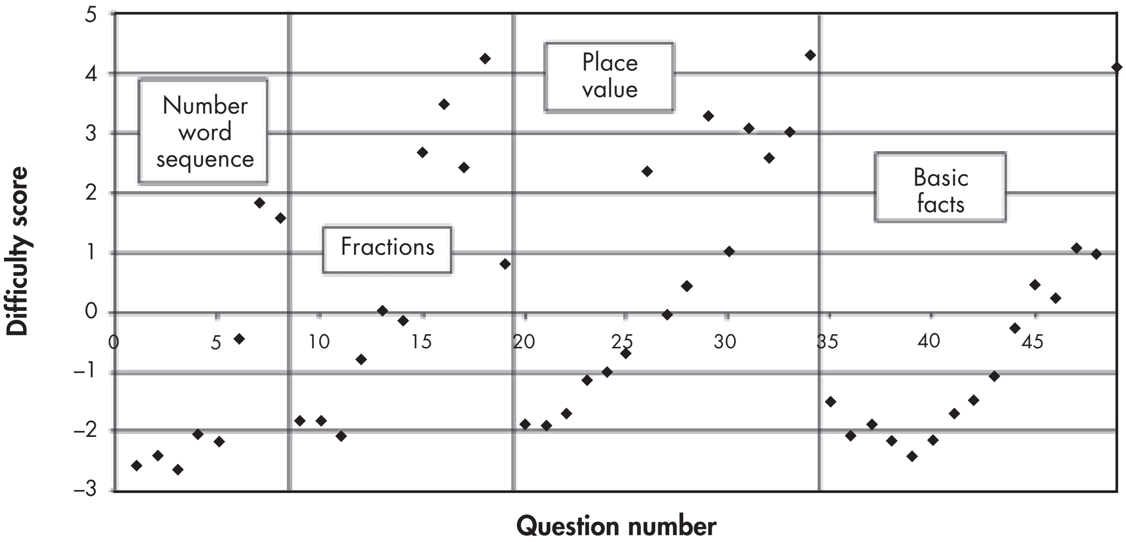

Figure 1. Difficulty scores for SNP knowledge test items

Discussion

The criteria for marking the test indicate that all items within a stage must be answered correctly for a student to be rated as having attained that stage, so it is reasonable to expect that the items within a stage should be relatively similar in difficulty. The item analysis shows some interesting results, with some items revealed as significantly easier than others within the same stage and others seemingly well out of place in terms of difficulty. It should be noted that the items for the test were chosen to be representative of the knowledge that students are thought to have at each of the described stages of the Number Framework. These decisions, while informed by professional experience, were not based on student data. This research represents the first attempt to quantify the results of individual elements of the Numeracy Project Assessment diagnostic tool (NumPA) testing. It is assumed for the following discussion that the intention of the knowledge test is solely to allocate the student with a stage for each knowledge domain and to identify the gaps in their number knowledge. Some of the items that are shown to be "misplaced" may provide additional information on students' knowledge if analysed independently.

Forwards and Backwards Number Word Sequence

It is important that teachers are able to identify the small proportion of secondary students who are not able to accurately state the number before or after a given number because an inability to do so will prevent them from making progress in their development of the most basic strategies for operating with numbers. Figure 2 illustrates the difficulty scores for the eight items in the knowledge test relating to this domain.

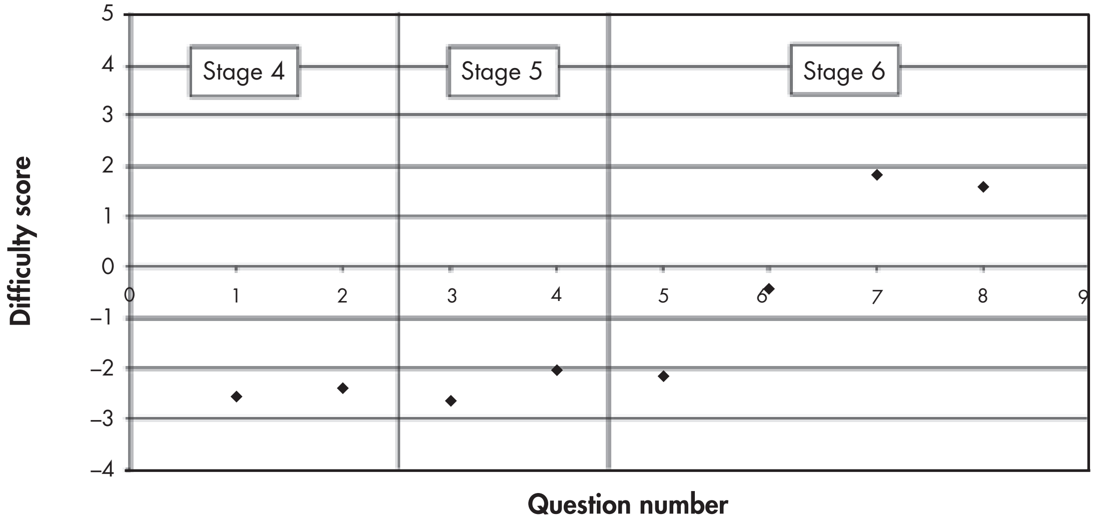

Figure 2. Difficulty scores for SNP knowledge test number word sequence items

Ninety-five percent of students tested answered the first four items in this domain correctly, giving them a rating of at least stage 5. It is the four items (5–8) that are grouped within stage 6 that are of interest; they have item difficulty scores ranging from –2.165 to 1.815. Items 5 and 6 (one more than 3049; one less than 2400) are considerably easier than items 7 and 8 (one more than 989 999; one less than 603 000) and are therefore unlikely to contribute to the outcome of the test because it is reasonably safe to assume that they will be answered correctly by all students who respond correctly to the more difficult items. In fact, a count of the data shows that 399 of the 485 students rated at stage 5 correctly answered items 5 and 6.

Fractions

The eleven items relating to the fractions domain seem to be reasonably well clustered. The difficulty scores of these items are illustrated in Figure 3.

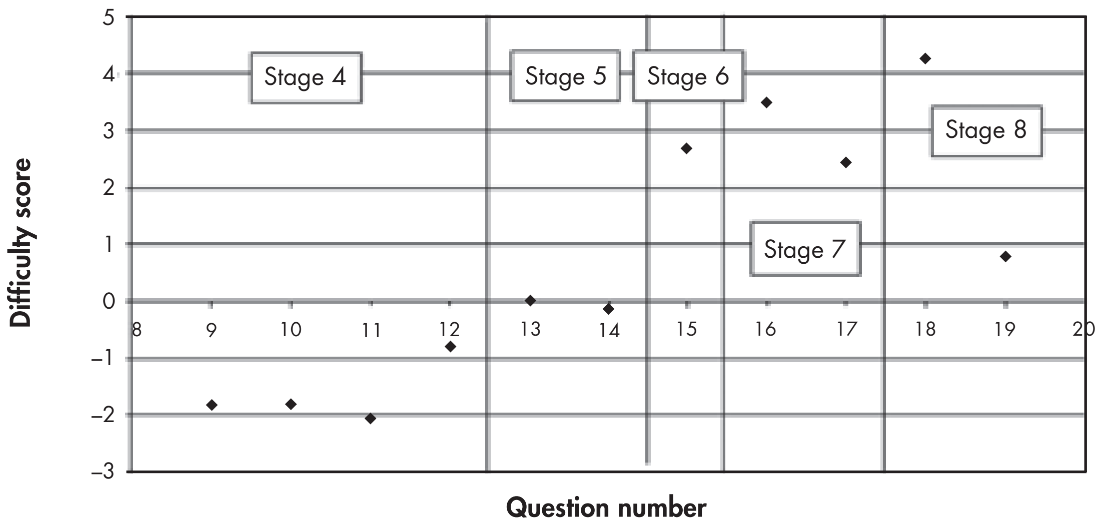

Figure 3. Difficulty scores for SNP knowledge test fractions items

There are three instances in which items in this domain do not fit well into the difficulty grouping. The relevant items discussed below are:

- item 12 Write one-quarter as a fraction.
- item 15 Which fraction is the same as eight-sixths? $\frac{6}{8}$, $\frac{12}{6}$, 1, $\frac{2}{14}$
- item 17 Which fraction is the same as three-quarters? $\frac{2}{6}$, $\frac{6}{8}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{3}{9}$
- item 18 Which of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{5}{16}$ is the smallest?
- item 19 Which of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{5}{16}$ is the largest?

Firstly, item 12 appears to be more difficult than the other three items in the same stage (Write one-half, one-sixth, and one-third as a fraction). Secondly, item 15, with a difficulty score of 2.666 despite being at stage 6, is at least as difficult as stage 7's item 17, which has a difficulty score of 2.417. Finally, while item 19, with a difficulty score of 0.792, is placed at stage 8, it is rated as easier than any of the stage 6 or 7 items, which had difficulty scores between 2.417 and 3.49. In fact, 107 of the 127 students rated at stage 7 answered item 19 correctly. Only 6 of the 127 students rated at stage 7 would have been rated at stage 8 if they had answered this item correctly, with the remainder also answering item 18 incorrectly.

Place Value

The 15 place value items are those for which the difficulty scores are least well clustered. These scores are illustrated in Figure 4.

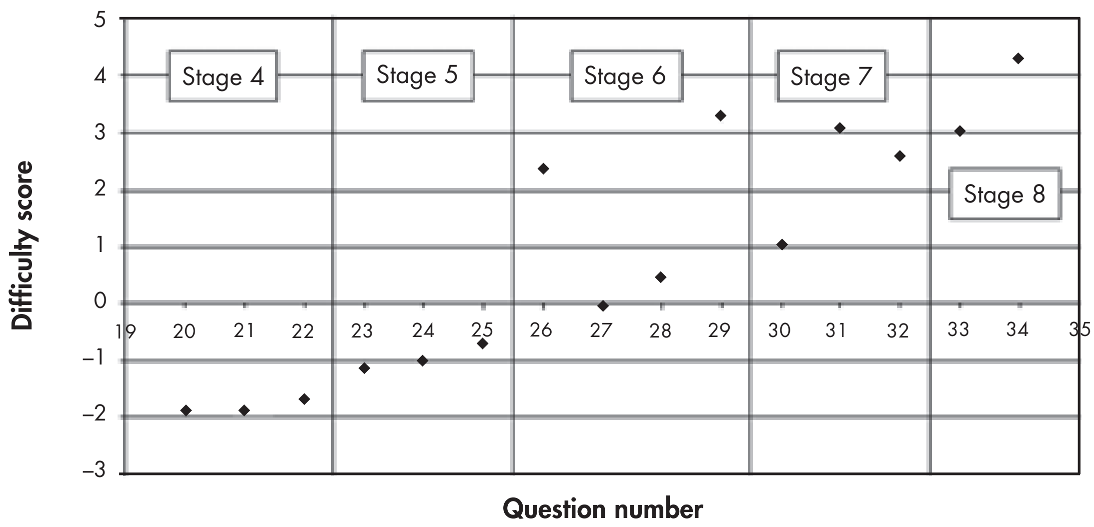

Figure 4. Difficulty scores for SNP knowledge test place value items

While the stage 4 and 5 items do seem to be grouped in terms of their difficulty, several of the other items appear to be misplaced. The relevant items discussed below are:

- item 29 Three-tenths more than 4.8.
- item 26 How many $1,000 notes in $2,408,000?
- item 30 Which of 0.478, 0.8, 0.39 is the largest?

In terms of misplaced items, item 29 in particular, which is placed at stage 6, has a difficulty score of 3.295, which is higher than any of the stage 7 items (difficulty scores between 1.019 and

3.078). Item 26, with a difficulty score of 2.366, also appears more suited to placement in stage 7. If these two items were placed in stage 7 instead of stage 6, 454 of the 578 students rated at stage 5 would have been rated at stage 6. If only item 29 were moved, 193 students would have been rated a stage higher. Conversely, item 30 appears too easy for stage 7, with a difficulty score of only 1.019, compared with 3.078 and 2.582 for the other two items in that stage. In fact, only two of the 89 students who were rated at stage 6 would have been rated at stage 7 if that item had been excluded.

Basic Facts

Figure 5 illustrates the difficulty scores of the 15 basic facts items.

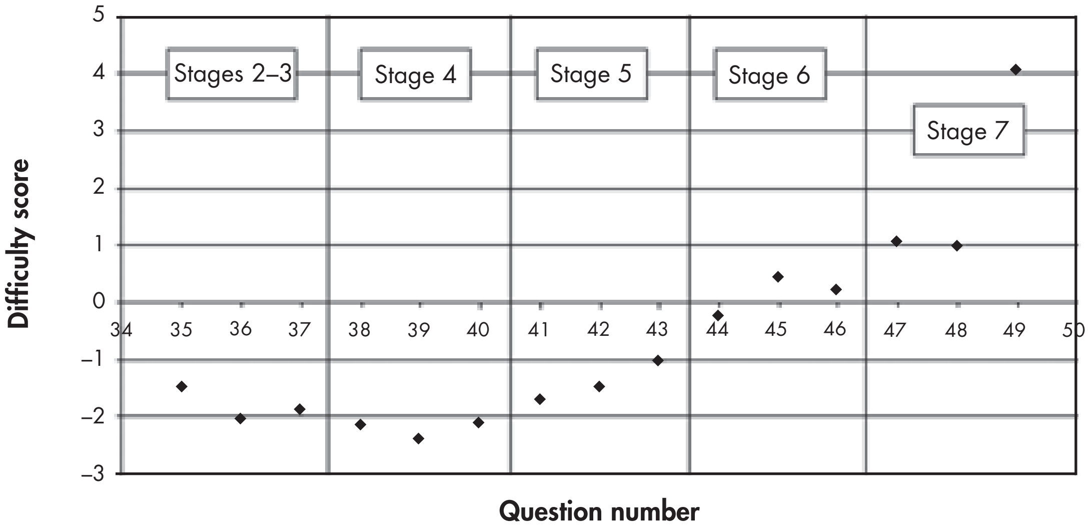

Figure 5. Difficulty scores for SNP knowledge test basic facts items

The difficulty scores of these items are particularly interesting in that they show that the three items defining stage 4 (items 38–40; difficulty scores between –2.122 and –2.404) are all easier than all of the three items defining stages 2–3 (items 35–37; difficulty scores between –1.492 and –2.081). This strongly suggests that, at least for year 9 students, addition with tens and doubles is “easier” than addition with and within 5 and within 10. In fact, of the 73 students rated at stages 0–1 because they answered at least one of items 35–37 incorrectly, 51 answered all of items 38–40 correctly. The items defining stages 5–6 have well-defined clusters of difficulty scores, with the exception of item 49 (45 divided by 6), which, with a difficulty score of 4.085, is significantly more difficult than any of the other basic facts items. The next most difficult has a difficulty score of 1.068. If item 49 were excluded, 325 of the 434 students rated at stage 6 would have been rated at stage 7.

Comparison of Domains

The criteria for rating students’ knowledge stages indicate that students should be rated at a stage if they correctly answer all of the items at that stage for that domain. Table 1 shows the percentages of students rated at each stage for each knowledge domain. While there is neither a requirement nor an expectation that students will be operating at the same stage for each domain, the stages are intended to be roughly equivalent.

Table 1
Percentages of Students at Each Stage of the Four Domains

	Domain			
Stage	FNWS	Fractions	Place value	Basic facts
0–1				7
2–3	3	12	8	3
4	2	10	13	7
5	49	37	58	20
6	47	21	9	43
7		13	6	20
8		6	7	

It is interesting to note the relatively low proportions of students who reached stage 6 or higher on the place value domain (219) compared with the other three knowledge domains (405 or more). This disparity could be corrected by moving item 29 (What number is three-tenths more than 4.8?) from stage 6 of the place value domain to stage 7. The relatively high proportion of students at the lower stages of the fractions domain, while of concern, confirms other research focusing on students' acquisition of fraction knowledge.

Results of Findings

Changes have been made to six items in the SNP knowledge test for 2006 as a result of the release of preliminary findings from the analysis of the knowledge tests.

Items 5 and 6 (one more than 3049; one less than 2400) were made more difficult by increasing the numbers to 30 099 and 24 000 respectively. It is hoped that this will make these items similar in difficulty to items 7 and 8.

The fractions in item 19 (Which of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{5}{16}$ is the largest?) have been changed to $\frac{2}{3}$, $\frac{5}{9}$, and $\frac{7}{10}$ respectively. The basis for this change is an assumption that students were answering the item correctly by applying a false assumption that a larger denominator always implies a smaller fraction. This would have resulted in students identifying $\frac{2}{5}$ as the largest fraction without taking the numerators into account. Interestingly, with the new selection of fractions, students could still answer correctly through a misconception. This time, if they believe that a larger numerator always implies a larger fraction, they will choose $\frac{7}{10}$, the correct answer, without taking the denominators into account.

The size of the numbers in item 26 have been reduced from "How many $1,000 notes are there in $2,408,000?" to "How many $100 notes are there in $26,700?". This change is intended to bring the difficulty of the item more in line with other stage 6 items.

Item 29 (What number is three-tenths more than 4.8?) has been changed to "What number is three-tenths less than 2?" This change is intended to reduce the item's difficulty, and it also, by allowing either "1.7" or "one and seven-tenths" as answers, removes the decimal symbolism from the problem.

The decimals in item 30 (Which of 0.478, 0.8, 0.39 is the largest?) have been changed to 0.76, 0.657, and 0.7 respectively. This change is based on an assumption that students were identifying 0.8 as the largest decimal by identifying it as eight-tenths, while the other two decimals were hundredths and thousandths. Knowing that tenths are bigger than hundredths or thousandths could lead to a correct answer without taking into account how many tenths, hundredths, or thousandths are involved. The change to the decimals used means that this misconception will no longer lead to a correct answer.

Concluding Comment and Key Findings

Rasch analysis of 1000 SNP knowledge tests indicated that:

- the items on the knowledge test were generally ordered from easiest to most difficult within each domain;
- there were several items that were significantly easier or more difficult than those with which they were grouped.

Six changes were made to the knowledge test for 2006 as a result of these findings.

References

Assessment Systems Corporation (1996). *RASCAL Rasch Analysis Program.* St Paul, MN: Assessment Systems Corporation.

Thomas, G., Tagg, A., and Ward, J. (2006, in press). Numeracy assessment: How reliable are teachers' judgments? *Findings from the New Zealand Numeracy Development Project 2005.* Wellington: Ministry of Education.

Performance on the Number Framework

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Introduction

In 2005, the Ministry of Education offered New Zealand secondary schools an opportunity to improve the teaching and learning of number concepts and skills through the Secondary Numeracy Pilot Project (SNP), a professional development programme for teachers.

The overall aim of the SNP is to develop teachers' knowledge of number concepts, student strategies for operating with number, and instructional practice in order to improve student achievement in year 9.

For the SNP, the Numeracy Development Project (NDP) diagnostic interview used to assess students against the Framework in both strategy and knowledge was modified to better suit the needs of secondary schools. SNP teachers have only one set of strategy questions to use in their individual interviews with students at the beginning of the project (term 1) and at the end (term 4). The knowledge domains are assessed as a pen-and-paper whole-class assessment.

A key aspect of the evaluation of the SNP is to quantify any improvement. This chapter aims to address the following research questions:

1. Do SNP students make progress on the Number Framework?
2. How does this progress compare to that of year 8 students in NDP schools?
3. What demographic factors impact on the progress and performance of SNP students?
4. Is there a relationship between students' number knowledge profiles and their progress on the multiplicative strategy domain?

The results in this chapter are divided into three sections:

- The performance of students on the strategy domains. This describes students' abilities to operate with number.
- The performance of students on the knowledge domains. This describes the key items of number knowledge.
- The relationship between students' use of strategies and their number knowledge.

Where overall differences are described between groups, a T-test has been carried out to verify a difference to at least the 95% confidence level. In addition, differences in percentages of students at particular levels of each domain are not reported unless they are greater than 5%. It needs to be noted that, in some instances, the figures show some significantly different mean gains that may be smaller than other gains in the same figures that are not statistically significant due to differences in sample size. In all tables, percentages are rounded. Percentages less than 0.5% are therefore shown as 0%, and where there are no students represented, the cell is left blank.

Sample

The results reported in this chapter were obtained by downloading data from the online Numeracy Database on 23 January 2006. Results from all students in SNP schools were included, providing these results included an initial and final entry for each of the seven domains assessed.

Of the 43 schools that participated in the SNP in 2005, results were available at this time for 3975 students from 31 schools. Table 1 comprises a breakdown of these students by ethnicity. Two-thirds (66%) of the students were of New Zealand European origin, 20% identified as Māori, and approximately 5% identified as each of Pasifika, Asian, or other. There were more male students than female (57% compared with 43%). (This gender imbalance in the results was caused by the data from several girls' schools not being available at the time that data for this evaluation was downloaded.)

Table 1
Profile of SNP Students by Ethnicity

Ethnicity	Percentage
NZ European	66
Màori	20
Pasifika	5
Asian	4
Other	5
Total	3975

Performance on the Strategy Domains

Additive Domain

Tables 2–5 and figures 1–3 present the results of SNP students on the additive domain, which describes students' strategies for solving addition and subtraction problems.

Table 2, comparing initial and final additive stages, shows that the percentage of students at the top two stages (6 and 7) of the additive domain increased from 45% at the start of the project to 69% by the final assessment. These students are able to use a range of mental part–whole strategies to solve addition and subtraction problems. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from 15% to 5% over the course of the project. There were still 5% of students unable to partition numbers mentally, and this is a cause for concern.

Table 2
Initial and Final Additive Stages

	% initial additive	% final additive
0: Emergent	0	0
1: One-to-one counting	0	
2: Counting from one on materials	0	0
3: Counting from one by imaging	1	0
4: Advanced counting	14	5
5: Early additive part–whole	39	26
6: Advanced additive part–whole	37	46
7: Advanced multiplicative part–whole	8	23
Number of students	3975	3975

Note: Percentages may not total to 100 due to rounding.

Table 3 shows the initial and final additive stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were more likely than Māori or Pasifika students to be at the higher stages of the additive domain, with 72%, 53%, and 57% respectively reaching the top two stages by the end of project.

Table 3
Initial and Final Additive Stage by Ethnicity

	NZ European		Māori		Pasifika	
Additive stage	% initial	% final	% initial	% final	% initial	% final
0: Emergent	0		0	0		
1: One-to-one counting	0					
2: Counting from one on materials	0	0	1	0	1	
3: Counting from one by imaging	1	0	2	1	2	1
4: Advanced counting	12	5	22	9	29	6
5: Early additive part–whole	38	23	45	37	42	36
6: Advanced additive part–whole	40	47	27	42	24	45
7: Advanced multiplicative part–whole	9	25	4	11	2	12
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 4 shows that male students (26%) were more likely than female students (18%) to reach the top stage of the additive domain. Similar percentages (5% and 6% respectively) remained at the advanced counting stage or lower.

Table 4
Initial and Final Additive Stage by Gender

	Male		Female	
Additive stage	% initial	% final	% initial	% final
0: Emergent	0		0	0
1: One-to-one counting			0	
2: Counting from one on materials	0	0	0	0
3: Counting from one by imaging	1	0	1	0
4: Advanced counting	12	5	18	6
5: Early additive part–whole	35	22	44	31
6: Advanced additive part–whole	42	46	30	45
7: Advanced multiplicative part–whole	10	26	6	18
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 5, a breakdown of additive performance by school decile band, shows that students in the high-decile bands performed better than students in either medium- or low-decile bands, with 30% reaching stage 7, compared with 19% of students in medium-decile and 14% of students in low-decile bands.

Table 5
Initial and Final Additive Stage by Decile

	Low		Medium		High	
Additive stage	% initial	% final	% initial	% final	% initial	% final
0: Emergent	1		0		0	0
1: One-to-one counting			0			
2: Counting from one on materials	0		0	0	0	0
3: Counting from one by imaging	1	1	1	0	1	0
4: Advanced counting	20	8	17	7	9	3
5: Early additive part–whole	47	33	39	28	37	22
6: Advanced additive part–whole	26	45	36	46	41	45
7: Advanced multiplicative part–whole	5	14	6	19	12	30
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Figure 1 compares the numbers of stages that SNP students gained on the additive domain with the numbers of stages gained from the same starting stage by year 8 students participating in the NDP. For both samples, the numbers of students initially rated at stages 0–3 were very small, so these results have been excluded from the following figures. The pattern of students' performance initially rated at stages 4–6 is similar for the two samples, with a higher proportion of SNP students initially at stage 5 progressing to at least stage 6 ($p < 0.01$) and a higher proportion of year 8 students initially at stage 6 progressing to stage 7 ($p < 0.001$).

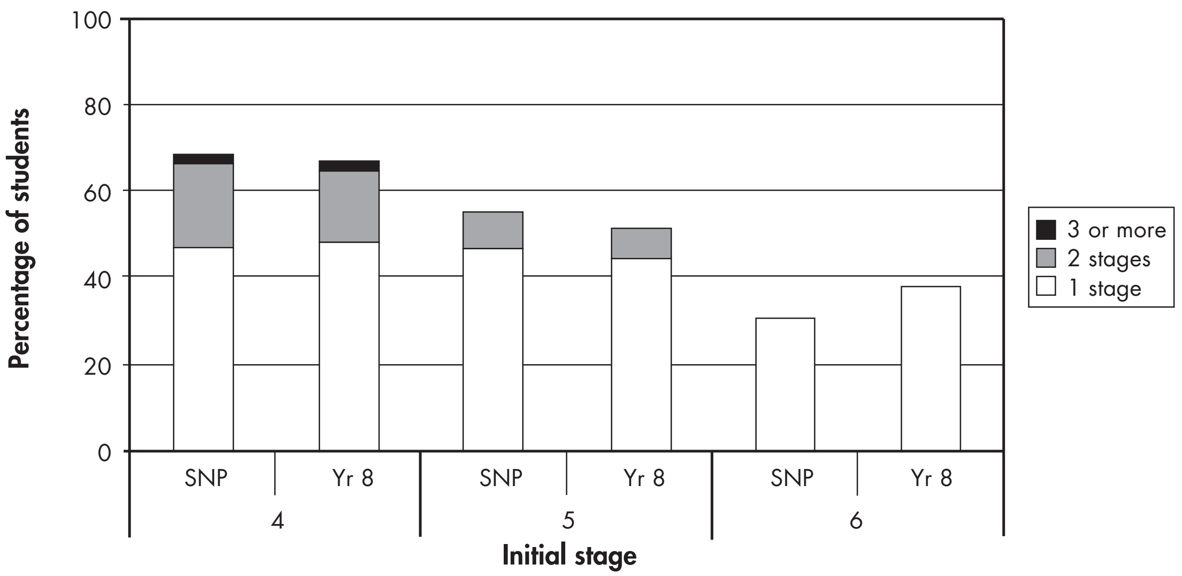

Figure 1. Number of stages gained by initial additive stage for SNP and year 8 students

Figure 2 shows the progress made on the additive domain by students' initial stage and ethnicity. A significantly higher proportion of New Zealand European students than Māori students progressed from stage 5 ($p < 0.001$) and from stage 6 ($p < 0.001$).

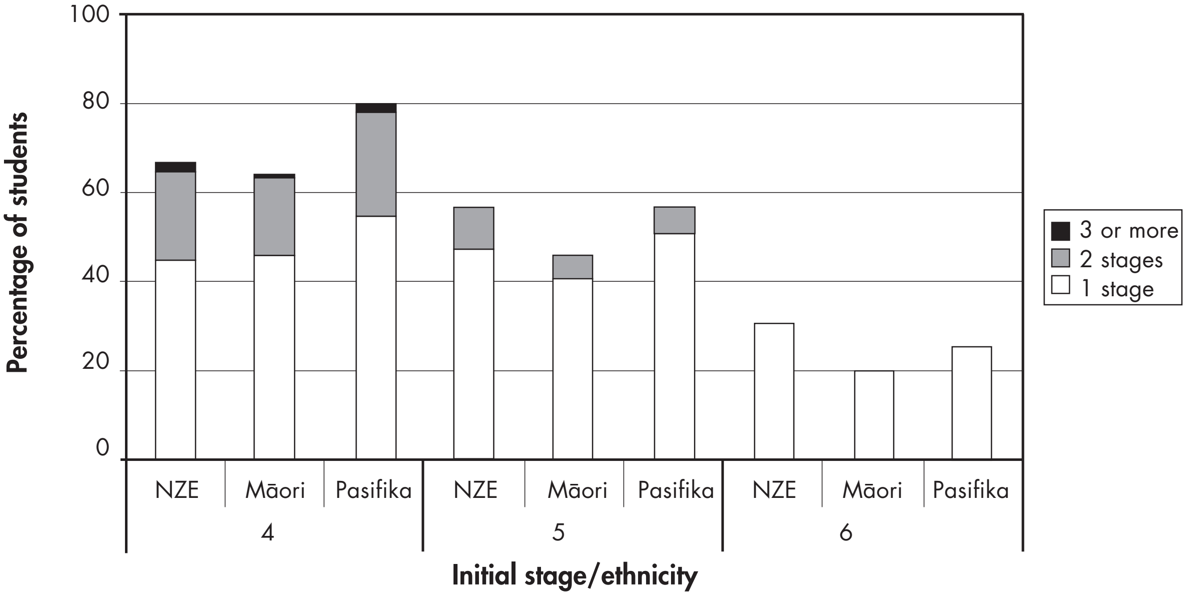

Figure 2. Number of stages gained by initial additive stage and ethnicity

Figure 3 shows that students from high-decile schools are significantly more likely than medium-decile ($p < 0.05$) or low-decile ($p < 0.01$) students to make progress from stage 5 on the additive domain. The same pattern applies to students initially at stage 6 ($p < 0.001$ and $p < 0.05$ respectively). The proportions of students from medium- and low-decile schools who made progress were similar.

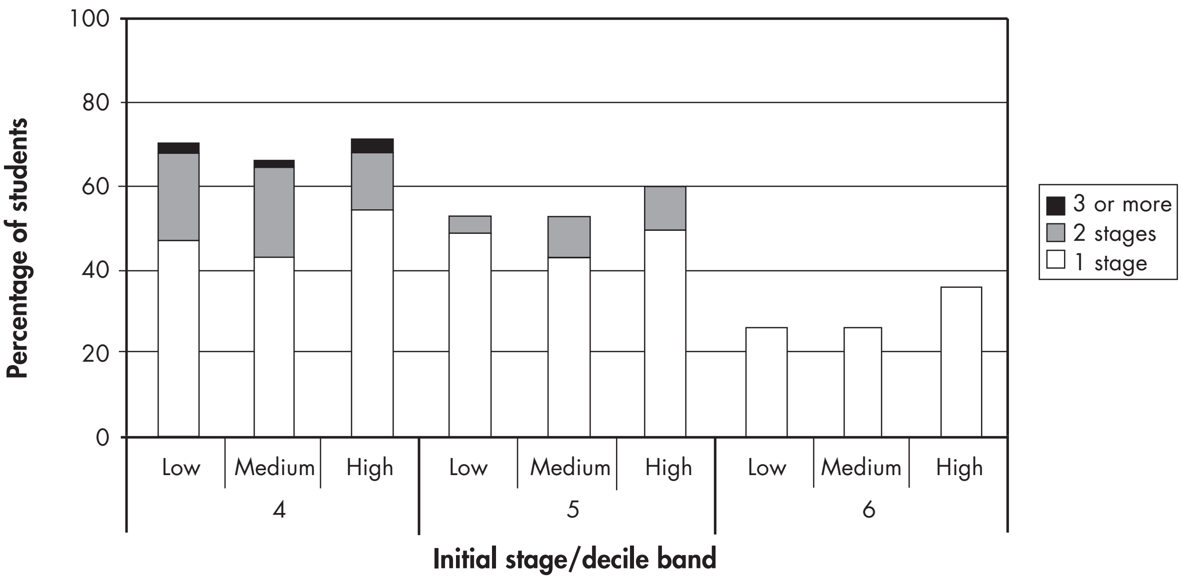

Figure 3. Number of stages gained by initial additive stage and decile band

Tables 6–9 and figures 4–6 present the results of SNP students on the multiplicative domain, which describes students' strategies for solving multiplication and division problems.

Table 6 compares the initial and final multiplicative stages and shows that the percentage of students at the top two stages (7 and 8) of the multiplicative domain increased from 25% at the start of the project to 46% by the final assessment. These students are able to use a range of advanced mental strategies to solve multiplication and division problems. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from 14% to 6% over the course of the project. There were still 6% of students who used skip-counting as their most advanced multiplicative strategy, and this is a cause for concern.

Table 6
Initial and Final Multiplicative Stage

	% initial multiplicative	% final multiplicative
2–3: Counting from one	2	0
4: Advanced counting	12	6
5: Early additive part–whole	27	16
6: Advanced additive part–whole	34	32
7: Advanced multiplicative part–whole	20	30
8: Advanced proportional part–whole	5	16
Number of students	3975	3975

Table 7 shows the initial and final multiplicative stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were considerably more likely than Māori or Pasifika students to be at the higher stages of the multiplicative domain, with 51%, 30%, and 27% respectively reaching the top two stages by the end of the project.

Table 7
Initial and Final Multiplicative Stage by Ethnicity

	NZ European		Māori		Pasifika	
Multiplicative stage	% initial	% final	% initial	% final	% initial	% final
2–3: Counting from one	1	0	3	1	5	1
4: Advanced counting	9	5	20	10	24	11
5: Early additive part–whole	25	14	33	23	36	20
6: Advanced additive part–whole	35	31	31	37	23	41
7: Advanced multiplicative part–whole	23	33	11	23	11	22
8: Advanced proportional part–whole	6	18	2	7	2	5
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 8 shows that male students (53%) were more likely than female students (39%) to reach the top two stages of the multiplicative domain. Similar percentages (5% and 6% respectively) remained at the advanced counting stage or lower.

Table 8
Initial and Final Multiplicative Stage by Gender

	Male		Female	
Multiplicative stage	% initial	% final	% initial	% final
2–3: Counting from one	2	0	2	0
4: Advanced counting	10	5	14	6
5: Early additive part–whole	23	13	33	19
6: Advanced additive part–whole	35	29	32	36
7: Advanced multiplicative part–whole	23	33	15	28
8: Advanced proportional part–whole	7	20	3	11
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 9 presents a breakdown of additive performance by decile band. Students from high-decile schools performed better than students from either medium- or low-decile schools, with 58% reaching stage 7 or higher compared with 41% of students from medium-decile schools and 32% from low-decile schools. At the lower stages of the domain, only 3% of students from high-decile schools and 6% of students from medium-decile schools remained at the advanced counting stage or lower, compared with 13% from low-decile schools.

Table 9
Initial and Final Multiplicative Stage by Decile

	Low		Medium		High	
Multiplicative stage	% initial	% final	% initial	% final	% initial	% final
2–3: Counting from one	4	1	2	0	1	0
4: Advanced counting	21	12	14	6	6	3
5: Early additive part–whole	32	20	29	18	23	11
6: Advanced additive part–whole	27	35	34	34	36	28
7: Advanced multiplicative part–whole	14	24	18	30	25	34
8: Advanced proportional part–whole	2	8	4	11	9	24
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Figure 4 compares the numbers of stages that SNP students gained on the multiplicative domain with the numbers of stages gained from the same starting stage by year 8 students from schools participating in the NDP (many year 8 students were in their second year of the Intermediate Numeracy Project). Again, the numbers of students initially rated below stage 4 were very low and have been excluded from the following figures. The pattern of performance of students initially rated at stages 4–7 was very similar for the two samples, with SNP students initially at stage 6 making significantly greater gains ($p < 0.01$) than year 8 students.

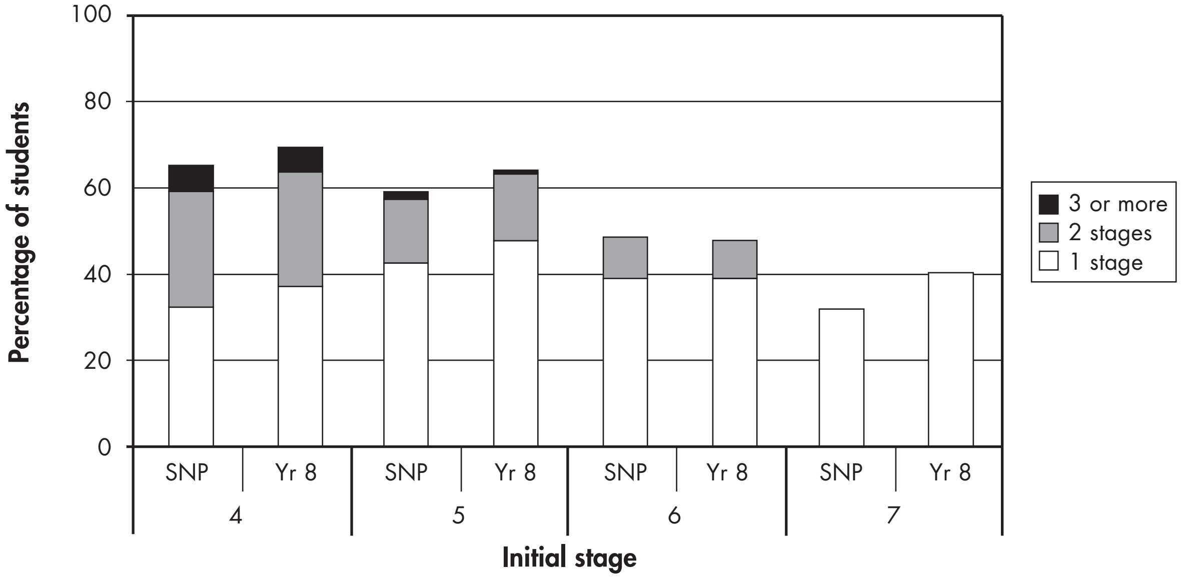

Figure 4. Number of stages gained by initial multiplicative stage for SNP and year 8 students

Figure 5 compares progress made on the multiplicative domain by students' initial stage and ethnicity. A significantly higher proportion of New Zealand European students than Māori students made progress from stage 6 ($p < 0.001$) and from stage 7 ($p < 0.01$).

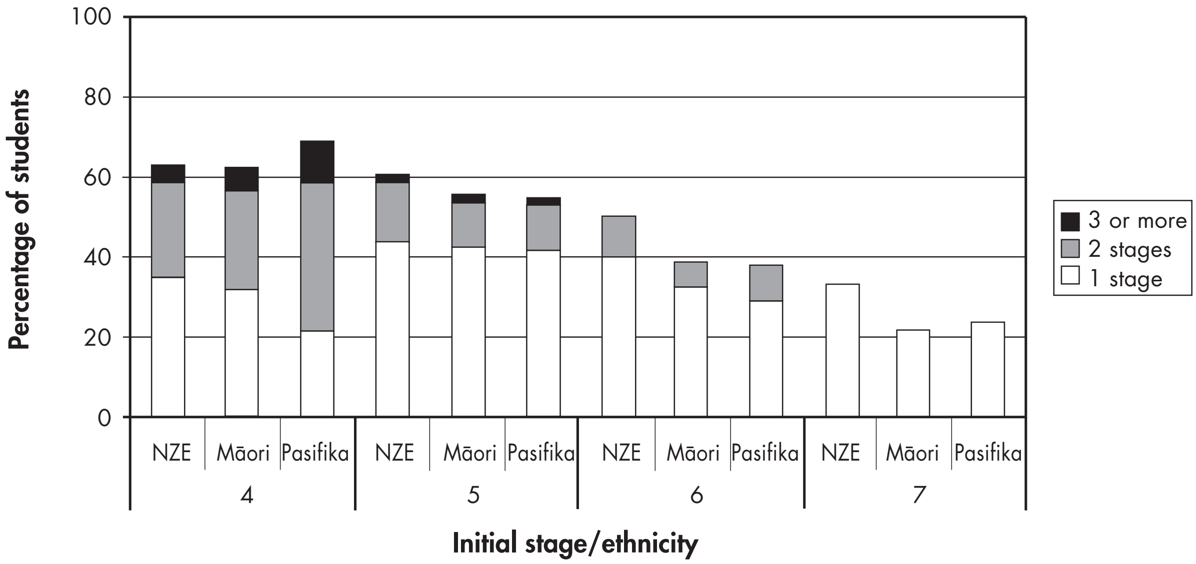

Figure 5. Number of stages gained by initial multiplicative stage and ethnicity

Figure 6 shows that a significantly higher proportion of students from high-decile schools made progress from stage 6 on the multiplicative domain than medium-decile ($p < 0.001$) or low-decile ($p < 0.001$) students. A significantly higher proportion of high-decile than medium-decile students also made progress from stage 7 ($p < 0.001$).

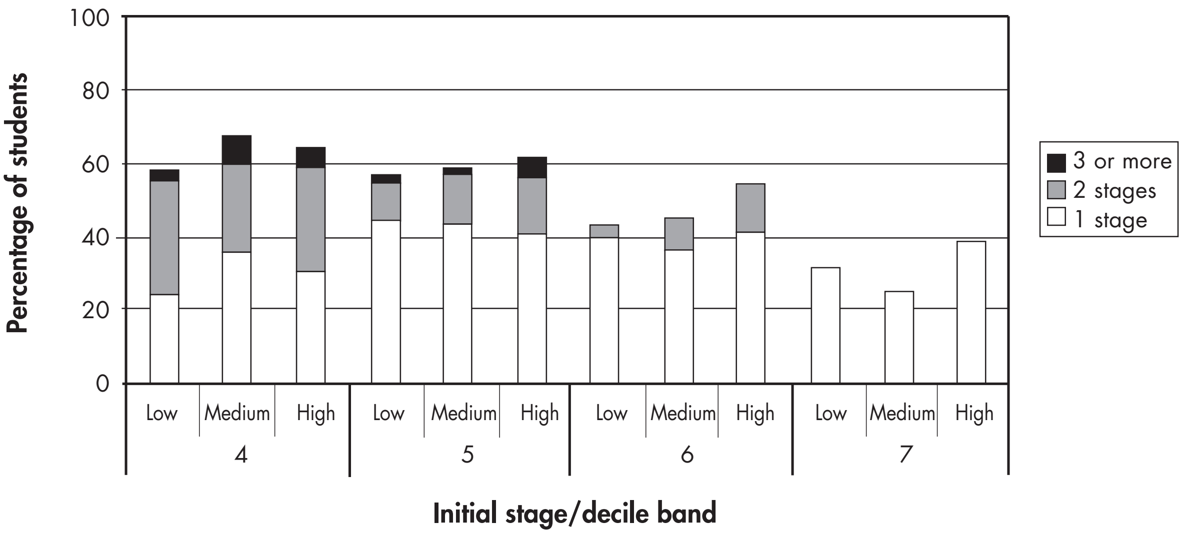

Figure 6. Number of stages gained by initial multiplicative stage and decile band

Proportional Strategy Domain

Tables 10–13 and figures 7–9 show the results of SNP students on the proportional domain, which describes students' ability to solve problems involving ratios and proportions.

Table 10, comparing initial and final proportional stages, shows that the percentage of students at the top two stages (7 and 8) of the proportional domain increased from 36% in the initial assessment to 53% by the end of the year. These students use multiplication and division to find fractions of numbers. Correspondingly, the percentage of students still exclusively using counting strategies (stage 4 or lower) decreased from 17% to 6% over the course of the project. There were still 6% of students who needed to share out objects to find fractions of a set, and this is a cause for concern.

Table 10
Initial and Final Proportional Stage

	% initial proportional	% final proportional
1: Unequal sharing	1	0
2–4: Equal sharing	16	6
5: Early additive part–whole	29	23
6: Advanced additive part–whole	17	17
7: Advanced multiplicative part–whole	31	41
8: Advanced proportional part–whole	5	12
Number of students	3975	3975

Note: Percentages may not total to 100 due to rounding.

Table 11 shows the initial and final proportional stages of students by ethnicity. Both at the start and at the end of the project, New Zealand European students were more likely than Māori or Pasifika students to be at the higher stages of the proportional domain, with 60%, 34%, and 26% respectively reaching the top two stages.

Table 11
Initial and Final Proportional Stage by Ethnicity

	NZ European		Māori		Pasifika	
Proportional stage	% initial	% final	% initial	% final	% initial	% final
1: Unequal sharing	1	0	2	1	2	
2–4: Equal sharing	12	5	27	12	33	12
5: Early additive part–whole	26	19	35	35	41	34
6: Advanced additive part–whole	18	15	15	17	11	27
7: Advanced multiplicative part–whole	36	46	19	30	11	22
8: Advanced proportional part–whole	5	14	2	4	1	4
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 12 shows that male students (59%) are more likely than female students (47%) to reach the top two stages of the proportional domain. Similar percentages (6% and 7% respectively) remained at the advanced counting stage or lower (stages 1–4).

Table 12
Initial and Final Proportional Stage by Gender

	Male		Female	
Proportional stage	% initial	% final	% initial	% final
1: Unequal sharing	1	0	2	0
2–4: Equal sharing	15	6	18	7
5: Early additive part–whole	27	20	32	26
6: Advanced additive part–whole	16	15	19	19
7: Advanced multiplicative part–whole	34	43	27	39
8: Advanced proportional part–whole	7	16	2	8
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 13 presents a breakdown of performance on the proportional domain by decile band, showing that students from high-decile schools performed better than students from either medium- or low-decile schools, with 65% reaching stage 7 or higher compared with 48% of students from medium-decile schools and 39% from low-decile schools. At the lower stages of the domain, only 4% of students from high-decile schools remained at the equal sharing stage or lower, compared with 7% of students from medium-decile schools and 16% of students from low-decile schools.

Table 13
Initial and Final Proportional Stage by Decile

	Low		Medium		High	
Proportional stage	% initial	% final	% initial	% final	% initial	% final
1: Unequal sharing	5	1	2	0	1	0
2–4: Equal sharing	21	15	20	7	10	4
5: Early additive part–whole	37	29	30	26	25	16
6: Advanced additive part–whole	12	16	19	18	17	15
7: Advanced multiplicative part–whole	23	33	26	39	40	47
8: Advanced proportional part–whole	2	6	4	9	7	18
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Figure 7 compares the numbers of stages gained on the proportional domain by SNP students with the numbers of stages gained from the same starting stage by year 8 students from schools participating in the NDP. The small numbers of students initially rated below stages 2–4 have been excluded from the following figures. The pattern of performance of students initially rated at stages 2–4 and higher is similar for the two samples. Because of the higher proportions of students gaining more than one stage, the mean gains of SNP students initially at stage 5 are significantly greater than those of year 8 students ($p < 0.05$).

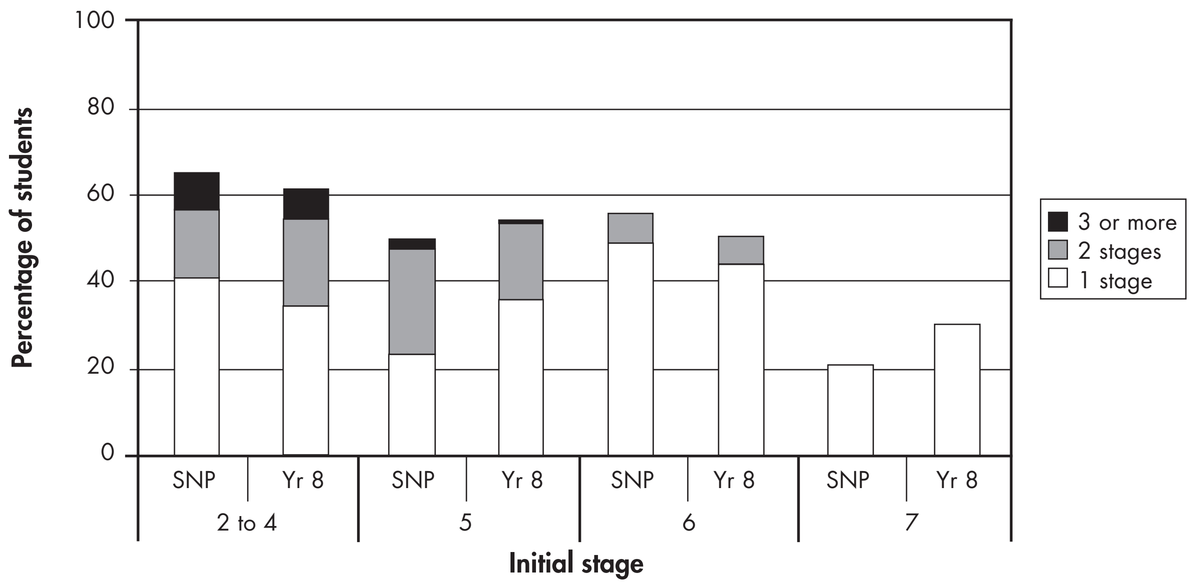

Figure 7. Number of stages gained by initial proportional stage for SNP and year 8 students

Figure 8 shows the progress made on the proportional domain by students' initial stage and ethnicity. A significantly higher proportion of NZ European students than Māori students made progress from stage 5 ($p < 0.001$) and from stage 7 ($p < 0.001$). A significantly higher proportion of Pasifika students than Māori students made progress from stage 5 ($p < 0.05$).

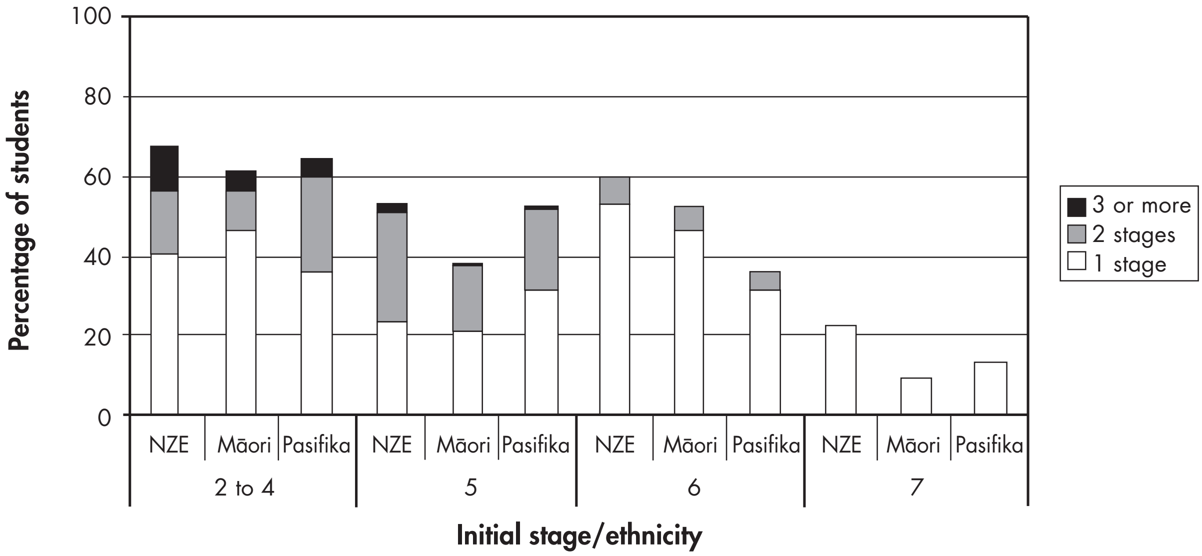

Figure 8. Number of stages gained by initial proportional stage and ethnicity

Figure 9 shows that a significantly higher proportion of students from high-decile schools than low-decile schools made progress from stages 2 to 4 ($p < 0.05$), 5 ($p < 0.01$), and 6 ($p < 0.05$). A significantly higher proportion of students from medium-decile schools than from low-decile schools made progress from stages 5 ($p < 0.01$) and 6 ($p < 0.05$). A significantly higher proportion of students from high-decile schools than from medium-decile schools made progress from stage 7 ($p < 0.05$).

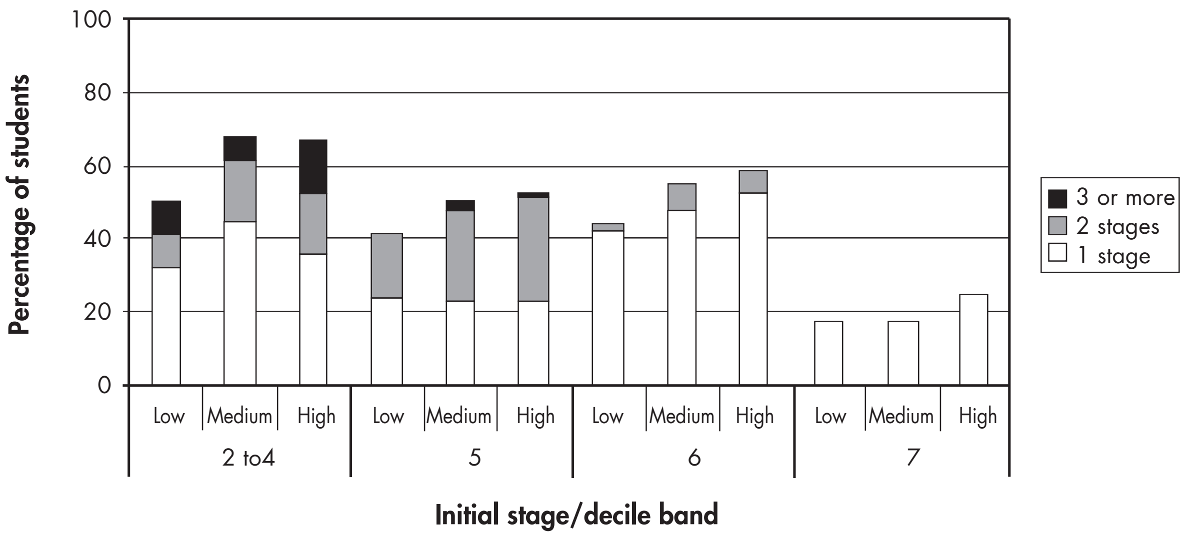

Figure 9. Number of stages gained by initial proportional stage and decile band

Comparison of Strategy Domains

Table 14 shows the percentages of students at each stage at the final assessment for all three strategy domains. It would be expected that students would be more likely to be at the higher stages of the additive domain than of the multiplicative domain and more likely to be at the higher stages of the multiplicative domain than of the proportional domain. While there were similar proportions of students still at the counting stages of each domain, the proportions at the higher stages were mixed, with only 23% of students rated at stage 7 of the additive domain, compared with 46% and 53% at either stage 7 or 8 of the multiplicative and proportional domains respectively.

Table 14
Final Strategy Stages – All Domains

	Domain		
Stage	% additive	% multiplicative	% proportional
< 4: Counting from one	1	0	1
4: Advanced counting	5	6	6
5: Early additive part–whole	26	16	23
6: Advanced additive part–whole	46	32	17
7: Advanced multiplicative part–whole	23	30	41
8: Advanced proportional part–whole	N/A	16	12

Note: Percentages may not total to 100 due to rounding.

Performance on the Knowledge Domains

Forward Number Word Sequence

Tables 15–18 present the results of SNP students on the Forward Number Word Sequence (FNWS) domain, which describes students' ability to identify the number after a given number.

Over the duration of the project, the proportion of students able to identify the number after a given number in the range 1 to 1 000 000 (stage 6) increased from 54% to 71% (see Table 15).

Table 15
Initial and Final FNWS Stage

	% initial FNWS	% final FNWS
0: Emergent	0	0
1: Initial to 10	0	
2: To 10	1	0
3: To 20	1	0
4: To 100	3	2
5: To 1000	41	27
6: To 1 000 000	54	71
Number of students	3975	3975

Table 16 shows that a higher proportion of New Zealand European students (74%) than either Māori (57%) or Pasifika (65%) students reached the top stage of the FNWS domain.

Table 16
Initial and Final FNWS Stage by Ethnicity

	NZ European		Māori		Pasifika	
FNWS stage	% initial	% final	% initial	% final	% initial	% final
0: Emergent	0			0		
1: Initial to 10	0					
2: To 10	0	0	1	1	4	3
3: To 20	1	0	3	1		1
4: To 100	2	1	5	3	6	2
5: To 1000	39	25	51	37	47	31
6: To 1 000 000	58	74	40	57	43	65
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 17 shows that a higher percentage of male (73%) than female (68%) students reached the top stage of the FNWS domain.

Table 17
Initial and Final FNWS Stage by Gender

	Male		Female	
FNWS stage	% initial	% final	% initial	% final
0: Emergent	0			0
1: Initial to 10			0	
2: To 10	1	1	1	0
3: To 20	1	0	1	1
4: To 100	3	2	3	1
5: To 1000	38	24	45	30
6: To 1 000 000	57	73	51	68
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 18 shows that a higher proportion of students from high-decile schools (74%) than students from medium-decile (69%) or low-decile (64%) schools reached the top stage of the FNWS domain.

Table 18
Initial and Final FNWS Stage by Decile

	Low		Medium		High	
FNWS stage	% initial	% final	% initial	% final	% initial	% final
0: Emergent			0			0
1: Initial to 10			0			
2: To 10	2	2	1	0	0	0
3: To 20	0		1	1	1	0
4: To 100	6	3	3	2	1	0
5: To 1000	46	31	40	27	40	24
6: To 1 000 000	46	64	53	69	57	75
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Fractions Domain

Tables 19–22 illustrate the performance of SNP students on the fractions domain, which describes students' ability to identify and order fractions.

Table 19 shows that the percentage of students able to identify equivalent fractions (stage 7 or 8) increased from 22% at the initial assessment to 40% by the final assessment. The percentage of students still unable to order unit fractions (stage 4 or lower) decreased from 18% to 9%.

Table 19
Initial and Final Fractions Stage

	% initial fractions	% final fractions
2–3: Non-fractions	5	2
4: Assigns unit fractions	13	7
5: Orders unit fractions	39	29
6: Co-ordinates numerators/denominators	21	23
7: Equivalent fractions	15	25
8: Orders fractions	7	15
Number of students	3975	3975

Note: Percentages may not total to 100 due to rounding.

Table 20 shows that New Zealand European students performed better than Māori and Pasifika students on the fractions domain, with 46% reaching the top two stages of the domain, compared with 22% and 23% for Māori and Pasifika students respectively. Fewer Pasifika (11%) than Māori (17%) students remained unable to order unit fractions by the end of the project.

Table 20
Initial and Final Fractions Stage by Ethnicity

	NZ European		Māori		Pasifika	
Fractions stage	% initial	% final	% initial	% final	% initial	% final
2–3: Non-fractions	4	1	11	3	8	4
4: Assigns unit fractions	11	5	19	14	15	7
5: Orders unit fractions	40	28	43	37	49	33
6: Co-ordinates numerators/denominators	22	21	17	24	20	34
7: Equivalent fractions	16	28	8	14	5	16
8: Orders fractions	8	18	2	8	2	7
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 21 shows that while a higher percentage of male (18%) than female (12%) students reached the top stage of the fractions domain, similar percentages (9% and 8% respectively) remained unable to order unit fractions (stage 4 or lower).

Table 21
Initial and Final Fractions Stage by Gender

	Male		Female	
Fractions stage	% initial	% final	% initial	% final
2–3: Non-fractions	7	2	4	1
4: Assigns unit fractions	12	7	13	7
5: Orders unit fractions	37	27	42	30
6: Co-ordinates numerators/denominators	19	21	23	25
7: Equivalent fractions	17	25	12	25
8: Orders fractions	8	18	5	12
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

As shown in Table 22, students from high-decile schools performed better than students from medium-decile schools on the fractions domain and both performed better than students from low-decile schools, with 71%, 61%, and 46% respectively reaching at least stage 6 by the end of the project.

Table 22
Initial and Final Fractions Stage by Decile

	Low		Medium		High	
Fractions stage	% initial	% final	% initial	% final	% initial	% final
2–3: Non-fractions	11	3	6	2	3	1
4: Assigns unit fractions	20	12	14	7	9	4
5: Orders unit fractions	40	38	43	30	35	24
6: Co-ordinates numerators/denominators	19	17	19	25	24	21
7: Equivalent fractions	7	17	12	23	20	30
8: Orders fractions	4	12	6	13	9	20
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Place Value Domain

Tables 23–26 present the results of SNP students on the place value domain, which describes students' ability to partition whole numbers and decimals using their place value.

Table 23 shows that the percentage of students at least able to identify the number of tenths in numbers and order decimals (stage 7 or 8) increased from 18% at the initial assessment to 35% by the end of the project.

Table 23
Initial and Final Place Value Stage

	% initial place value	% final place value
0–1: Non-counting	0	
2–3: Counts by ones	2	1
4: 10s to 100, order to 1000	11	5
5: 10s to 1000, order to 10 000	49	35
6: 10s, 100s, 1000s, orders whole numbers	19	25
7: Tenths in and orders decimals	10	18
8: Tenths hundredths and thousandths	8	17
Number of students	3975	3975

Note: Percentages may not total to 100 due to rounding.

Table 24 shows that a higher proportion of New Zealand European students (39%) than Pasifika (27%) or Māori (17%) students reached the top stages of the place value domain.

Table 24
Initial and Final Place Value Stage by Ethnicity

	NZ European		Māori		Pasifika	
Place value stage	% initial	% final	% initial	% final	% initial	% final
0–1: Non-counting	0				1	
2–3: Counts by ones	2	0	4	1	4	3
4: 10s to 100, order to 1000	9	4	16	9	17	4
5: 10s to 1000, order to 10 000	48	32	56	46	58	41
6: 10s, 100s, 1000s, orders whole numbers	20	24	16	28	14	25
7: Tenths in and orders decimals	11	20	5	11	5	22
8: Tenths hundredths and thousandths	10	19	2	6	2	5
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 25 shows that while a higher percentage of male (37%) than female (32%) students reached at least stage 7 of the place value domain, a similar percentage of male (6%) to that of female (4%) students remained at stage 4 or lower.

Table 25
Initial and Final Place Value Stage by Gender

	Male		Female	
Place value stage	% initial	% final	% initial	% final
0–1: Non-counting	0		0	
2–3: Counts by ones	3	1	2	0
4: 10s to 100, order to 1000	11	5	11	4
5: 10s to 1000, order to 10 000	47	34	52	36
6: 10s, 100s, 1000s, orders whole numbers	18	23	20	27
7: Tenths in and orders decimals	10	17	10	19
8: Tenths hundredths and thousandths	11	20	5	13
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 26 shows that students from high-decile schools were more likely to reach the higher stages of the place value domain than students from medium-decile schools and that both were more likely than students from low-decile schools.

Table 26
Initial and Final Place Value Stage by Decile

	Low		Medium		High	
Place value stage	% initial	% final	% initial	% final	% initial	% final
0–1: Non-counting	0		0			
2–3: Counts by ones	4	1	2	1	2	1
4: 10s to 100, order to 1000	21	8	10	5	9	4
5: 10s to 1000, order to 10 000	51	43	51	36	46	31
6: 10s, 100s, 1000s, orders whole numbers	17	24	20	27	17	21
7: Tenths in and orders decimals	6	17	9	17	12	20
8: Tenths hundredths and thousandths	2	7	6	14	13	24
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Basic Facts Domain

Tables 27 to 30 illustrate the performance of SNP students on the basic facts domain, which rates students' ability to quickly recall basic number facts.

Table 27 shows that the percentage of students who knew at least their subtraction and multiplication basic facts (stage 6 or higher) increased from 71% at the initial assessment to 82% by the final assessment. At the end of the project, 4% of students were still unable to recall their multiplication facts for 2, 5, and 10 (that is, were still below stage 5).

Table 27
Initial and Final Basic Facts Stage

	% initial basic facts	% final basic facts
0–1: Non-grouping	1	0
2–3: Within and with five	2	1
4: Within 10, doubles, and teens	6	3
5: Addition, multiplication for 2, 5, 10	21	14
6: Subtraction and multiplication	47	44
7: Division	24	38
Number of students	3975	3975

Note: Percentages may not total to 100 due to rounding.

Table 28 shows that New Zealand European students performed better than either Māori or Pasifika students on the basic facts domain. While a slightly higher proportion of Pasifika (78%) than Māori (73%) students knew their subtraction and multiplication facts (stage 6 or higher) by the end of the project, a similar proportion (7% compared with 5%) remained below stage 5.

Table 28
Initial and Final Basic Facts Stage by Ethnicity

	NZ European		Māori		Pasifika	
Basic facts stage	% initial	% final	% initial	% final	% initial	% final
0–1: Non-grouping	1	0	1	0	1	
2–3: Within and with five	1	0	3	1	2	2
4: Within 10, doubles, and teens	5	3	8	4	9	5
5: Addition, multiplication for 2, 5, 10	21	13	27	20	23	16
6: Subtraction and multiplication	48	45	46	45	45	44
7: Division	24	39	16	28	21	34
Number of students	2642	2642	783	783	193	193

Note: Percentages may not total to 100 due to rounding.

Table 29 shows that female students performed similarly to male students on the basic facts domain, with 84% knowing their subtraction and multiplication basic facts (stage 6 or higher) by the end of the project and only 2% still unable to recall their multiplication facts for 2, 5, and 10 (stage 4 or lower), compared with 80% and 5% respectively of male students.

Table 29
Initial and Final Basic Facts Stage by Gender

	Male		Female	
Basic facts stage	% initial	% final	% initial	% final
0–1: Non-grouping	1	0	0	0
2–3: Within and with five	2	1	1	0
4: Within 10, doubles, and teens	7	4	4	2
5: Addition, multiplication for 2, 5, 10	22	15	21	12
6: Subtraction and multiplication	45	41	50	46
7: Division	24	39	23	38
Number of students	2248	2248	1727	1727

Note: Percentages may not total to 100 due to rounding.

Table 30 shows that students from high-decile schools performed better than students from medium-decile schools (86% compared with 81% at stage 6 or higher by the end of the project) on the basic facts domain and that both performed better than students from low-decile schools.

Table 30
Initial and Final Basic Facts Stage by Decile

	Low		Medium		High	
Place value stage	% initial	% final	% initial	% final	% initial	% final
0–1: Non-grouping	2	1	0	0	1	0
2–3: Within and with five	3	1	1	1	1	0
4: Within 10, doubles, and teens	8	5	6	3	5	2
5: Addition, multiplication for 2, 5, 10	30	16	22	15	17	12
6: Subtraction and multiplication	42	47	47	45	48	40
7: Division	15	29	22	36	28	46
Number of students	435	435	2059	2059	1481	1481

Note: Percentages may not total to 100 due to rounding.

Comparison of Knowledge Domains

Table 31 presents the percentages of students at each stage of the four knowledge domains at the end of the project. That a small proportion of students remain at stage 4 or lower on each domain should be a cause for concern for teachers in SNP schools. At the higher stages, it can be seen that a similar percentage of students reached stage 7 and 8 on the fractions (40%), place value (35%), and basic facts (38%) domains.

Table 31
Final Knowledge Stages – All Domains

	Domain			
Stage	% FNWS	% Fractions	% Place value	% Basic facts
<4	1	2	1	1
4	2	7	5	3
5	27	29	35	14
6	71	23	25	44
7	N/A	25	18	38
8	N/A	15	17	N/A

Note: Percentages may not total to 100 due to rounding.

The Relationship between Strategy and Knowledge

Tables 32 and 33 explore the relationship between students' gains in strategy stage and their number knowledge. The tables present the knowledge results on the fractions, place value, and basic facts domains of students who were initially rated as advanced additive (stage 6) on the multiplicative domain. This subgroup of the students was chosen for two reasons. Firstly, because this is the stage at which the highest proportion (34) of SNP were initially assessed. Secondly, there was a relatively even split of students between those who made progress from stage 7 and those who did not. The students are separated into two groups: those who progressed to at least stage 7 (advanced multiplicative), and those who did not.

Table 32 compares the end-of-project number knowledge profile of the two groups. In each knowledge domain, over half (57, 51, and 52 respectively) of the students who progressed were rated as at least stage 7. Contrastingly, less than a third (30, 25, and 32 respectively) of the students who did not progress were rated at stage 7 or 8 on the knowledge domains.

Table 32
Comparing the Number Knowledge Profile of Initially Advanced Additive Students Who Progressed to Advanced Multiplicative with Those Who Did Not

Number of students	680	661
	% remained advanced additive	% became multiplicative
Fractions		
2–3: Non-fractions	1	0
4: Assigns unit fractions	6	2
5: Orders unit fractions	33	19
6: Co-ordinates numerators/denominators	29	22
7: Equivalent fractions	21	33
8: Orders fractions	9	24
Place value		
2–3: Counts by ones	1	0
4: 10s to 100, order to 1000	4	1
5: 10s to 1000, order to 10 000	40	23
6: 10s, 100s, 1000s, orders whole numbers	31	25
7: Tenths in and orders decimals	18	24
8: Tenths, hundredths, and thousandths	7	27
Basic facts		
0–1: Non-grouping	0	0
2–3: Within and with five	0	0
4: Within 10, doubles, and teens	3	2
5: Addition, multiplication for 2, 5, 10	15	5
6: Subtraction and multiplication	49	40
7: Division	32	52

Note: Percentages may not total to 100 due to rounding.

Table 33 presents the same results, but in this case, the percentages represent the proportions of students finishing the project at each knowledge stage who did or did not progress on the multiplicative domain. Over half of those students who were rated at stage 7 or 8 on each knowledge domain progressed on the multiplicative domain, while over half of those who were rated below stage 7 on each knowledge domain remained advanced additive.

Table 33
Comparing the Proportions of Initially Advanced Additive Students at Each Knowledge Stage Who Progressed to Advanced Multiplicative with Those Who Did Not

	Number of students	% remained advanced additive	% became multiplicative
Fractions			
2–3: Non-fractions	10	80	20
4: Assigns unit fractions	50	78	22
5: Orders unit fractions	351	64	36
6: Co-ordinates numerators/denominators	345	58	42
7: Equivalent fractions	364	40	60
8: Orders fractions	221	29	71
Place value			
2–3: Counts by ones	7	86	14
4: 10s to 100, order to 1000	29	86	14
5: 10s to 1000, order to 10 000	423	64	36
6: 10s, 100s, 1000s, orders whole numbers	373	57	43
7: Tenths in and orders decimals	279	43	57
8: Tenths, hundredths, and thousandths	230	22	78
Basic facts			
0–1: Non-grouping	2	100	0
2–3: Within and with five	3	33	67
4: Within 10, doubles, and teens	34	59	41
5: Addition, multiplication for 2, 5, 10	134	75	25
6: Subtraction and multiplication	603	56	44
7: Division	565	39	61

Concluding Comment and Key Findings

Students in schools participating in the SNP made progress on all three strategy domains. More specifically, the findings were as follows:

- The percentages of students rated in the top two stages of the additive, multiplicative, and proportional domains increased from 45 to 69, 25 to 46, and 36 to 53 respectively.
- The percentages of students still rated stage 4 or lower on the additive, multiplicative, and proportional domains decreased from 15 to 5, 14 to 6, and 17 to 6 respectively.
- Greater percentages of New Zealand European students than Māori and Pasifika students were at higher initial and final stages across all three domains.
- A higher proportion of male students than female students reached the top stages of each strategy domain; similar proportions remained at the bottom stages.
- Students from high-decile schools performed better than students from medium-decile schools, who in turn performed better than students from low-decile schools.

Students also made progress on the four knowledge domains as shown below.

- New Zealand European students performed better than Māori or Pasifika students; the performance of Pasifika students was overall slightly better than that of Māori students, though this varied between domains.
- Male students generally performed slightly better than female students, the exception being in the basic facts domain, where they performed similarly.
- Students from high-decile schools performed better than students from medium-decile schools, who in turn performed better than students from low-decile schools.

A comparison of knowledge profiles of initially advanced additive students showed that those students who made progress on the multiplicative domain were more likely to be at the higher stages on the knowledge domains than those who did not.

References

Ministry of Education (2005). *Book 1: The Number Framework.* Wellington: Ministry of Education.

Ministry of Education (2005–). "The Diagnostic Assessment." Unpublished (SNP strategy diagnostic test and whole-class knowledge test)

Evaluation of Secondary Numeracy Pilot Project 2005

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Introduction

In 2005, the Ministry of Education offered New Zealand secondary schools an opportunity to improve the teaching and learning of number and algebraic concepts and skills through the Secondary Numeracy Pilot Project (SNP), a professional development programme for teachers.

The overall aim of the SNP is to develop teachers' knowledge of number and algebraic concepts, student strategies, and instructional practice in order to improve student achievement in year 9. A key part of the project is presenting teachers with a framework of broad stages describing students' numerical and algebraic thinking. Each stage is characterised by the range of strategies that students use to solve calculation problems. Teachers use a diagnostic interview to assess the stages of their students' thinking. Teachers are also introduced to number problem-solving strategies, activities, and equipment to use when working with students. There is a particular emphasis on ways of developing increasingly sophisticated strategies for solving number and algebra problems.

This report evaluates the impact of the 2005 SNP on 320 teachers across 11 regions: Northland, Auckland, Bay of Plenty, Wanganui, Manawatu, Hawke's Bay, Wellington, Nelson, Canterbury, Otago, and Southland. The report identifies changes in student achievement and teacher knowledge and practice that can be attributed to the professional development provided by the SNP.

The research addressed the following main questions:

Impact on teachers

1. Has the 2005 SNP pilot had an impact on teachers' professional knowledge? If so, what are the changes?
2. What experiences and factors do teachers report as influencing these changes?
3. Do teachers perceive that changes in their professional knowledge (including knowledge of the subject, the pedagogical content, and learners' cognitions) have an impact on their classroom practices? If so, how?
4. What impact has the shift from staff member to in-school facilitator had on existing relationships within the school?

Impact on facilitators

1. What is the impact on external and in-school facilitators of developing teacher subject and pedagogical content knowledge at the year 9 level?
2. What knowledge have the in-school facilitators developed through their participation in the SNP?

3. What are the key qualities and skills required for effective in-school facilitation of the SNP at the year 9 level?

In-school facilitation model

1. What evidence is there that an in-school facilitation model is effective in building teacher capability?
2. What have been the benefits and drawbacks of in-school facilitation, according to different members of the school mathematics community?
3. Is there any evidence of enhanced collegial support within the mathematics department?

Key Findings

Impact on teachers

- There was little change in the teachers' mathematical knowledge, as reported by the teachers or facilitators. Data from facilitators reported growth in teachers' knowledge of teaching mathematics. Data from teachers suggested more modest growth.
- Both teachers and facilitators reported an improvement in teachers' knowledge of how students learn mathematics across the duration of the year.
- Many teachers reported that achievement data gathered had influenced subsequent mathematics teaching by giving them a level at which to plan lessons and helping them to group students.
- The overwhelming view of teachers and in-school facilitators was that teachers' knowledge of number was the most important factor in teaching mathematics at year 9. This belief ran through responses to all questions, being strongest in the requirements for year 8 to bring to year 9 (99% of teachers and 33% of facilitators). It was seen as a key element by 49% of teachers in the initial survey and 77% in the final survey.
- Teachers saw time as the greatest limiting factor to teachers in fulfilling the SNP. Initially, 30% of facilitators and 39% of teachers saw it as an issue, and at the end of the year, 41% of facilitators and 33% of teachers saw it this way.

Impact on facilitators

- Most in-school facilitators felt that they had developed their knowledge of facilitation during the year. They generally felt more knowledgeable about the teaching of mathematics, how to prepare materials, and how to put their ideas across to teachers. They were more confident about running professional development sessions, although many would welcome more support on this in the initial stages of the project.
- People skills were seen as a very important quality for in-school facilitators to possess, for example, being approachable, having the ability to listen, and being able to communicate effectively. A belief in, and enthusiasm about, numeracy and the role of the facilitator were also important, as was the need to know mathematics.
- Many in-school facilitators noticed little impact in changing their role from being a staff member to being a facilitator. Many had been heads of department (HODs), so they knew teachers well and were used to discussing issues and working in a mentoring role. For others, the role provided opportunities to develop their work in mentoring staff.

In-school facilitation model

- There is evidence to show that the in-school facilitation model is effective in building teacher capability in a significant way. Teachers were demonstrating an increased knowledge of how to teach calculation and were thinking more about what they were doing. The model had impacted on classroom practices, and teachers were more willing to try new things and take risks in areas such as group teaching and the creative use of materials.
- Having a facilitator in school was mostly seen as beneficial. For example, there was someone for teachers to take their problems to on a daily basis, which made dealing with problems arising in the context of specific schools easier and also made for better professional development and elevation of the status of mathematics in the school. Having a member of the school's mathematics department lead the programme gave the project credibility because that person was seen as being knowledgeable about the complexities of mathematics teaching and learning in their own school. In some schools, it took time to build trust so that colleagues were comfortable with in-class support. Workload was an issue for many in-school facilitators, particularly those with HOD responsibilities. Some teachers had problems seeing the facilitator in a different role.
- There is evidence that collegial support increased during the period of the SNP. Teachers were working together better with more discussion and were feeling more at ease within the school and with the facilitators.

Recommendation

- SNP schools should be provided with ongoing support so that they can consolidate and extend the advances made in mathematics teaching.

Background

The 2005 SNP pilot built on exploratory work carried out in secondary schools in 2001, 2002, and 2003 and on the introductory Numeracy Development Project (NDP) workshops provided throughout New Zealand in 2004. The evaluation investigated the impact of external and in-school facilitators on teachers' development of subject and pedagogical content knowledge at the year 9 level of schooling as well as considering the project's impact on the development of professional mathematics communities in the departments of participating schools.

The teacher development model for the 2005 pilot project was different from that used in all the other numeracy projects in its use of in-school facilitators supported regionally by an external facilitator. The in-school facilitator's role underpins the whole-school approach by involving all the members of the mathematics department who teach year 9 students, as well as having the goal of developing a professional mathematics community in the school. The in-school facilitator is typically a person holding a position of responsibility for mathematics (either the HOD or the management unit (MU) holder responsible for year 9 mathematics) and as such, should be better able to facilitate modifications to school systems as well as ensuring agreement across department staff for the timing of meetings and coaching and in-class modelling of effective mathematics teaching. The in-school facilitator is provided with training both nationally and regionally and is supported by the regional co-ordinator through regular meetings and visits to the school.

Rationale for the Project

Recent evaluations of the exploratory numeracy projects in secondary settings found that student achievement at year 9 has been variable. The challenge of changing teachers' practice at the secondary level has also been noted by a number of commentators. In the evaluation of the Numeracy Exploratory Study in 2001, Irwin and Niederer (2002) noted the "unexpected weaknesses, especially in the understanding of fractions, the ability to find a fraction of a whole number, and the meaning of large numbers" (p. 97). While noting the possibility of a number of factors for this, one factor identified was the weak pedagogical content knowledge of teachers in the area of place value. In 2002 and 2003, students also performed weakly in the National Certificate of Educational Achievement (NCEA) algebra achievement standards, with about 50% of students being unable to solve straightforward algebraic equations at levels 1 and 2[1]. It is worth noting here that the standards were comprised of largely knowledge-based questions, with very few drawing on students' mathematical generalisations and strategy development.

Specific Aims of the Evaluation

This evaluation investigated the impact of external and in-school facilitators on developing teacher subject and pedagogical content knowledge at the year 9 level of schooling through participation in the SNP Pilot 2005 and the extent to which professional mathematics communities have developed in each mathematics department.

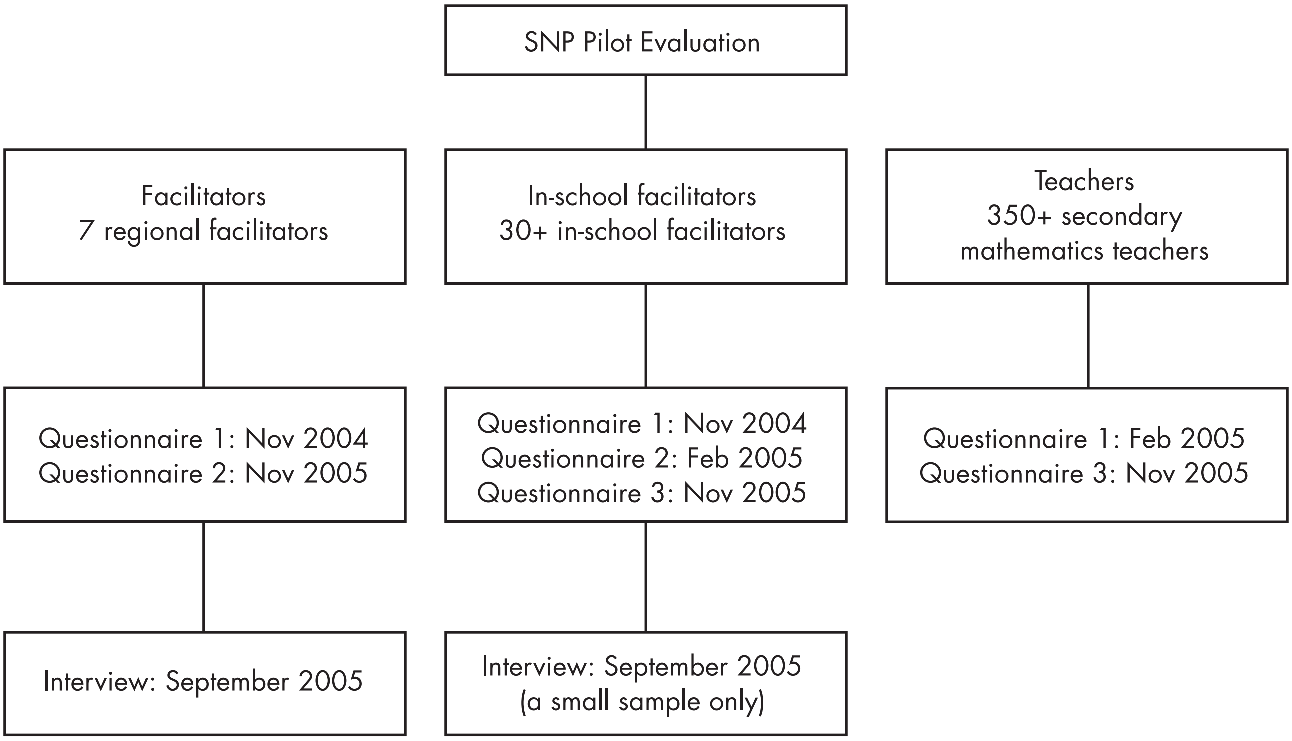

Figure 1. Overview of the evaluation of SNP Pilot 2005

1 Level 1: Use straightforward algebraic methods and solve equations: Not achieved: 50.1%.
Level 2: Manipulate algebraic expressions and solve equations: Not achieved: 48.2%.

Design and Methodology

The methodology took the form of an extended case study (Stake, 1998), involving about 30 teachers at selected schools nationally who were participating in the SNP Pilot 2005 as in-school facilitators and the 360–400 year 9 teachers with whom they were working. Feedback was sought from the external facilitators on the mentoring process used with these in-school facilitators.

To determine whether changes to teacher knowledge and practices have occurred as a result of their involvement in the SNP Pilot 2005, data from two research tools was used: semi-structured interviews and concept mapping. The concept mapping approach was used to examine the impact of these resources on the professional knowledge of teachers in New South Wales (Bobis, 1999). More recently, this approach has been used to evaluate the pilots of the Early and Advanced Numeracy Projects (Thomas & Ward, 2001; Higgins, 2001).

The research involved the collection of base-line data from the 30 in-school facilitators in November 2004. The data included:

- biographical details (number of years of teaching; qualifications in mathematics);
- beliefs about year 9 mathematics in terms of what should be taught;
- beliefs about what year 8 students bring to year 9;
- beliefs about how mathematics should be taught at year 9;
- knowledge and views of facilitation, including the skills they already have and the skills they need to develop.

At the first national training for in-school facilitators (November 2004), the participants were asked to draw a concept map entitled "Key elements of effective professional development" as a starting point for revealing their beliefs about the focus of the professional development to be facilitated with the teachers in their school. They were invited to redraw their concept map at the end of the first year in the project (November 2005) as a means of identifying changes they had made to their beliefs and knowledge about teaching mathematics at year 9. A small number of the in-school facilitators were also interviewed to gather more in-depth comments.

The analysis was informed by the teacher-centred model of professional development outlined in the 2001 evaluation of the Advanced Numeracy Project (Higgins, 2002).

Overview of Participants

Demographic data of the schools involved in the project

A total of 43 schools over 11 regions of New Zealand were involved in the project. The schools involved ranged in size from 201 students in the smallest school to 2247 students in the largest school. The schools ranged in decile from 1 to 10.

Table 1
Demographic Data of Schools

Category	Details	Number of schools	Percentage (N = 43)
Region	Northland	2	5
	Auckland	9	21
	Bay of Plenty	8	19
	Manawatu–Wanganui	4	9
	Hawke's Bay	2	5
	Wellington	4	9
	Nelson	1	2
	Canterbury	6	14
	Otago	6	14
	Southland	1	2
Size of school (number of pupils)	101–500	10	23
	501–1000	13	30
	1001–2000	18	42
	2001+	2	5
Decile of school	1–3	10	23
	4–7	16	37
	8–10	17	40

Demographic data of teacher respondents

Of the 319 questionnaires sent out to teachers, 177 were returned, resulting in a 55% response rate. Of these 177, two arrived too late for inclusion. The following analysis is based on the remaining 175.

Table 2
Demographic Data of Teacher Respondents

Category	Details	Number of Teachers	Percentage (N = 175)
Years of teaching experience*	1–5	37	21
	6–10	18	10
	11–15	25	14
	16–20	17	10
	21–30	34	19
	31+	16	9
	No response	28	16
Highest mathematics qualification*	Secondary school	18	10
	University stage 1	15	9
	University stage 2	30	17
	University stage 3/Bachelor's degree	62	35
	Postgraduate level 1 (including Masters, PhD)	22	13
	No response	28	16

*Not all respondents completed this question.

Impact on Teachers

Teachers' Mathematical Knowledge

Pre and post-comparisons are useful for assessing the impact of a programme of professional development. Teachers were asked to rate their mathematical knowledge at the beginning and end of the year, using a likert scale where 1 was strong and 5 was weak. As illustrated in Figure 2, teachers reported little change in their own knowledge of mathematics as a result of their involvement in the SNP. This is not surprising, given that, as specialist mathematics teachers, they would start with a high level of mathematical knowledge anyway.

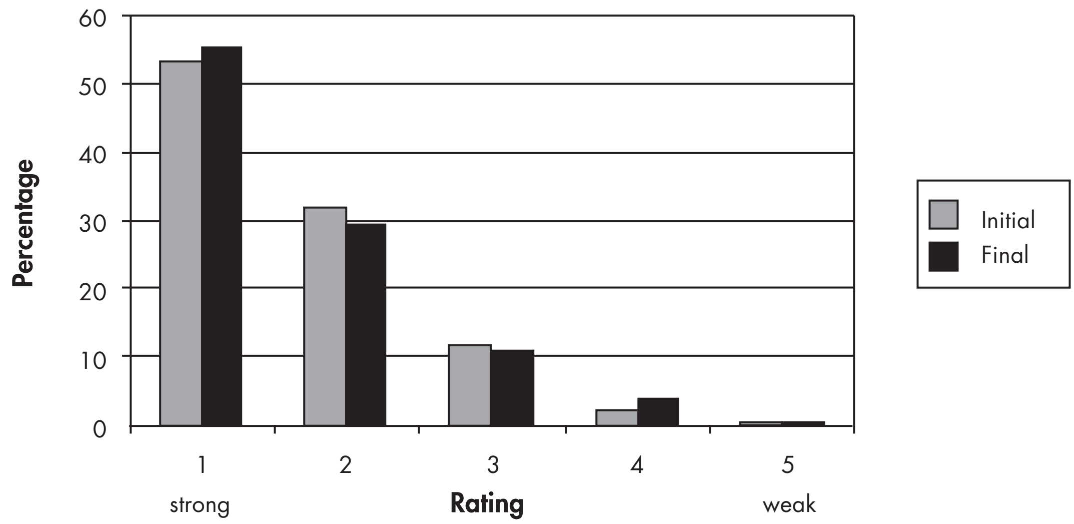

Figure 1. Teachers' self-rating mathematical knowledge

Facilitators were also asked to rate teachers' mathematical knowledge at the beginning and end of the year. Their rating of teacher knowledge was slightly lower than that of teachers' rating of themselves. The facilitators reported little change in teachers' mathematical knowledge over the year.

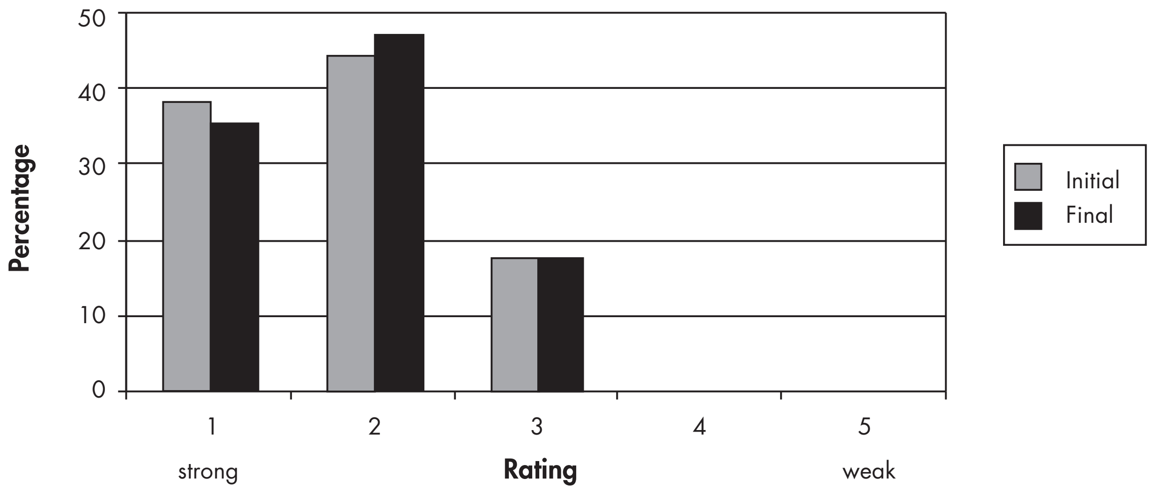

Figure 2. Facilitators' rating of teachers' mathematical knowledge

Teachers' Mathematical Practice

Similarly, teachers were asked to rate their mathematical teaching practice at the beginning and end of the year. The pattern that emerged here was different from that of knowledge. Initially, teachers rated their teaching practice more highly than they did at the end of the year (see Figure 3). This may have been because they shifted from instruction based on textbooks to instruction based on discussion and thinking.

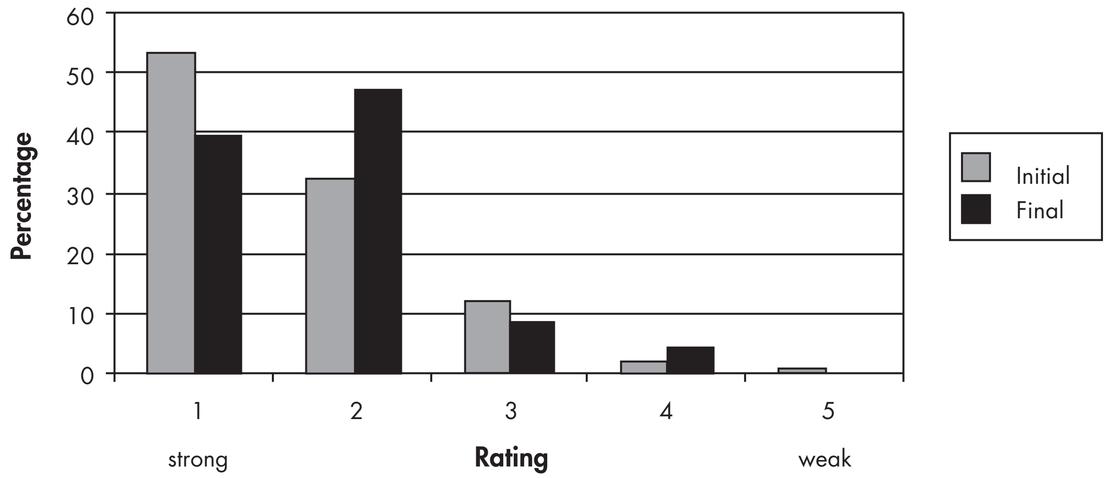

Figure 3. Teachers' self-rating of mathematics teaching practice

Facilitators were also asked to rate teachers' knowledge of teaching mathematics. The facilitators regarded teachers' knowledge of teaching mathematics as being not as strong as teachers rated themselves. However, an improved rating over the year is evident.

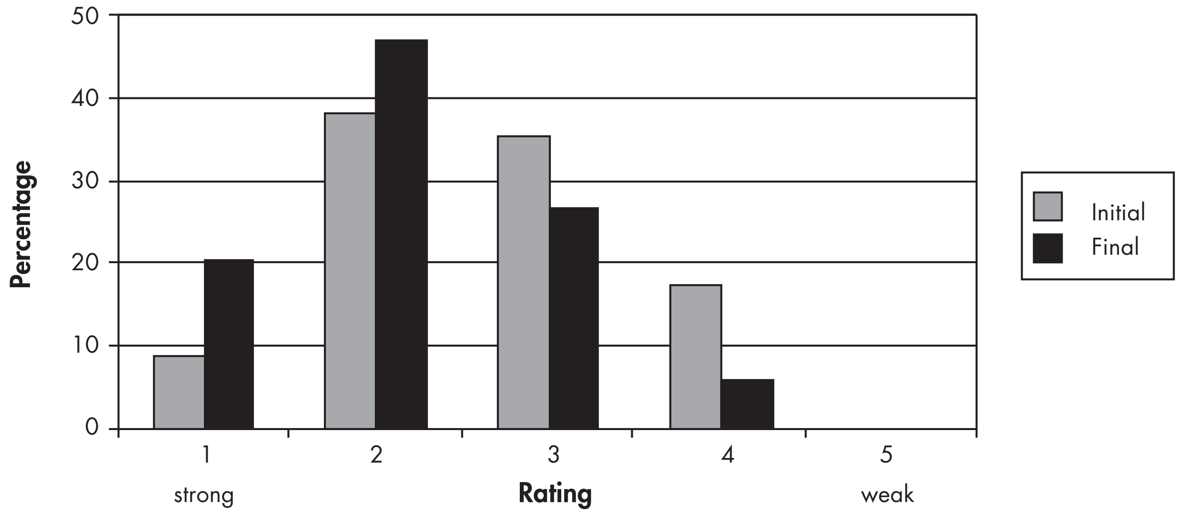

Figure 4. Facilitators' rating of teachers' knowledge of teaching mathematics

Teachers self-rated their knowledge of how students learn mathematics before and at the completion of the professional development. They did not rate this knowledge as highly as they rated their own knowledge of mathematics. However, an improvement over the year is noted.

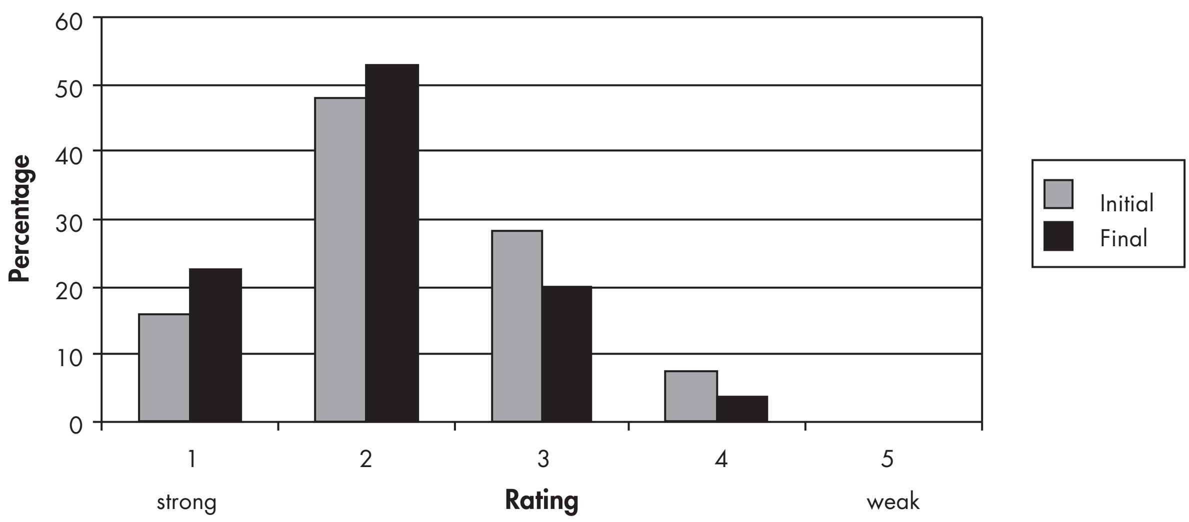

Figure 5. Teachers' self-rating of how students learn mathematics

Facilitators were also asked to rate teachers' knowledge of how students learn mathematics. Over the course of the year, they indicated that this aspect of teacher knowledge improved.

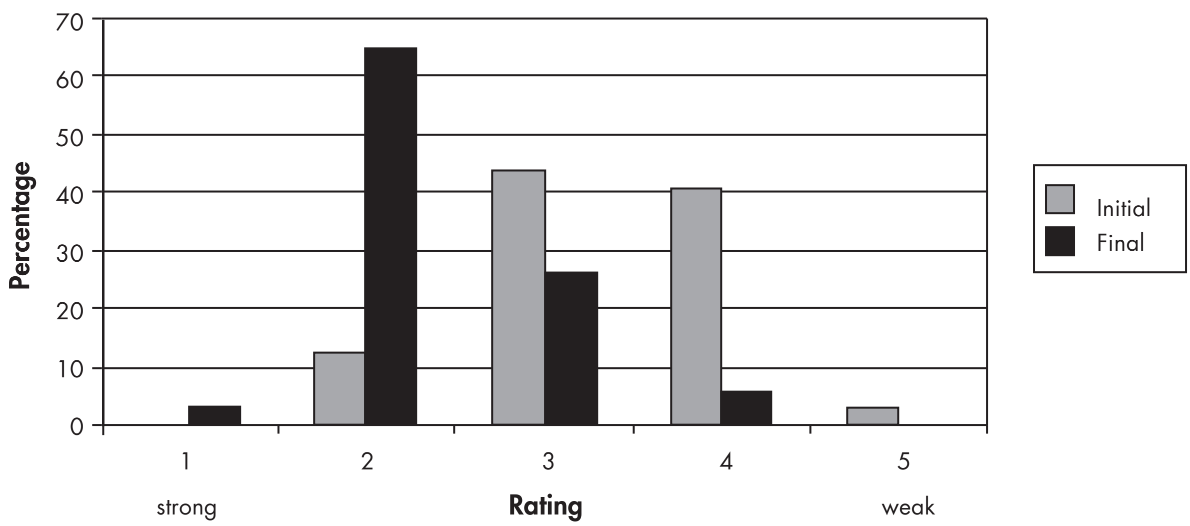

Figure 6. Facilitators' rating of teachers' knowledge of how students learn mathematics

Year 9 Mathematics Programme

One way of gauging the impact of professional development on classroom practice is to investigate teachers' views of the key elements of teaching and assessing of a mathematics programme in the first year of secondary schooling.

Initial responses to the question "What were the key elements of a year 9 mathematics programme?" were wide-ranging and varied, with most teachers listing several elements they felt were important. By far the largest number of responses included number/numeracy as a key element (49%). Typical comments were:

> The programme should also consolidate and build number skills, the basis of most other strands.
>
> Emphasis on number skills, a feel for number. Little else can be taught in mathematics without an understanding of number.
>
> Numeracy, as it is the foundation – needs to be the basic number one key element.

Many teachers mentioned algebra, often in connection with number (26%). Some saw algebra as being important to future mathematics learning. Typical comments were:

> Algebra. Students struggle in algebra even when they move to higher classes. The problem is that the topic becomes more difficult during year 12 and they have less time to practise and grasp the concept of algebra.
>
> Algebra – the cornerstone of so much subsequent maths learning.
>
> Transition from mainly about numeracy to looking at what algebra is and how it's useful.

Strands that were also rated by larger groups of respondents were geometry (12%) and measurement (10%). Comments about geometry included:

> Geometry ... should be emphasised more.
>
> Geometry – shapes, angle types, no theoretical work like corresponding angles.
>
> Geometry – the ability to visualise spatially in 2D/3D is a powerful tool.

Comments about measurement included:

> Thorough understanding of ... measurement
>
> Measurement (units, perimeter, area, and volume)

Statistics were mentioned by 6% and fractions by 3%. A further 16% talked about the basics (skills, facts). Comments on these areas included:

> Making sure they have the basics and moving them on from there in all areas of mathematics.
>
> Proceeding until students have a solid grounding in the basics.
>
> Consolidating all basic work and building towards future mathematical learning.

The fact that students should be able to use strategies they had learned to solve problems was seen as a key element by 20% of teachers. Typical comments were:

> The programme should enable students to acquire knowledge and to use their knowledge to solve problems.
>
> <u>Skills</u>! Students need to know why to do things, not <u>how</u> to do them. If they know why, then how comes naturally.
>
> Problem solving – need to design own methods, strategies, etc.

Many other topics were mentioned by small numbers of teachers: student understanding (9%); the curriculum – including one response about cross-curricular activities (5%); patterns and prior knowledge (both 4%); calculators, student enjoyment, and variety (3% each); time, learning styles, and levels, *Mathematics in the New Zealand Curriculum* (*MiNZC*), NCEA, and group work (2% each); basic building blocks, monitoring, year 8, 9, and 10 students, exploration, and testing (1% each); and concepts, computers, listening skills, resources, feedback, and streamed classes (just under 1% each).

There was no response from 11% of teachers, with a further 2% of answers saying nothing relevant.

Final responses by teachers to the question "Have you changed your views of what the key elements of a year 9 mathematics programme should be?" are listed below. Of the teachers who responded to this question, the same number of outright "yes" and "no" answers were received (26%). Almost as many responses (23%) gave no comment on any view change but rather offered general explanations. Comments included:

> Key elements should be more basic knowledge rather than rushing through the curriculum.
>
> The key elements change, depending on my group – some numeracy, specifically decimals, percentages, fractions, has to be included (not always the case in the past) as skills – they don't always have the level expected at year 9.
>
> Number knowledge and strategies are the key to a successful programme. The other strands give context to numbers.

Comments from 13% of teachers indicated that there was no change to their view of the key elements, because they already put a high value on numeracy. Examples of these comments are:

> I have always felt that basic numeracy is essential for ultimate success with mathematics, and I'm pleased to see the emphasis returning to this.
>
> Not really – number work or place value has always been high.
>
> I still believe the key elements should be a strong grounding in the basics and number.

A small group (8%) did not address this question, and 4% of teachers were unsure or saw a slight change.

The subject of numeracy rated highest in the explanations of all answer groups. Of the "yes" answers (51%), the "no" answers (26%), and the remainder, 31% mentioned numeracy, using the terms "numeracy", "number", "numerical", and "numerate". Typical comments from the "yes" group were:

> Yes, I have recognised the importance of number manipulation for all students.
>
> Yes, numerical knowledge underpins all mathematics ...
>
> Yes. More numerate. Students are getting more knowledge of basic mathematical principles.

From the "no" group, the comments included:

> No, but maybe the emphasis has changed – greater time spent on numeracy.
>
> No. I have always believed number skills and processes to be the key to learning in maths but believed, and still do, that the other areas of the curriculum enhance student number skills.

From the remainder, the comments included:

> The basic skills/knowledge of numeracy are fundamental to all other areas of maths. If a class is weak in maths in general, number should be the area of concentration.
>
> Not really – still a need to ensure that basic calculation/number skills underpin everything we teach.
>
> It is important to teach the number components of the programme at the right level.

Several other points were raised in a minor way in teachers' explanations. Taken as a percentage of the total number of respondents, these comments included: the learning of basic skills (10%); students' prior knowledge and understanding (each at 9%); the use of calculators (7%); fractions and decimals (6%); tables and mental arithmetic (5%); and geometry and algebra (7%).

Teaching Year 9 Mathematics

When teachers were asked about how they thought year 9 mathematics should be taught, initially they offered a wide range of opinions, with only three opinions being voiced by more than 10%

of the teachers. These three options were: groups and group work (20%), a variety or mix of teaching methods and learning experiences (18%), and hands-on or practical experiences (11%). Many answers included several points. Comments relating to groups and group work included:

> Groups (ability grouping), so that I can help like-minded students to progress on to the next step.
>
> In small groups, as the Numeracy Project is trying to do.
>
> Ideally, students should be grouped according to the stages they are at. Each group should receive instruction at the appropriate level for their development.

Views on variety or mix of teaching methods and learning experiences included:

> With a variety of activities – practical, written, investigations, textbook, fun activities, plenty of practice at basic skills, using equipment where necessary.
>
> A mix of teacher-directed and student-directed teaching should be employed. Lessons need to be a mix of textbook, investigations, co-operative learning activities, games, and fun.
>
> Using a variety of skills and approaches, from problem solving, investigating, through to formal "up front" teaching.

Typical responses about hands-on or practical experiences were:

> Students should have as much hands-on experience as possible.
>
> In well-equipped classrooms, using as much hands-on material as possible.
>
> I feel that year 9 maths should be hands-on problem solving ...

The large range of other points raised in response to this question included: exploration and experimentation, teaching at students' level, number and basic skills, real contexts, and reinforcement and repetition (7% each); individually based work (6%); calculators, prior knowledge, whole-class teaching, and problem solving (4% each); resources (3%); use of technology, interactive teaching, traditional teaching, structure, the curriculum, class size, time, and the use of trained maths teachers (2% each); teacher and student enjoyment, explanation, assessment/testing, discussion, and concepts (1% each). Year 9 as an introduction to secondary maths, the syllabus, individual needs, questioning, and thinking all fell below the 1% mark.

Only 5% of teachers offered answers of a more general nature, and 10% of teachers did not respond to this question.

At the end of the year, teachers were asked if their views about how to teach mathematics had changed. More teachers gave outright "yes" responses (27%) than "no" (23%) to this question. In 28% of cases, the teachers gave no outright "yes" or "no" responses but offered an explanation of what the teacher was doing or thought needed doing or described the difficulties the teacher was experiencing. For example:

> Greater use of concrete equipment would help some students. I like the idea of allowing students to work out their own ways of getting to the answer and being able to explain it. In the past, it tended to be "do it my way".
>
> We have to find out the numeracy level of our students and work from there. Hopefully, as the programme makes a difference in the primary and intermediate schools, we will change our emphasis to teaching the levels that the students are meant to be at.
>
> It is my belief that primary and intermediate mathematics should follow the Numeracy Project guidelines. I have to admit that I found it difficult to change my teaching practice/methods. I feel that my teaching was more effective last year.
>
> The Numeracy Project has provided material to use more group work, and the students have enjoyed this. (I have too.)

Some 7% of teachers felt that their views had changed to a degree. A smaller number of responses (4%) indicated no change to the teachers' views. Only 2% of respondents were unsure, and 8% did not address the question of change to their views.

The comments from all groups in response to this question were varied. In the "yes" group, numeracy was once again the highest area of change (25%). Comments from this group included:

> Yes. A large emphasis needs to be placed on raising the strategy levels for numeracy for each student, so a greater emphasis on the number topics is needed. I used to assume the students arrived from year 8 knowing more than they do!!!
>
> Yes. The way I now teach algebra has changed. Now I teach it based a lot more on number. Students take to it better this way and have better understanding.
>
> Yes. Across the board I believe that placing greater emphasis on numeracy skills will be beneficial, although in years to come I hope many of these skills will be coming in [with the students].

However, in the "no" group, numeracy didn't feature. In this group, 35% of responses had no explanation at all. Of the remaining responses, 16% talked about numeracy. Comments included:

> Reinforcing number strategies and key knowledge is necessary.
>
> The very significant improvement in their numeracy made me think there was a lot in the curriculum already that reinforced those skills.
>
> Ensure sound number strategies are in place before other topics are introduced, especially algebra.

Other comments, taken as a percentage of the total number of respondents, included these topics: students' prior knowledge and understanding, activities, learning experiences, and resources (9% each); basic knowledge (8%); equipment and materials (both 7%); and use of calculators, teaching strategies, group work, and comments about year 9 (all under 5%).

The Use of Assessment Data in Mathematics Teaching

Teachers were asked to describe how information about individual student achievement (for example, observations, interactions, and test scores) informed their subsequent mathematics teaching. Many responses to this question contained a variety of points about how achievement information had informed teaching. The main focus was the use of achievement data to help in planning subsequent lessons (43%). Typical responses were:

> Feedback is essential to decide at what level to pitch the next lesson. All factors are considered.
>
> It helps me plan lessons to meet the needs of individual students. Without these assessments, my lessons would perhaps have been generic and in fact a total waste of the students' time.
>
> All used to aid the planning of each lesson, to help with the differentiated work of each group.

Some of these responses (6%) indicated that the information was also used to alter the level and pace of teaching according to "the way the students interact and their body language". Although these responses explicitly mentioned changes to planning and/or delivery, the nature of the change was seldom made clear.

Another way in which information was used by teachers (12%) was in giving instructions and setting of tasks at different levels for individuals and groups within the class. For example:

> A guide to how far I need to extend the difficulty level each way, to accommodate students of different abilities.
>
> Helps in deciding group formation.
>
> It helps me to group children according to their needs – and so address them individually.

Another way information was used by 8% of teachers was to help group students for instruction. A few respondents (6%) said that the information enabled them to set their teaching levels to give an appropriate variety of work for students. Re-teaching was the prime use suggested by 5% of teachers, with an additional 1% listing both re-teaching and varying the programme. Under 1% of teachers talked about student misconceptions and feedback. The fact that data on individual achievement was not used in any significant way was indicated by 6% of respondents. Some answers (18%) were ambiguous.

When asked at the end of the year if the project had changed the way in which information about individual student achievement (for example, observation, interactions, and test scores) informed their subsequent mathematics teaching, the number of teachers answering "yes" (29%) was markedly more than the teachers answering "no" (16%). Comments that indicated the teachers had noticed a change in their mathematics teaching, without saying "yes", stood at 16%. Comments included:

> Observations and the interview(s) help with individuals. Test scores themselves are a more coarse tool. Both can be used to form groups as well as select resources.
>
> I am beginning to use individual information as I become confident with teaching the subject.
>
> I found it very interesting in the interviews to find out which strategies students use to solve problems and do numerical calculations. This affected the way I taught as I tried to mention several strategies, rather than just teaching one method.

A smaller group of teachers indicated no change in their teaching (10%) and made comments such as:

> Not greatly. Having ascertained where students are at, one is still required to move through the curriculum, covering certain requirements. I have also worked to take students from where they are at and move them on as far as possible.
>
> Not really. Have still had a real need for traditional testing.
>
> Not really, as I have always used information about students and watched class reactions to monitor my teaching.

A further 15% made no comment about any change but gave general statements. For example:

> Potential for more detail in this method.
>
> We need to teach to where they are at – start from what they know and go from there.
>
> A complete picture is built up using all the different assessments. The numeracy testing demonstrates how students solve problems. However, it is difficult to get the time at the start of the year and at the end of the year because secondary teachers have 3–4 other classes to teach.

There was no response to this question from 9% of the teachers, and 5% saw some change or had their views confirmed.

The explanations given by all the above groups were numerous and varied. In the "yes" group, by far the greatest number of teachers (28%) commented on a change to teaching to students' needs and levels. Typical comments were:

> Yes, to a point at least. It has forced me to teach students in ability groups and aim appropriate work at their level.
>
> Yes – I have seen the need to teach groups of students with same stage level together.
>
> Definitely – as far as grouping according to their level of ability – has definitely been a huge factor and definitely assisted my teaching and the students' learning.
>
> Yes. It was easy to identify the students' strength/weakness and then plan the lessons to address those issues.

An equal percentage of teachers (11% in each case) felt that they had made changes in their understanding of how students process knowledge and approach problems and in their use of testing and of interviews. Comments here included:

> Yes, more information about how students approach a problem from interview.
>
> Yes. Pre-test of sections of the syllabus used to determine subsequent teaching.
>
> Yes. In a way, testing highlights the numeracy issues to a much greater extent.
>
> Yes. The interview process was particularly useful in determining how my students arrive at the answers.

The remainder of the "yes" respondents covered a range of topics, including information acquired, checking/rechecking, and assessment (4% each); and teaching aids (2%).

In the "no" group, the biggest number of comments was once again in the area of teaching to students' needs and levels (12%), although these comments were not always favourable. They included:

> No. The project has given me a better idea about pupils' needs, but practical/logistical issues prevent us from adjusting our teaching methods, for example, large numbers, discipline, resources, planning ahead.
>
> No. The data collected initially in my view gave some indication of students' ability and prior knowledge, but the assessing technique, timing, scoring, etc., I think, introduced in many cases meaningless data.

The remaining comments from this group of respondents (8%) related to gaps, with testing, assessment, and the provision of more information at 4% each. Acquiring more information rated highest in the remaining responses (13%), with comments such as:

> Project gave us much more information about their achievement (the area where they do well and do not do well).
>
> I feel I know much more about what each student can do. It has been a very useful part of the programme, helps me to target the teaching.

Other comments in this remaining group mentioned testing (9%), improvement for/of students, and interviews (6% each); grouping (2%); and planning (1%).

In-school facilitators were asked to comment at the end of the year on whether or not the project had changed the way in which teachers in their school used information gathered about individual student achievement. As with teacher responses, the "yes" answers (23%) were higher than the "no" (14%). Positive comments that included the word "more" sat at 14%. Facilitators who gave no view but offered an explanation numbered 18%. Nearly one-quarter (23%) of facilitators saw some change or were unsure, compared with 5% of teachers.

As with teacher responses, facilitator answers included a range of topics. However, by far the largest group discussed teaching to students' needs – this fell into the group of answers that mentioned knowledge and understanding (14%). Comments included:

> Yes, teachers understand and know about individual student achievement ... modified the schemes and used them to teach to the needs of different classes.
>
> More in-depth information about student knowledge and understanding.
>
> We are more aware of the vast array of ways students misunderstand maths!

The largest group of responses from facilitators was about testing/assessing (27%). Comments included:

> No, but I think it will as they have the knowledge that multi-assessment is a better reflection of a student ...
>
> Yes, in conjunction with asTTle [Assessment Tools for Teaching and Learning] assessments, pre- and post-testing is now dominating.
>
> No. The initial testing was very informative.
>
> Yes, feedback from other teachers on the programme has led us to trial asTTle for diagnostic purposes. Diagnostic strategy testing is used to broadly group year 9.

Two other larger groups of responses (14%) were about school reports and the Numeracy Project Assessment diagnostic tool (NumPA). Comments about school reports included:

> We include the results in the school reports.
>
> We used the numeracy testing (second time) to do our reports this year.
>
> Used results in school reports to inform parents of student progress.

Likewise, comments about NumPA included:

> Most of the teachers have used NumPA data to help with their teaching.
>
> We had begun to use NumPA to assess needs of students ...
>
> Teachers made use of NumPA results.

The remainder of the explanations were spread over the following topics: interviews, class teaching, asTTle, and grouping (each 9%), and time and ways to fix shortcomings (both 4.5%).

Addressing Teachers' Needs

Meeting the needs of participating teachers is an important aspect of any professional development. The shifts in classroom practice being suggested have been of a more fundamental nature the higher up the school the project has moved. The needs that teachers and in-school facilitators expressed must be seen in the context of wider issues in secondary schooling.

Initially, teachers were asked to explain what they saw as their greatest needs as a mathematics teacher. Two major areas were mentioned – time (39%) and resources (35%). Some teachers talked about both. Many of the time responses just had the word "time", but some examples of longer explanations were:

> More contact time with my classes. To see the students on a daily basis.
>
> Time to discover the skills level of individual students and the time to work with small groups or even individuals in the classroom.
>
> Time – for planning, resource preparation, reading/assessing student work – for reading/researching current issues in maths and maths education – for searching for suitable materials for students.

Comments about resources included:

> Interesting and stimulating resources. I simply do not have the time to put together the resources I would like to use.
>
> Resources that target specific areas of a topic besides workbooks and homework books. Too much of these are geared towards "reading" about how to do a problem, rather than doing a multitude of problems ...
>
> Meaningful resources. In order to define your classroom as interactive, you need to have something other than books. Therefore if a student has a weakness in number banding, I require strategies (topic plans) and resources to hit this objective.

One issue discussed by 12% of teachers was class size; they wanted smaller numbers and a smaller range in ability. Streaming was favoured by 1%. Comments included:

> Smaller classes – just can't give them the one to one time they need.
>
> Class of smaller ability range so you could teach to one level.
>
> Smaller classes. With 32–35 students in a class, it's almost impossible to teach properly.

Comments about more personal needs were made by 11% of teachers. Comments included:

> To provide consistency and continuity of learning and not to be giving the students sudden spurts of irrelevant skills and techniques.
>
> Being organised to best target each student and knowing that the students are engaged enough to gain something out of the activity.
>
> As this is my first year teaching maths, I could use help to set quality targets for student achievement. Developing strategies to teach.

Other topics that were mentioned by smaller groups of respondents were professional development and a range of difficulties with students (7%); strategies and ideas for teachers (6%); support from the school (4%); use of computers and calculators (3%); use of technology and variability (both 2%); number, prior knowledge, the curriculum, and understanding of student learning (1% each); and classroom help, attendance, content, audio visual, change, and trained teachers, each 1%. A few (3%) gave no specific answer to the question, and 2% did not answer the question at all.

At the end of the year, teachers were asked to comment on how the project had addressed some/all of their needs. The responses looked at here all refer to the part of the question regarding some needs, as no teacher said that all their needs were met. Of the total number of answers, 13% responded with a definite "yes" and 22% with a definite "no", using statements such as:

> Yes. The Framework gives confidence that when a student presents with a range of abilities you have the necessary information to find the baseline to start progress from.
>
> No, my greatest need is to teach year 9s four days a week instead of three.

Of the responses that did not say "yes", 28% were positive. For example:

> The project has given me the opportunity to refine my delivery more specifically to students' needs and provided resource material that can be trialled and adapted.
>
> Given me different ways of looking at number, and thus it is easier to be on the same wavelength as kids.
>
> It has addressed needs and provided valuable professional development. My greatest need is more time with my classes – 3.5 periods a week at years 9 and 10 is not enough.
>
> It has addressed a need to provide basic numeracy skills and has provided useful resources to do so.

Responses that were more negative were given by 6% of teachers. These responses were often about lack of time and resources.

A small group of teachers (13%) said they saw some of their needs addressed:

> It has addressed some needs – the ability of how, what, where the students do/learn maths – but teaching numeracy is taxing and very difficult, but we will persevere.
>
> Some – I am a better teacher of arithmetic. I had always assumed it was taught at primary school and students would have more feeling for number by secondary level.
>
> Some. Time spent working with other maths teachers is invaluable.

Unanswered questions made up 9% of the total. There were a small number of answers that indicated no change, lack of certainty, or neutrality (3% each).

There were three major ways in which teachers saw the project addressing their needs, these being the provision of resources and activities (14%); a better understanding of students (9%); and the improvement in number strategies (9%). Comments from the resources and activities group included:

> The introduction of equipment to year 9 classes has been a real need. Games to make maths more fun are also a plus. But there's always room for more tips, ideas, etc., and the preparation of resources would be beneficial.
>
> I used some activities presented at workshops, which I couldn't find in a maths textbook. It was helpful.
>
> It has provided me with a range of teaching material that is stimulating and fun for the students.
>
> The resources and suggested activities have been a great asset.

Minor ways in which the project addressed needs were revitalisation and teaching strategies (both 4%); groups, testing, calculators, and professional development (2% each); and resourcing and pedagogy (both 1%).

Other more negative comments remarked on the heavy workload and lack of time (12%). For example:

> No. It has caused me endless hours of planning and preparation and tripled my normal workload at this year level. With other classes and responsibilities this year, the load has been unsustainable.
>
> The need for time to deliver a content-heavy curriculum – even more time with a "numeracy" based approach.
>
> The project created a lot more work for me ...

A very small number of teachers criticised the lack of resources (6%), the project (2%), class size, and lack of support (both 1%).

Both in-school and regional facilitators were asked at the beginning of the year what they anticipated would be the greatest needs of the teachers in the project.

The majority of in-school facilitators mentioned several needs in their response to this question. Time was the need most mentioned (30%), with a range of explanations given. For example:

> Time to think about and complete the changes. Time to reflect, assess, and self-appraise what they are doing.
>
> Time to think about the information the pre-testing shows them and then to plan effective teaching strategies to develop the students' understanding.
>
> Time – will there be enough time to teach the rest of the syllabus?
>
> Time: to understand the procedures/reasons for the work they need to do, to implement, to network.

Three needs that were seen as being an issue by 24% of in-school facilitators were teacher understanding/knowledge, resources, and support for teachers. Comments about teacher understanding/knowledge included:

> To understand how students form their ideas in number and to be able to help them advance their understanding.
>
> The knowledge of what is required in the project ...
>
> Build an understanding of the Number Framework and how to transition students from one stage to the next.

Comments about resources included:

> Resources to present to students.
>
> Materials with which to work and feel confident using.
>
> Resources to use to develop hands-on activities in the classroom.

Comments about teacher support included:

> Guidance with programme planning to help them let go of what is traditionally done.
>
> Encouragement to use new/different strategies.
>
> Support from both regional and in-school facilitators.
>
> Help with evaluation process (support will be needed).

The only other need mentioned in a significant number of responses was the need for confidence (15%). Typical comments were:

> Confidence in looking at the teaching of numeracy in a different manner.
>
> Confidence to use the resources.
>
> That they are confident in what they have to do ...

Other anticipated needs were planning, risk taking, and group work (9% each); programme implementation, strategies, teaching skills, professional development, and the need to focus (6% each); and class control and modelling (both 3%).

All the regional facilitators responded to this question. Likewise, the biggest anticipated need they saw teachers requiring was time (43%). Comments included:

> Time to discuss, make sense of ideas, experiment.
>
> Time – there is a huge amount to absorb ...
>
> Time to reflect, discuss, think, plan ...

Another 29% mentioned the need for resources. A range of other needs was mentioned by 14%, including the need for a paradigm shift, management issues, assessment, focusing on maths education, support/reassurance, group work, use of materials, and mental strategies.

In-school facilitators were asked two questions about needs in the final round of questioning: how the project had addressed teacher needs and what they anticipated would be ongoing needs.

Almost half (45%) of those who responded to the first question on addressing teacher needs used language that indicated there had been some anticipated needs met. Comments included:

> Partly, in terms of resources and workshops ...
>
> Too early to say, but there has been a slight change in the teaching practice of a large number of teachers.
>
> It may have given teachers more options for starters or ways to introduce topics.
>
> To some extent. However, I simply did not have the time to do the project justice.

Only one of the respondents in this group used the word "yes". The remainder of responses (50%) gave general explanations about the project and staff involvement. Typical comments were:

> The project highlighted the fact that everyone works towards problems in different ways.
>
> Running a numeracy classroom requires a major shift for teachers, and the workshops and support we gave each other was invaluable.
>
> Practical help and advice to learn a new style of teaching.

One respondent to the questionnaire did not answer this question.

A large group (55%) commented on how teachers have responded to the project. Comments included:

> The project is not fully accepted by all staff, although one or two would not be prepared to admit that it has [been worthwhile].
>
> However, there has been an overwhelming (at times) amount of information and material for them to process, use, etc., so while they have been positive about the PD sessions, there has always been more to do.
>
> For new teachers, I think it has made them aware of the importance of number in all aspects of maths. For staff who have taught maths for many years, some have taken to it enthusiastically, others have been reluctant.
>
> Some have been prepared to try new activities. Others have reverted to their traditional teaching style. Most have tried something new.

Resources and materials were talked about by 32%. Comments included:

> Yes, good materials.
>
> There was enough material towards the end, although initially we were preparing our own.
>
> There were plenty of paper resources – almost too many to sort through.

An increased awareness of students' thinking/knowledge was commented on by 27%. For example:

> There is an awareness and use of children's thinking at different stages ...
>
> Staff are not only aware of problem areas of pupil knowledge but now have a way of addressing it.
>
> It has given a framework or stages at which children learn numeracy (number/mathematics) and how they can be moved on to the next stage.

Other anticipated needs that were addressed were time, the ongoing nature of the project, and number (9% each); and workshops, starters, support, observation, and pedagogy (each 4.5%).

All of the in-school facilitators answered the question about anticipated ongoing needs. Once again, most answers contained many points, and five of them were mentioned by several facilitators – resources (50%), time and support/guidance (both 23%), observation and the facilitator themselves (both 18%). Comments about resources included:

> Help in resource building.
>
> Resources ... materials, practice worksheets – and a scheme that includes the "numeracy" strands and stages relevant to our clientele.
>
> More resources (that they don't have to create or look for).
>
> Materials support, use of materials, and also material for use in classroom (worksheets, etc.).

Comments about time included:

> Time! To reflect on different activities, etc.
>
> Time for planning, preparation, and review.
>
> Time to do assessing. If this is not provided, it will not be possible to continue strategy assessment.

Comments about support/guidance included:

> Support in introducing new ideas/methods.
>
> Ongoing ... support to develop skills in this area (formerly done by primary teachers).
>
> More in-class support. (My own timetable has been changed next year to be able to provide this.)

Comments about observation included:

> Need exposure to some excellent examples of teaching (for example, video) both whole-class and in groups (some teachers don't have the confidence to try things cold).
>
> More help in observing "good" numeracy classrooms.
>
> In-class support/observations.

Comments about the facilitator's own role included:

> I am trying to organise this for 2006 [observation in classrooms].
>
> More support from the regional facilitator (I'll do what I can).
>
> My own timetable has been changed next year to be able to provide this [in-class support].

Other areas that in-school facilitators saw as being ongoing needs for teachers included pedagogical knowledge, discussion/reflection, and professional development (9% each) and workshops and class size (both 4.5%).

Impact on Facilitators

Introduction

Facilitators' knowledge of mathematics, mathematics teaching and learning, and facilitation underpins any professional development initiative.

Key Elements of a Year 9 Mathematics Programme

Both in-school and regional facilitators were asked what they thought were the key elements of a year 9 mathematics programme. Initially, many responses from in-school facilitators to this question gave several key elements. Answers were varied and covered a range of topics. Number was the most emphasised (15%), then number together with algebra (9%), followed by these two plus measurement and number with statistics (3%). Comments about number included:

> Students without basic number understanding don't like maths, so do not progress. Before abstract concepts are covered in later years, it is essential that they have a good grasp of what a number is and how numbers interrelate.
>
> Students should be able to be confident about understanding if any answer is feasible or not, for example, simple adding, subtracting, multiplying things together.
>
> Number and algebra are the most important elements as these two are very important for all the other maths strands.

Mathematics processes/concepts and curriculum strands were mentioned by 9% of in-school facilitators:

> The focus should be on students understanding concepts and learning through solving problems that are meaningful to them. Other curriculum strands should incorporate understanding and efficient use of number/algebraic thought.

Some other elements mentioned by one or two respondents included NCEA, group work, a two-year programme, transition from primary to secondary school, building on previous knowledge, and maintaining enthusiasm.

All regional facilitators answered this question, offering a variety of key elements and all listing several in the one answer. Number was most commonly mentioned (71%), with algebra next (57%), and comments about the strands, problem solving, and developing/extending students each rating 43%. Comments covering these elements included:

> Experimenting and learning about effective teaching of algebra driven by the number strand.

> Problem-solving experience.
>
> Developmental – move them from where they are at.
>
> Consolidate students' number understanding – extend their familiarity with fractions, decimals, percentages, integers. Extend their facility with symbol representation.
>
> Problem solving – across strands. Coverage of other strands.

Meeting/understanding students' needs and diagnosing student ability were seen as a key element by 29% of regional facilitators. A further 14% saw the following elements as important: enjoyment, materials, geometry, stage 7, mental strategies, and effective teaching.

When asked at the end of the year if they had changed their views on the key elements, in-school facilitators' answers ranged from one word to large paragraphs containing many ideas. The majority (68%) responded with "yes" answers (included in this group were answers that used language such as "more", "now", and "I did not realise", indicating an affirmative response); 14% responded with "no" answers or an explanation implying "no"; and 4.5% or were unsure.

Explanations for in-school facilitators' viewpoints were wide ranging and varied. As with the teachers, numeracy was the subject mentioned most (27%). Comments included:

> One thing we do know is that the pupils should be coming in with a better numeracy base.
>
> Yes, I see the need for number in all strands.
>
> Yes. The emphasis needs to be on number knowledge and strategies with a view to moving on to algebra. The other strands can be taught as usual, but the number concepts need to be emphasised all the time.
>
> I'm more focused on incorporating number skills and strategies into every strand. Kids can't do the strands if they can't do the number work.

The other main explanation concerned the teaching of strategies (23%). Comments included:

> Strategies to equip them to tackle problems with a clear understanding of what they are doing rather than just memorising an algorithm.
>
> Yes, strategy techniques especially mental should be part of all year 9 programmes.
>
> Much of what we previously assumed that students knew, we are now aware that they do not know always. Therefore we have gone back a step to teaching strategies for dealing with number that they need for further progress.

Other explanations discussed algebra, fractions, decimals, percentages, basic knowledge, and the way in which students construct knowledge/learn (9% each) and equations, geometry, measurement, statistics, prior knowledge, use of materials, group work, diagnostic information, and worksheets (4.5% each).

Teaching Year 9 Mathematics

When asked initially to explain how they thought year 9 mathematics should be taught, in-school facilitators mentioned a range of ways. Just over half (52%) gave variety (choice, mixture) as a way to teach mathematics. Comments on this included:

> Variety of approaches – activities to promote appreciation of relationships, etc.
>
> Using a variety of techniques that will enthuse all types of students, for example, co-operative learning strategies.
>
> In a variety of ways, depending somewhat on the personality of the teacher. A variety in styles of delivery is important to meet different learning styles and keep interest.
>
> I think there needs to be a mix of teaching strategies.

Group work, including groups based on ability, was mentioned by 39% of in-school facilitators. Typical comments included:

> Group work where students get time to discuss problems.
>
> Should be taught in groups of similar ability with classes that are not too large.
>
> Grouped with similar levels of ability.
>
> Group work – opportunity for discussion/co-operative learning.

Students' needs, involving/understanding students, numeracy, and a hands-on approach were mentioned by 18%. Comments about student needs included:

> Will vary, I think, to suit the students' needs – academically and socially.
>
> Appropriate to the needs of the pupils.
>
> By measuring current stage and appropriate teaching to needs.

Comments about involving/understanding students included:

> Delivery of the knowledge is probably best achieved by involving the pupils in the lessons as much as possible – get them thinking.
>
> To encourage understanding and find relationships with what students presently understand and then build on this.
>
> Asking good questions of students, challenging their thinking and giving them a chance to explain their thinking.

Comments about numeracy included:

> By topic, but with numeracy skills incorporated into each lesson by way of (1) starter questions, (2) games, (3) competitions.
>
> Many students who have gaps with numeracy need to have this addressed, incorporated into the current programme or focus on numeracy.
>
> I like the numeracy approach of materials – image – properties.

Comments about a hands-on approach included:

> Main focus on number/algebra units with hands-on manipulatives.
>
> Hands on – greater use of equipment.
>
> Hands-on activities. Some teaching of basic skills leading to problem-solving structures to develop these.

A further 15% of in-school facilitators talked about activities/materials and assessment. Comments included:

> Lots of discovery – activities to promote appreciation of relationships, etc.
>
> So we should spend more time playing in the materials (for example, physical manipulations, diagrams).
>
> Assess where individual students are in a group.
>
> Formative assessment (for example, asTTle and NumPA) and each objective within the strands checked for student understanding of concept.

Some of the in-school facilitators also mentioned a range of other ways that year 9 should be taught mathematics. These alternatives included textbooks, problem solving, and enjoyment (9% each); topic based teaching, context, and teachers (6% each); and mental skills, NCEA, individual teaching, whole-class teaching, curriculum strands, rote learning, practice, and class size (3% each).

The majority of regional facilitators (57%) also discussed multiple ideas, while 29% stated "in a variety of ways" and 14% did not respond. Of those who listed several points, the most mentioned idea was teaching to students' levels and needs (57%). Some comments were:

> Start where children are at.
>
> Appropriate teaching to needs.
>
> To encourage understanding and find relationships with what students presently understand and then build on this.

Three other points were raised by 29% of regional facilitators – diagnostic testing, use of materials, and practice. A further 14% saw group work, assessment strategies, morning classes, and daily classes as ways year 9 mathematics should be taught.

At the end of the year, all in-school facilitators responded to the question asking if they had changed the way in which they thought year 9 mathematics should be taught. The number of "yes" responses was the highest at 46%. "No" responses (which included "not really") were given by 18%, and 36% gave only an explanation.

Once again, explanations were varied, but all the in-school facilitators supplied them, often covering several points. At 27%, number, lessons/teaching, and materials were the most mentioned topics. Comments on number included:

> There is a real potential for parts of number to be integrated into other strands or in a thematic approach.
>
> No, it has, however, validated the need to explore number, develop number sense, etc.
>
> Yes, after this year – rearrange school scheme to push "numeracy" to the front of year 9 and 10 – for slower kids, it is dominating the year.
>
> Yes, for about 40% of students who are not multiplicative, numeracy is the priority and needs to be emphasised when covering the other strands.

Comments on lessons/teaching included:

> No, a mixture of traditional lessons, textbook lessons, activity lessons, and numeracy/discussion type lessons.
>
> The structure of a lesson needs to be planned more carefully to include knowledge, strategy, and practice.
>
> Teach more to the ability of the class, rather than teaching something because "it is in the scheme".
>
> A bit more creativity in the classroom, less "drill and kill".

Comments on materials included:

> Yes, I place much more emphasis on mental strategies and connections to concrete materials.
>
> Need for more "building work" done through pictures, graphs, diagrams, materials, etc.
>
> Yes, year 9 programme should be taught by using appropriate materials to introduce the concept, imaging, and bridging until abstraction has been achieved.

Group work and grouping was mentioned by 14%. For example:

> I think there is some room for group work.
>
> Where possible, different levels of understanding should be targeted, using group work of various types.
>
> The programme can be taught in a traditional way, although the use of groupings (not necessarily groups) assists with management.

Other comments included discussion about: basics and knowledge/understanding (9% each) and mental strategies, levels, processes, calculators, streaming, and discussion (5% each).

Key Qualities and Skills of Facilitation

At the start of the year, regional facilitators were asked to rate their own knowledge of mentoring other professionals. Figure 7 shows that most rated their knowledge of mentoring others as strong to very strong.

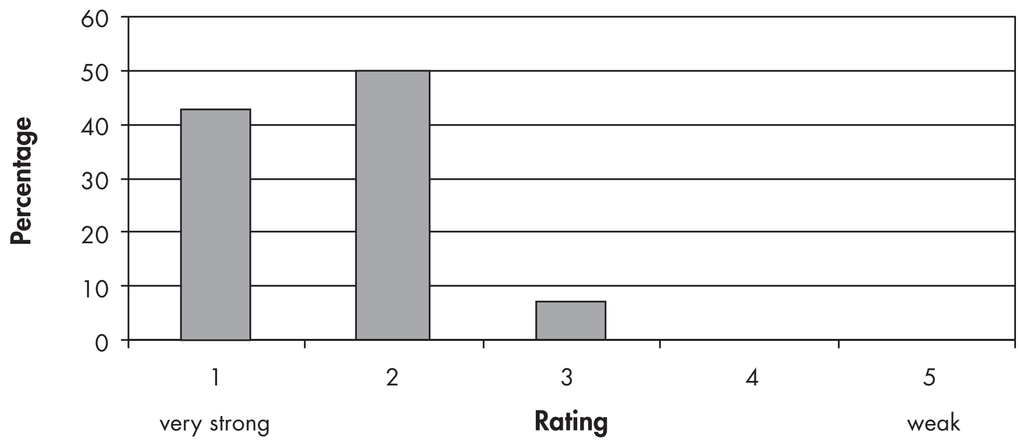

Figure 7. Regional facilitators' knowledge of mentoring other professionals

In-school facilitators were asked at the beginning of the year how they would rate their own knowledge of facilitation. The results can be seen in Figure 8, in which fewer in-school facilitators rated their knowledge as being very strong.

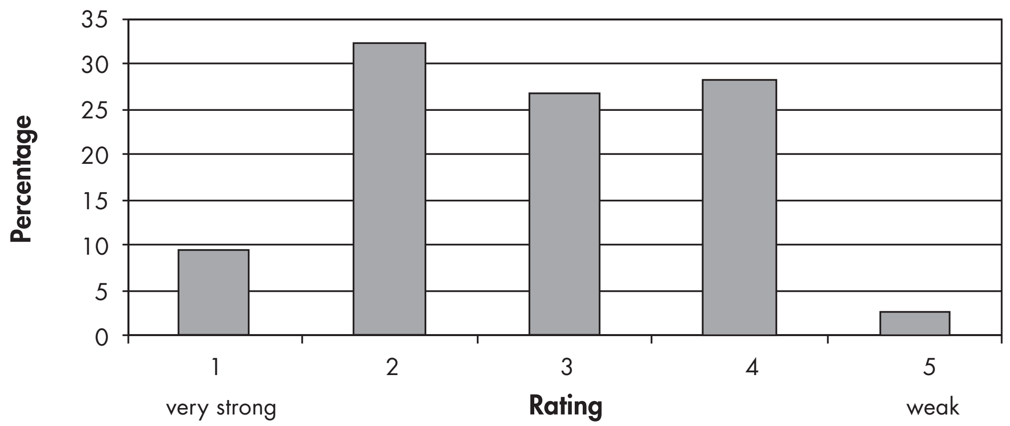

Figure 8. In-school facilitators' rating of their own knowledge at the beginning of the year

When asked what skills they already possessed, in-school facilitators gave a range of skills that they thought would be useful in their SNP role, many giving more than one. People skills were mentioned the most (27%). Typical comments were:

> Ability to develop good rapport with colleagues.
>
> Good relationships with the members of my present maths department.
>
> I have good communication skills and interpersonal skills.
>
> Ability to get ideas across to others.

Previous facilitation work was mentioned by 21%, some from NCEA experience (9%) and some from other areas (12%). Comments from this group included:

> NCEA facilitator for 101 [students in levels] 1–3.
>
> Facilitation of groups on NCEA training day.
>
> In-school facilitation of Maths/ICT [Information and Communications Technology] of staff in an area school and a large high school.

A further 21% cited skills in leadership and management as useful experience, some as HODs (15%) and the remainder in other roles. Their comments included:

> As an HOD, I am able to run effective meetings and encourage others to contribute and participate.
>
> In my work as HOD, I have been involved in working with teachers to develop their classroom teaching skills.
>
> Leading large faculty. Chairing meetings.
>
> Co-running a large department.

Knowledge and understanding of numeracy and number were cited by 18%. Comments included:

> Detailed understanding of number sense ... Some understanding of the Number Framework. Sound subject knowledge – passionate about the subject and strong interest.
>
> An insight into the project. I have helped to implement numeracy skills to year 9 for two terms.

Previous work with less academic learners and pupils who lacked confidence was seen as a useful skill by 12%. Their comments included:

> Spent many years working and developing programmes within our school for "slow learners" ... Have an affinity with students who struggle with maths and their own confidence.
>
> The needs of students who often struggle.

A range of other skills was mentioned by smaller numbers of in-school facilitators. These skills included mentoring and maths projects/programmes (9% each); group work, presentation, professional development, and the willingness to try new things (6% each); and problem solving, teaching adults, employee relations, and timetabling (3% each).

In-school facilitators were also asked to state what skills they thought they needed to develop. The range of skills given was fewer than those the facilitators felt they already had. Some respondents were unsure (6%) or made comments that did not relate directly to the question (6%). The skill that they felt most needed to be developed was confidence (18%). Comments included:

> I need to feel confident about the Numeracy Project so I can facilitate.
>
> Confidence to talk to larger groups.
>
> Sometimes can be not quite so confident – when unfamiliar with material.
>
> Confidence with my own ability to convert others to the value of the Numeracy Project.

Three skills were mentioned by 15% – knowledge of the project, testing/assessing, and motivating teachers. Typical comments on knowledge of the project included:

> In-depth knowledge of this project.
>
> Knowledge of Numeracy Project and how students learn.
>
> To improve my own picture of the aims and objectives of a particular project, that is, become more of an expert before embarking.

Comments on testing/assessing included:

> Practice at diagnostic testing to evaluate/recognise strategy and knowledge levels.
>
> Using the diagnostic test to group students.
>
> Finding ways of assessing the progression the students make.

Comments on motivating teachers included:

> The salesman side of facilitation – the need to sell this project to a large department that will contain several new teachers next year, especially in the face of all other changes taking place at school.
>
> Putting across what must be done in a way that will enthuse and educate.
>
> Ways to transfer own knowledge and teaching strategies to other teachers who are not comfortable with group work or with very limited students.

A further 12% felt that it was important to develop communication/presentation skills. Comments included:

> Communicate clearly.
>
> Presentation – use of tools available, sequential delivery.
>
> Presentation skills – selecting and delivering appropriate strategies to deliver numeracy strategies to team members.

Other skills named by 3% respectively were computer, facilitation, observation, modelling, concept maps, organisation, examples, and programmes.

Regional facilitators were also asked at the beginning of the year how they rated their own knowledge of facilitation, and they all answered this question. Figure 9 demonstrates on a scale of 1–5 (with 1 being the strongest and 5 the weakest) regional facilitators' rating of their own knowledge at the beginning of the year.

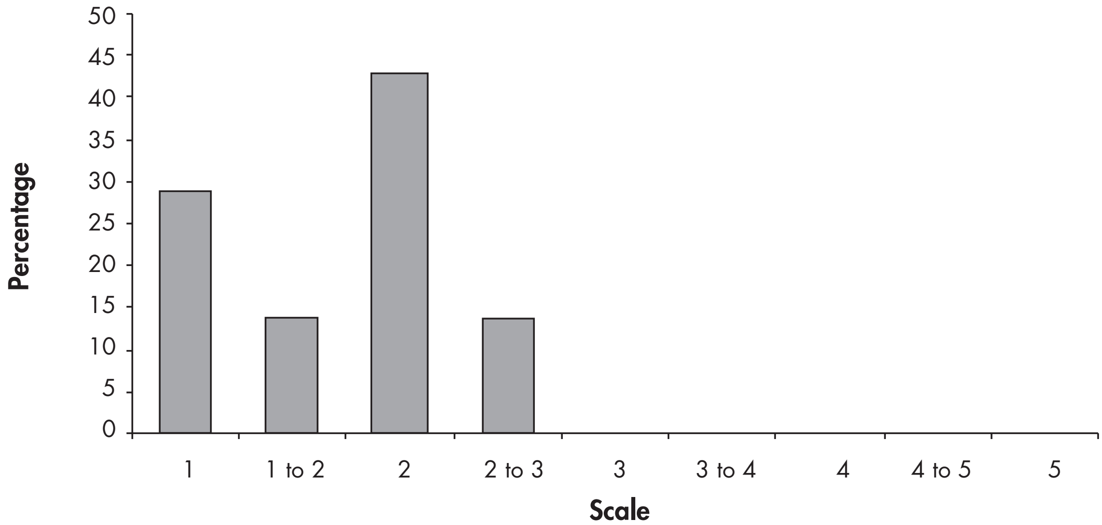

Figure 9. Regional facilitators' rating of their own knowledge at the beginning of the year

When asked what skill they thought they already had, 86% gave a range of skills, with the remaining 14% giving no answer. NCEA was mentioned by 43%, with experience as a mathematics adviser being given by 29%. Some comments about these skills included:

> Several years of NCEA training.
>
> NCEA facilitator.

A year as maths adviser.

Maths adviser for two years.

A wide-ranging number of skills were given by 14% of respondents. These skills included professional development co-ordinator, INP (Intermediate Numeracy Project)/SNP experience, teaching/communicating, research, NumPA, ability to listen, understand, and encourage, and knowledge of materials. There was no response from 14%.

A range of skills were seen as needing to be developed by 43%. For example:

> Always can develop improved ways to encourage teachers' engagement, develop independent-based strategies for sustainability, feedback.
>
> More of everything. In-class modelling, better organisation, presentation skills. More reading of research papers.
>
> All of the same continue to need development [talking about skills already listed].

A further 14% saw ICT skills and conflict handling as areas needing improvement. There was no response to this question from 29%.

At the end of the year, in-school facilitators were once again asked to rate their own knowledge of facilitation. All the facilitators answered this question. Figure 10 demonstrates on a rating of 1–5 (with 1 being the strongest and 5 the weakest) these results.

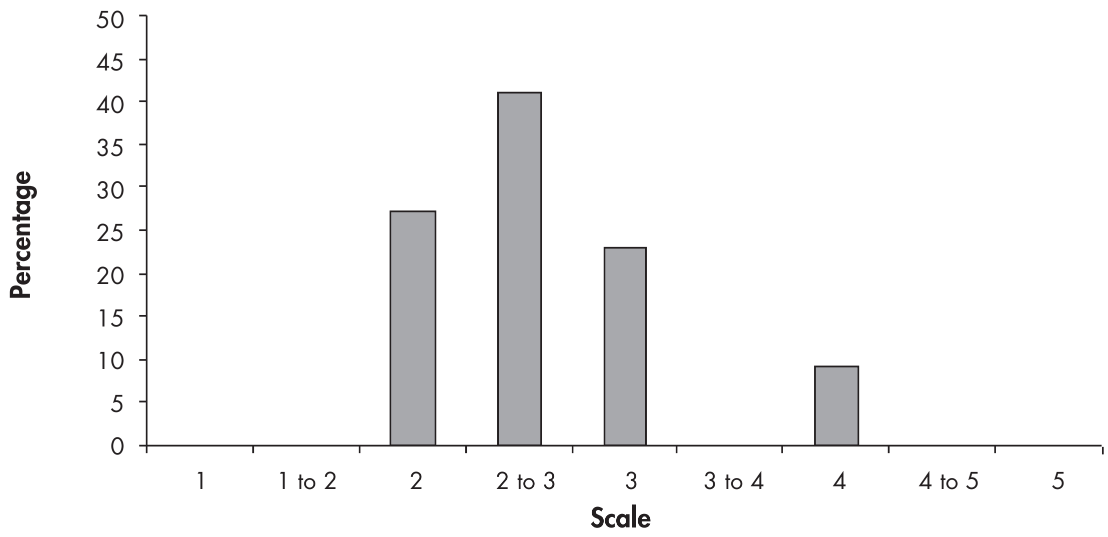

Figure 10. In-school facilitators' rating of their own knowledge – end of the year

Once again, in-school facilitators were asked to describe the skills they felt they still needed to develop. One facilitator did not respond to this question. The remaining facilitators offered a wide range of varied comments, with almost all facilitators talking about more than one issue. Things that affected facilitators on a more personal level featured in a large number of answers (37%). Typical comments from this area were:

> Organising and putting into practice what I have learnt this year. Not confident enough in first year to teach/facilitate effectively.
>
> Time management, setting realistic goals for change.
>
> Dealing with doubters. ... Confidence that I am doing the right thing. I am being harsh on myself.

The next most mentioned skill was the need to give good feedback (18%). Comments included:

> Constructive feedback.
>
> I need to develop my reporting back to teachers after observations.
>
> Effective observation/feedback.

Addressing issues with staff, involving and enthusing others, and modelling were represented equally in the range of skills listed as needing developing (14%). Typical comments about addressing issues with staff were:

> More help with techniques to encourage reluctant "older" staff to try new strategies.
>
> Dealing with those who want to be given the resources and not create their own, or to make their own choices.
>
> Addressing issues with teachers.

Comments about involving and enthusing others included:

> Probably would like to involve others more in the running of in-service courses.
>
> Better at delegating our tasks.
>
> To try and enthuse the rest of the Y9 group.

Comments about modelling included:

> I will need to do more work on ... modelling lessons.
>
> Need to develop more confidence at "modelling" the teaching.
>
> In-class modelling.

Other skills that 9% of in-school facilitators mentioned included leadership, in-service, pedagogy, and in-class support, while 4.5% commented on coaching, reflective process, levels, group work, and resources.

In-school Facilitation Model

On the whole, both teachers' and in-school facilitators' comments about the in-school facilitation model were favourable. Initially, many in-school facilitators (43%) were excited about and looking forward to being involved in the project. They saw it as an opportunity to provide better professional development and student outcomes in mathematics. Comments included:

> It is an exciting project to be involved in and will open doors for us and gives us the chance to provide our students with a more valuable learning experience.
>
> Looking forward to it. My own personal development as well as better student outcomes in mathematics.

Quite a large group of in-school facilitators (36%) had concerns about some aspects of facilitation. For example:

> I would like there to be provision for me to visit primary/intermediate schools where the project is running successfully.
>
> I am also aware of the stress being faced by my colleagues at present and hope we introduce the project slowly and with time to allow them to investigate how the project will work best for them.

One regional facilitator commented:

> Facilitators coming in fresh from a busy secondary school need TIME to do it properly – DO LESS BETTER.

By the end of the year, most of the in-school facilitators were still enthusiastic about the project, saying such things as:

> I feel it has been invaluable ... Very worthwhile!!!
>
> Eureka moments.
>
> Overall, the Numeracy Project is excellent.

Some in-school facilitators (36%) felt that, for one reason or another, they could have done a better job. For example:

> Circumstances make me feel that I could have done a much better job of facilitation and will do in 2006. I will be much more proactive.
>
> On the whole, the teachers have been positive but needed more leadership from me. The expectation of weekly meetings is unrealistic, and the team meetings were too far apart. A balance needs to be found there.
>
> Attempted too much this year. Trying to learn new information and have confidence to impart it to others.

The Professional Community

An important aspect of sustaining change is building a professional community. One indicator of whether this has been successful is the degree to which departmental practice includes and supports professional dialogue and student learning.

The graph below shows how teachers estimated the amount of time spent on administration, assessment/reporting, professional and curriculum support, and other topics in department meetings. They estimated that more time was spent on administration than on other topics. However, the estimated amount of time on professional and curriculum support increased by the end of the professional development programme.

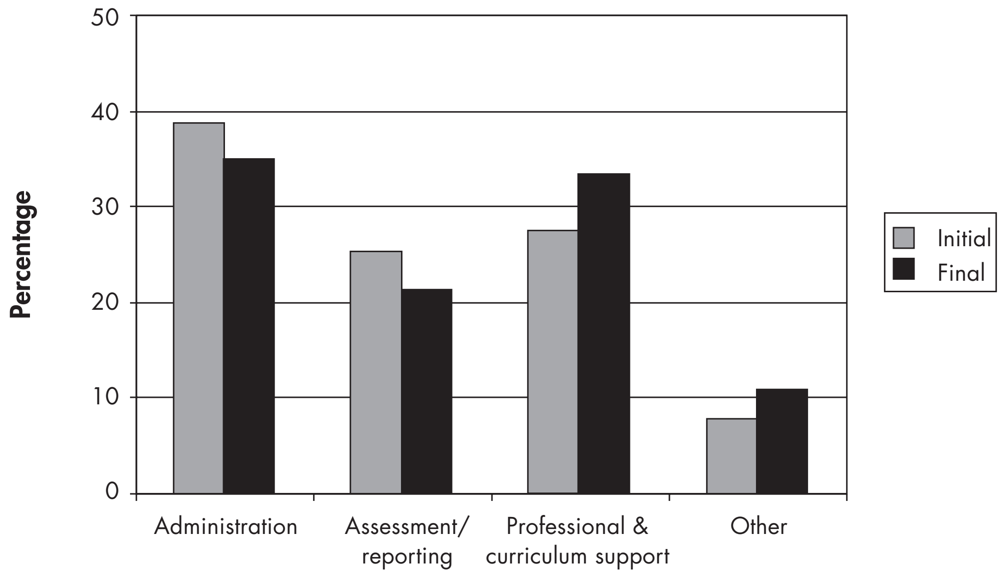

Figure 11. Teacher estimates of time spent at departmental meetings

In-school facilitators were also asked to estimate the amount of time spent on different activities at department meetings. Their estimates were similar to those of teachers. However, in contrast to teachers, they thought that there was an increase rather than a reduction of time spent on administration-type activities, and they did not estimate that the professional and curriculum support increased over the year.

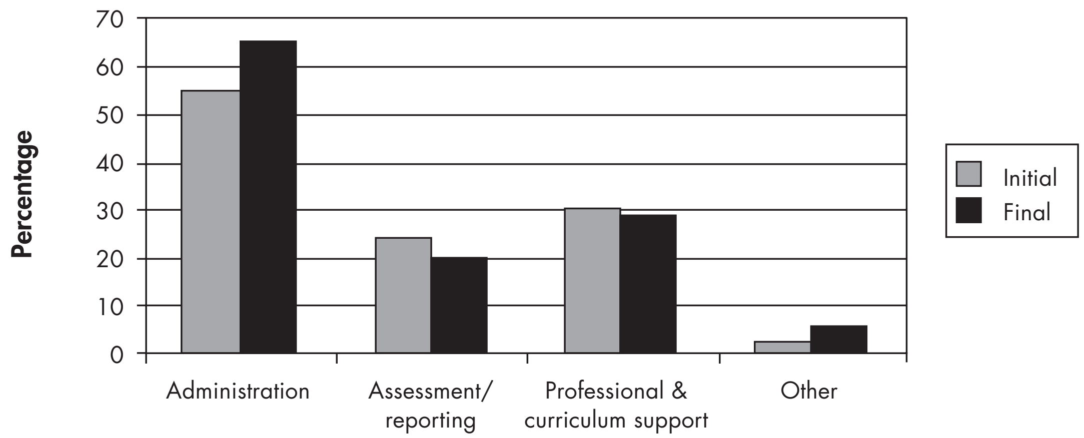

Figure 12. In-school facilitators' estimates of time spent at departmental meetings

Additional evidence that was gathered suggested that dialogue about the way that mathematics is taught and learned is taking place in schools. The following comments were made by regional facilitators:

> I think the biggest success that I see is that the departments are talking about teaching and learning ...
>
> The success ... the atmosphere in the department work area, the fact that they're talking together.
>
> It's the engagement and professional discussions about teaching mathematics that they all comment on.
>
> I think the successes have been the talk.

Comments from in-school facilitators and teachers were less direct but indicated that professional dialogue was taking place. In-school facilitators talked about using results from testing in school reports:

> We used the numeracy testing (second time) to do our reports this year. Very effective too!
>
> Used results in school reports to inform parents of student progress.

Teachers used the words "us" and "our", which indicate that dialogue had taken place:

> It has broadened our knowledge – this is stored on our database for future use.
>
> Numeracy testing during the project helped us monitor progress and refocus at times.

It may be that teachers were making a distinction between department meetings at which organisational matters were sorted out and separate numeracy meetings at which professional matters were discussed.

Concluding Comments

Most teachers taking part in the SNP saw number/numeracy as a critical component of the year 9 programme. There was a wide range of ways in which teachers felt that year 9 mathematics should be taught. Many teachers saw a need for an increased emphasis on number/numeracy as a result of the project. Many teachers used achievement data to support planning of subsequent lessons. Nearly one-third of teachers reported making greater use of assessment data as a result of the project. The NumPA interview gave many participating teachers greater knowledge of their students. However, pressures of time/work limited some teachers' use of achievement data. Time generally was identified as a pressure on teachers both at the start and at the end of the project.

The SNP gave some teachers ways to address the numeracy skills for individual students. Although the resources supplied were found to be useful, there is an ongoing demand for continued development of resources specifically for secondary school students.

Regional facilitators reported an increase of professional dialogue within mathematics departments. Qualities regarded as important for in-school facilitators included being enthusiastic, supportive, energetic, organised, realistic, and willing to take risks. To be successful, in-school facilitators felt that they needed to have good knowledge of mathematics and the project, be prepared to lead staff, and able to give effective feedback.

In general, the model of in-school facilitation was seen as successful. In part, this was due to the fact that the in-school facilitators were intimately aware of the context of the particular schools and were able to modify the programme to meet local circumstances. Their availability for ongoing dialogue assisted the project's implementation.

Overall, many participants in the SNP saw the first year of the project as successful in the way that it impacted on teachers' and in-school facilitators' knowledge, skills, and practice.

Acknowledgments

Grateful thanks to the teachers, in-school facilitators, and regional facilitators for participating in this evaluation.

Special thanks to Susan Kaiser and Julie White for their help with the project.

References

Bobis, J. (1999). *The impact of Count Me In Too on the professional knowledge of teachers.* A report prepared on behalf of the NSW Department of Education & Training, December 1999.

Higgins, J. (2001). *An evaluation of the year 4–6 Numeracy Exploratory Study:* Exploring issues in mathematics education. Wellington: Ministry of Education.

Higgins, J. (2002). *An evaluation of the Advanced Numeracy Project:* Exploring issues in mathematics education. Wellington: Ministry of Education.

Irwin, K. C, & Niederer, K. K. (2002). *An evaluation of the Numeracy Exploratory Study, years 7–10:* Exploring issues in mathematics education. Wellington: Ministry of Education.

Stake, R. (1998). Case studies. In N. Denzin & Y. Lincoln (Eds), *Strategies of qualitative inquiry* (pp. 86–109). Thousand Oaks, CA: Sage.

Thomas, G., & Ward, J. (2001). *An evaluation of the Count Me In Too Pilot Project:* Exploring issues in mathematics education. Wellington: Ministry of Education.

An Evaluation of the CAS Pilot Project

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Six schools were involved in a pilot scheme to evaluate the use of CAS calculators (Computer Algebraic Systems). Twelve teachers underwent professional development on effective ways to use CAS calculators in algebra and geometry. They then integrated the CAS hand-held calculators into these strands in their mathematics teaching. The project aimed to improve the teaching and learning of mathematics through the use of new generation graphics calculators that include CAS technology. Indicators of effectiveness that were monitored included changes in teacher practice, changes in student attitudes towards mathematics, and changes in student learning. Other areas that were evaluated included the professional development provided, the impact of CAS technology on assessment, attitudes towards technology, and issues of sustainability for the CAS initiative. The project shares many common values with the Numeracy Development Projects, and synergies between the two should be explored.

Background

During the first three years of the implementation of the National Certificate of Educational Achievement (NCEA) in mathematics, it became apparent that there was a need for support for teachers in the use of technology to improve teaching and learning in the classroom. Although a considerable number of teachers were using graphics calculators, there was little evidence that they had made changes in their teaching practice. *Mathematics in the New Zealand Curriculum (MiNZC)* (Ministry of Education, 1992) endorses the use of technology in mathematics and "assumes that both calculators and computers will be available and used in the teaching and learning of mathematics at all levels" (p. 14).

Reports from teachers and from research suggest that the use of this technology can improve students' conceptual understanding of mathematics. It was decided to initiate a pilot project in 2005 that would look at ways to improve the teaching and learning of mathematics in New Zealand schools using calculators that included CAS technology. CAS (Computer Algebraic Systems) refers to mathematical software that has the capability of manipulating algebraic expressions, solving equations, and differentiating and integrating complex functions.

This pilot project focused on the junior secondary school level (two year 9 classes in six schools around New Zealand were involved in the 2005 pilot) so that any effects of NCEA examinations in New Zealand schools on classroom practice would be kept to a minimum. Other countries are using CAS technology in the teaching of mathematics, but that teaching is at senior secondary level.

Methodology

Six schools took part in the pilot project, four in the North Island and two in the South Island. While there was a range of decile ratings among the schools selected, they were predominantly in the high range. The schools were all state schools located in urban areas, with four being in major population centres and two in smaller provincial centres. All but one were co-educational. The schools ranged in size from 770 to 2250 students.

An important aim of this research was to capture the stories of students, teachers, and professional development (PD) providers in the CAS Pilot Project, so that elements of effective practice could be identified and replicated. The information is based upon case studies in each of the pilot schools and includes some quantitative data. Asking common questions has allowed triangulation between the teachers, students, and PD providers.

Two mathematics teachers and two classes from each school took part in the study. In each school, we undertook:

- a face-to-face interview with each teacher involved in the project;
- a self-reflective questionnaire to each pilot teacher, asking them to contrast their teaching practices prior to and near the end of the 2005 CAS Pilot;
- lesson observations in each of the classrooms;
- a focus group of students who reflected on their experience of the project.

As well as this, both of the PD providers and the PD co-ordinator were interviewed. While most of this data was of a qualitative nature, quantitative data was also available, particularly from the teacher questionnaire. Quantitative data of students' attitudes and experiences of the project was also gathered in the student focus group. The findings are a meld between these qualitative and quantitative measures.

Results

Pedagogy: A New Paradigm For Teaching

Teachers, PD providers, and the literature are all in agreement; it is how the CAS is used in teaching that makes the difference. The PD co-ordinator went so far as to say "The technology [CAS] is an excuse to impact pedagogy where teachers have not taken on the philosophy of the curriculum."

MiNZC (1992) gives approaches to teaching and learning that emphasise good problem-solving skills in real-life situations, often on open-ended questions that are amenable to a range of different mathematical techniques. Students are encouraged to:

> search the information for clues, and to make connections to the various pieces of mathematical and other knowledge and skills which they have learned (p. 11).

This more exploratory approach is an essential ingredient of the teaching styles encouraged by the CAS Pilot Project. *MiNZC* also endorses the use of technology in mathematics. The technology, however, is intended to be the servant of the pedagogy. The two providers who facilitated the PD training days were promoting a new way of teaching and learning mathematics based on these principles. One of them stated:

> It's not to do with CAS at all. The stuff I've prepared is only 5% CAS. If you think [about it] there is a different pedagogy. I've used all these techniques for years without technology; then I used spreadsheets and graphics calculators, and now CAS. It is easier with CAS to move between the different representations.

Several of the teachers in the pilot made similar reflections. One said, "The pilot is all about pedagogy, and what learning we want to occur." Another attributed the changes to the pedagogy more than to the CAS. A third teacher stated, "The whole project is redesigning the way we do mathematics." Further evidence of the recognition of the importance of the new style of teaching was that about half of the teachers commented that they were already using some of the pedagogical approaches in their other classes. In addition to this, a number said that other teachers at their school were being influenced by the teaching ideas in the pilot project.

The literature also emphasises that it is not the technology that will bring about change, but the teaching and learning methodology in which it is embedded. Many authors discuss appropriate pedagogies for CAS use in the classroom. Often these papers referred to the "white box–black box" metaphor introduced by Buchberger (1990, cited in Cedillo & Kieran, 2003, p. 221). This distinguishes between using the technology blindly to perform routine mathematical tasks (black box), and using it to help students construct meaning for mathematical concepts and procedures (white box). Kutzler (2003) explores pedagogical approaches to CAS usage. He postulates, "The reason that so many students are at odds with mathematics may be related to their lack of experimentation" (p. 61). Kutzler cites Freudenthal (1979), who said, "We should not teach students something they cannot discover for themselves" (p. 61). Kutzler also recounts a telling personal communication from Heugl (the director of Austrian projects on CAS usage), who said, "If it is not pedagogically justified to use CAS, it is pedagogically justified not to use CAS" (p. 54). Hence, all indications are that CAS should only be used within a pedagogically sound framework.

So Has Teaching Changed?

Nine of the 12 teachers reported making changes to their teaching with their CAS pilot classes. The remaining three reported that their teaching remained largely the same. This is consistent with the self-reflective questionnaire results, which showed there had been a shift in teacher practices. Students had also noticed changes in many of the same areas that teachers had identified. Teaching had become:

- *more student led.* Around half the teachers commented on a shift away from teacher-led classrooms ("talk and chalk" as one described it) towards a far more student-focused approach. One said, "I let the students play with ideas and come up with their own ideas." Some students had also picked up on this. Students in the focus groups made remarks such as:

  I have time to work it out for myself.

  We work out how to get the answer.

- *more interactive.* A teacher from one school changed his teaching to "let the students see me solving the problem 'live' without [me doing] any preparation".

  This allowed the students to observe and respond to the teacher's problem-solving strategies. One student shared that "The teacher does it with us now. With the book, he acted like he's right, [but] he learns CAS like us." High levels of teacher–student interaction were seen in all of the lessons, with these being initiated by both teachers and students. These episodes addressed mathematical issues as well as details of how to use the CAS technology. All the teachers were roving the class for substantial parts of the lesson. A student noted, "There is more walking around [by the teacher]."

- *more explorative.* Both teachers and students reflected that the approach involved far more activity, exploration, and self-discovery, with far less emphasis on a rules-based approach to algebra or geometry. A teacher described letting the students tussle with a problem by themselves for longer before offering help. A student in a focus group remarked that they had to find the mathematical rules for themselves rather than being shown or told them. Another student said, "[We] explore to find out the formula." All the observed classes had a strong activity-based approach, with a focus on the explorations that were provided in the PD. These employed multiple methods of problem solving and multiple representations of mathematical concepts.

- *more discussion and questioning.* Several teachers commented on this:

  We are getting more mathematical discussions now.

  We are asking more open-ended questions and giving students more time to contemplate and debate issues.

Students observed that they had to do more explaining of ideas or answer questions such as "How does it work?" Teachers commented:

> Discussion on all units has been great. [It has] helped students generate ideas for themselves.
>
> Students were taking greater responsibility for learning.
>
> My class got into discussions of metacognition, discussions of how we learn.

The implication is that in a more student-led, discussion-based approach, students become more aware of their learning.

- *more collaboration.* A strong focus on either group work or working together in pairs aided this exploratory style. Most teachers reported that they were already using group work, but some reported an increase in this or in the frequency of peers helping each other to learn. One teacher stated, "There is a completely different feeling in the class. There is much more peer learning and helping each other." A student said, "When you teach others, you learn it more." Several students also commented that they shared ideas more now. One said that there was more sharing of answers as well. In all the observed lessons, many students were helping and explaining things to each other.

  Students were asked for their responses to the statement "Lessons using the CAS calculator are just like any other mathematics lesson." They were asked to place their response on a continuum from strongly disagree to strongly agree. These are shown in Figure 1. Students saw clear differences between the lessons in the pilot and other mathematics lessons.

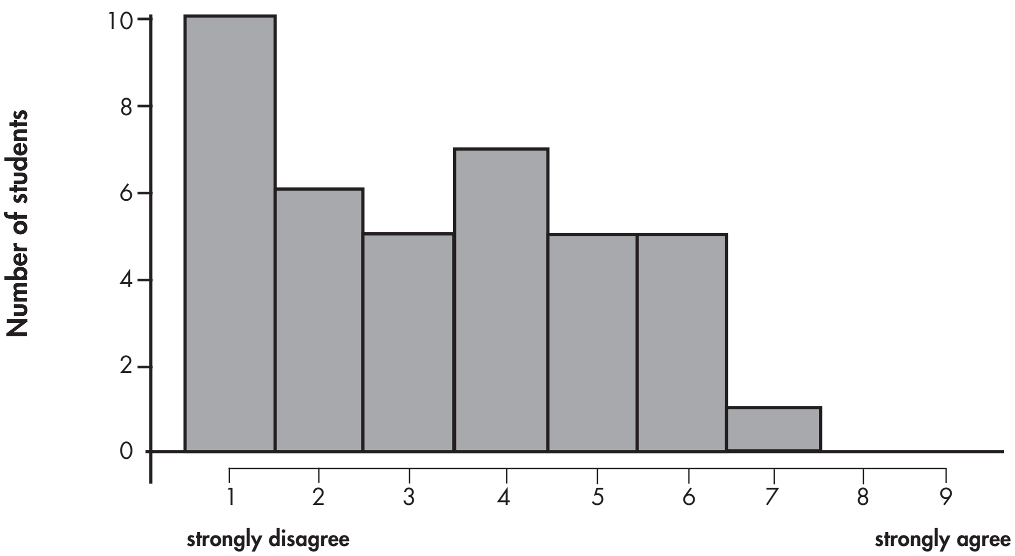

Figure 1. Student response to "CAS lessons are just like other maths lessons."

Perceptions of Teaching Style

In an Australian study of CAS teaching, the practices of three teachers were followed over a period of time (Kendal et al., 2005). Their practices were summarised as:

A: I ensure students have mastery of the rules and procedures of algebra.

B: I focus on the learner's personal construction of algebraic ideas.

C: I emphasise understanding of algebraic concepts.

Figure 2 shows the teachers' perceptions of their own teaching style pre- and post-CAS (teachers were allowed to select a combination of styles if they wished, so there are more than 12 responses both pre- and post-CAS). The graph shows a definite movement away from a rules and procedural style of teaching towards an exploratory and constructivist approach, or an understanding-based style. Teachers primarily saw their style as leaning most heavily towards teaching for understanding. Students, on the other hand, saw their teachers as employing a constructivist approach (style B).

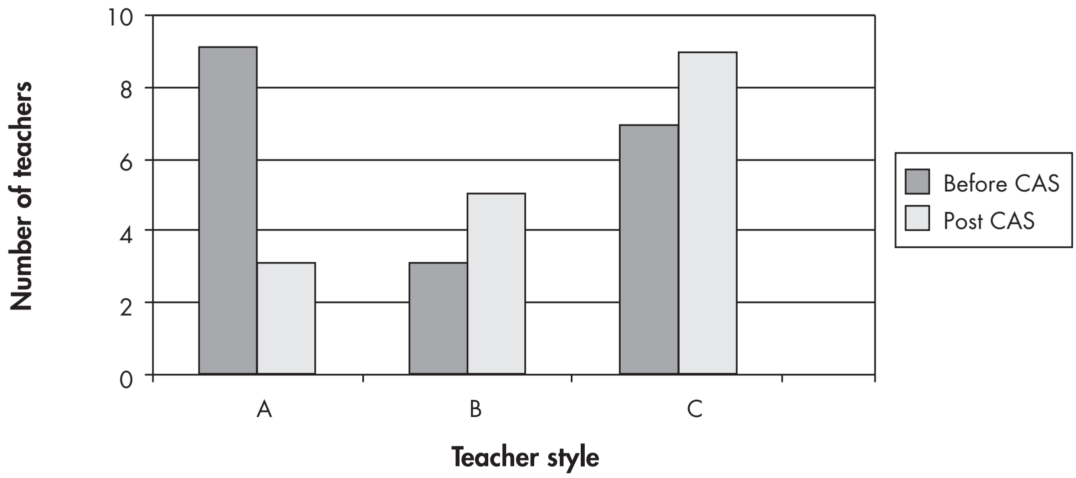

Figure 2. Teachers' perception of their teaching style pre- and post-CAS

Stories of Teaching

Altogether, nine classes were observed in the six pilot schools. The different styles of teaching in the observed lessons have been categorised into several distinct teaching methods. In some of the classes, the teaching was a mix of more than one method, while other classes primarily followed one pattern. These patterns are displayed in Figure 3. The stories that follow describe four quite different teaching approaches. These are arranged from most heavily student-led to those with a more teacher-led approach.

- *Story 1.* The major emphasis in this approach was using expert groups that each performed an exploration that helped them to discover a specific geometric property. The expert groups then returned to their "home" groups and explained to them what they had discovered. This teaching strategy has been referred to as the expert jigsaw method (see Kagan, 1994).
- *Story 2.* This was typified by students performing explorations at their own pace and in their own way. Students could choose to work individually, in pairs, or as a group. At times, we observed mathematical interactions between groups in quite different parts of

the classroom. During the independent explorations, the teachers roved and interacted with the students, asking and answering questions. The amount of off-task behaviours in each of these classes was the highest observed.

- *Story 3.* The emphasis of this teaching approach was a dialogue between the teacher and the whole class. While this method was teacher-centred, students were constantly responding to challenges from the teacher to perform mathematical activities and explain them. There was dialogue between teacher and students in a whole-class setting. The teachers in these classes used the strategy mentioned by Kutzler (2003) of "sequentially using and not using technology to achieve certain learning goals" (p. 53). Discussion was lively, and students were willing to defend their ideas even when they were in the minority.
- *Story 4.* This approach most closely resembles the traditional method of teacher instruction followed by students performing tasks. Generally, the period of whole-class teaching was short, as was the time when students performed the tasks. This pattern was then repeated several times during the lesson.

"a" to "i" are codes for each individual lesson observed.

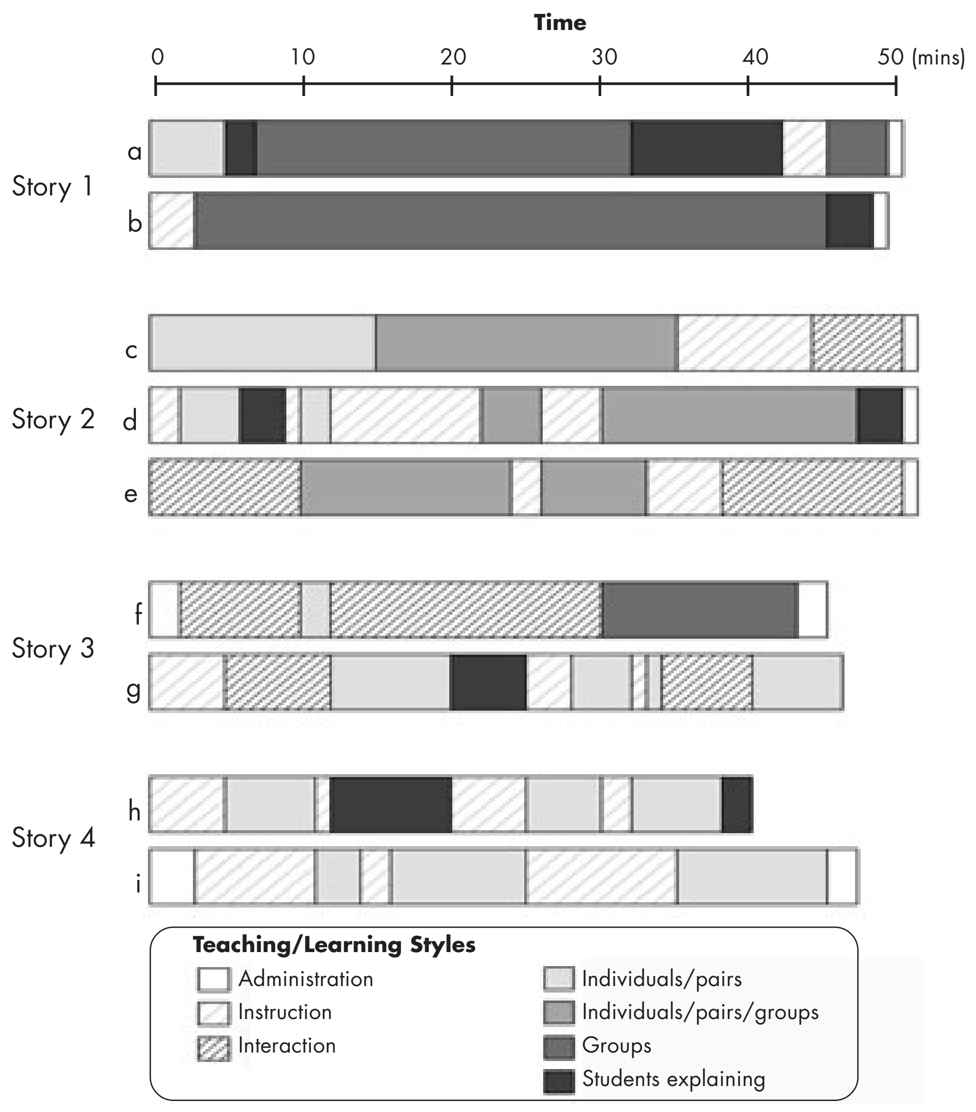

Figure 3. Observed style of teaching in lesson observations

Learning Issues: It Is Not Like Other Maths

Student learning could not be directly measured because no baseline data existed on the students in the pilot. The literature suggests that student learning can be enhanced by appropriate use of CAS. Heid (2003) noted that, after using CAS, "students' mathematical development seemed to proceed more rapidly" (p. 40). Heid and Edwards (2001) found that weaker students were able to "examine algebraic expressions from a more conceptual point of view ..." (p. 131). Noguera (2001) also commented that "[students'] cognitive development in algebra improved" (p. 263).

To evaluate this, teachers were asked whether they thought students were learning at a deeper level or at a faster pace and to give anecdotal evidence of any such improvements. Students were also asked what effects the pilot was having on their learning.

Student understanding

Over half the teachers believed that the CAS pilot had led to students having a deeper understanding of mathematical concepts. One teacher suggested that the learning might be broader as well as deeper, while another commented that the students were going further in their mathematics. Another said, "They are actually thinking now."

The remaining teachers were still unsure, apart from one who thought that the learning would be of the same depth. None of the teachers saw the CAS pilot experience leading to understanding that was not as deep. While teachers found it hard to quantify the improved depth of learning, the fact that the students were asking more sophisticated questions convinced one teacher that students' understanding had improved.

Students in the focus groups were asked whether using the CAS calculators had helped them understand mathematics better. The distribution of student responses is shown in Figure 4. This is clearly bimodal, with a significant group stating their understanding was worse since the CAS pilot and a similarly sized group viewing it positively.

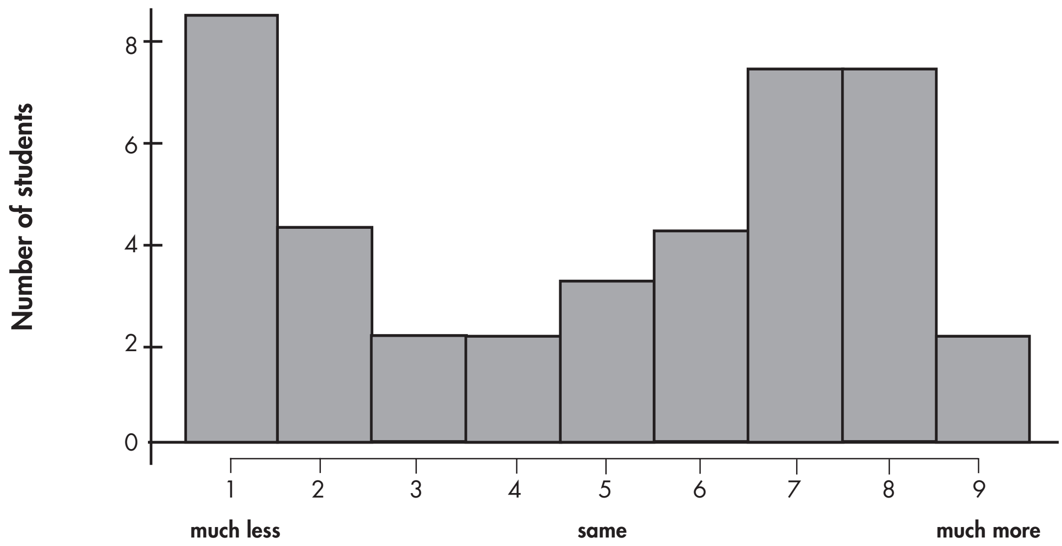

Figure 4. Students' views on their mathematical understanding in the CAS pilot

A number of issues seem to be affecting students' perception of whether their understanding was better, worse, or about the same. The main contributor to the negative responses related to the non-traditional approach to teaching mathematics that the project took. A number of comments of this type were made in the focus groups:

> What's this [way of teaching] got to do with maths? The calculator gets in the way.
>
> Sometimes I can't see the point.

Lagrange (1996, cited in Pierce & Stacey, 2002) said that in his experience:

> not all students wanted to use CAS. They did not want to be relieved of pen and paper work and that many, in fact, enjoyed doing routine calculation. (p. 576)

One student liked the fact that they were learning how to use the CAS calculator but thought they weren't learning as much maths. In the negative responses, it seemed to be a case of either students or parents believing "It's not real maths" or at least "It's maths, but not as we know it!"

Student attitudes

Students were largely positive in their attitudes towards the CAS project. Their confidence in using the calculator had grown, they found it reasonably easy to use, and they enjoyed using it. The teachers agreed that student attitudes were generally more positive.

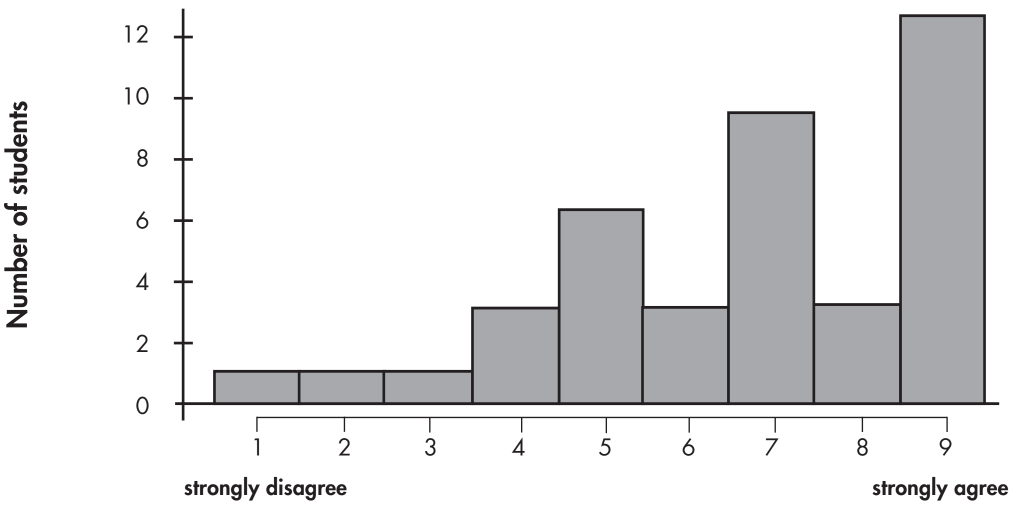

Figure 5. Student response to "I enjoy using the CAS calculator."

There were many aspects of the CAS pilot that students had enjoyed. These included the animations and the games or that the calculator was like a computer. A couple of students liked having less bookwork or writing. The ease of doing things, because the CAS is "doing it for you", was also appreciated. One said, "You can have fun and learn at the same time."

There was a real mix of views from students on whether they felt more positive towards mathematics. Some students felt much more positive, while others were less positive after the pilot than before it. Some saw it as a "dumbing down" of mathematics or as "not being real maths". A number of students had commented that maths was no longer boring. Some students who felt more positive about maths attributed it to the novelty effect of the CAS calculator:

> It's something new. We've done maths since primary school.

Some students shared why they felt less positive:

> [I] really used to like maths, but now it's a mass of confusion.

> I really enjoy CAS, but I ... am not learning as much maths.

Professional Development

The key driver in the CAS Pilot Project has been the provision of quality PD. The training sessions were run by PD providers. The underpinning aims and philosophies of the PD professionals have provided much of the intellectual thrust for the project. In the words of the PD co-ordinator for the project, "It's about teaching and learning mathematics."

Both PD presenters modelled an exploratory style of teaching, where real problems were posed and the CAS was used to explore them. This reflected their belief that the project was about pedagogy rather than technology. One teacher commented, "The initial emphasis was philosophical rather than technological. Technology is an assistant rather than a driver."

Teachers' perceptions of the PD

Most of the teachers found the PD very helpful. Some of the benefits they saw included:

- Getting together with teachers from other schools to share experiences and the pooling of resources. For many of the teachers, this was one of the most significant parts of the PD.
- The strong modelling of the appropriate pedagogy, which was the key ingredient of the project. While the pedagogy was modelled, teachers commented that the presenters did not reduce their professionalism as teachers.
- Having lesson plans and "off the shelf" resources, which they could take with them.
- Learning how to use CAS was seen as being an important feature by many teachers. They liked the way this had been blended with ways to teach using the CAS as a learning tool.
- Having two teachers per school involved in the PD.
- Being challenged mathematically and as teachers. Quotes illustrating this included:

  > [There are] challenging titbits. [It has] stimulating thinking.

  > [We have] achieved deeper understanding.

  > [We have been] given material that challenges what we have done in the past and how we have done it.

- Having time to reflect:

  > [I] didn't initially realise the value of the PD. [Now it has] crystallised and affected [my] teaching.

- A more integrated approach to mathematics was seen as being easier to achieve:

  > [We are] developing greater links between the strands, doing maths rather than the strands, [and it is] great making connections. They [students] suddenly realise that it is all connected and the teachers were wrong [to partition it].

Some areas for improvement of the PD were identified, along with areas for future PD requirements. These are covered in the full report (see Neill & Maguire, 2006, in press).

Assessment Of Learning, For Learning

One of the issues that this evaluation addresses is "What are the implications for assessment of student learning as a result of using the CAS technology? Do current forms of assessment need to change, and if so, in what ways?"

Both formative and summative assessment were explored when addressing this issue. Formative assessment, especially informal formative assessment, clearly supports the quality pedagogical approach that the CAS pilot employs. The challenge is that the summative assessments, especially the high-stakes ones, also need to work in harmony with an exploratory, understanding-focused approach to mathematics, rather than encouraging a more procedural approach.

Formative assessment

There has recently been an increasing emphasis on the fundamental role that formative assessment plays in effective teaching and learning. Authors such as Black and Wiliam (1998) have written on this subject. Ironically, Black and Wiliam's publication is entitled *Inside the black box: Raising standards through classroom assessment*. For Black and Wiliam, the black box is the classroom. They state: "present [UK] policy seems to treat the classroom as a *black box*. Certain *inputs* from the outside are fed in or make demands. Some *outputs* follow ..." (p. 1).

This *inputs–outputs* model of the classroom, which lacks a critical examination of what goes on in the process of teaching, is the very reservation that many have with CAS technology: feed in functions to it and out come solutions, without students having any understanding of the underlying mathematical concepts. The black box cannot be escaped by merely ignoring technology. What goes on inside the classroom is what matters, regardless of whether or not technology is used. Algorithms can be taught in a black box way either with the CAS or with pencil and paper. Black and Wiliam emphasise that quality formative assessment illuminates the black box, helping to change it into a white box.

Formative assessment was deeply embedded in the teaching and learning process of the CAS Pilot. One of the PD providers commented:

> If you let the teaching and learning process occur like we've been saying, formative assessment is happening every second of the day. ... [Teachers] will be assessing continually.

The other provider made comments of a similar kind, noting that:

> Good professional teachers can assess roughly where a student is at without doing a formal test. Often [teachers] don't place enough value on what kids are doing in the classroom.

If the pedagogy is of an exploratory nature with an emphasis on understanding, rather than procedural or algorithmic with an emphasis on performance, then formative assessment naturally follows.

The strong influence on formative assessment was borne out by the observations of classrooms, where many of the hallmarks of formative assessment were occurring, especially the informal types of assessment. High levels of student–teacher interaction were observed. Teachers were roving the classroom, observing students' work and mathematical conversations, initiating conversations, questioning students about their understanding, and giving them feedback on their learning.

Ironically, teachers thought that they were doing less formative assessment than they had been prior to the pilot. This was probably because their focus had moved from formal to informal assessment, which is at the heart of formative assessment. One PD provider suggested, "Sometimes [teachers] won't know that they are doing [informal assessment]."

Summative assessment

Summative assessment must not lose sight of the fact that it also influences teaching and learning. It helps define what happens in the classroom. Summative assessment must be for learning, not just of learning. This is particularly true of high-stakes assessment, which sends powerful messages to teachers about the learning that is valued and hence about the teaching that should occur.

Schools in the pilot had a number of different strategies for summative assessment. Four schools reported different forms of common tests across all of year 9. Of these, two thought their students were doing better, with one commenting that the CAS pilot "stretches students to be excellent".

The third school said the students were doing at least as well on the procedural skills, and the fourth made no comment. The form of the common tests varied from school to school. One format used was that some of the paper was in common across the school and some was just for the CAS students. Two other schools had a common test that was used across all year 9 classes, regardless of whether or not they were in the CAS pilot. The remaining two schools had no common test but did a series of teacher-written tests at different points within the pilot. In at least one class, these results were used to refine the teaching. One school said that they would be having an end-of-year exam, but that this would need to be different for CAS students.

Teachers in the CAS pilot schools felt they needed to address a number of issues around their own summative assessments. Many of these same issues also need consideration before CAS-enabled high-stakes assessments, at levels 1–3 of NCEA, become a reality. Summative assessment should be supportive of effective teaching and learning, not just a valid and reliable way of measuring student learning. It is not just formative assessment that supports good pedagogy: summative assessment should as well. If it is valued in teaching, it needs to be reflected in the high-stakes assessment (Harlen, 2005).

Technology in the Classroom

The CAS pilot aims to give a more structured and supportive approach to the introduction of the CAS technology than that which occurred when the graphics calculator was introduced into teaching and assessment. While the primary focus is on teaching and learning, the pilot is also interested in the role that technology plays in the classroom.

The effective and appropriate use of CAS depends on a number of factors. Key factors include the attitudes of teachers, PD providers, students, and their parents/caregivers towards technology in the classroom and its effective use.

Attitudes to technology

The extent of the impact that technology has in the classroom depends to a large extent on the teachers' attitudes, views, or philosophies concerning its use. The following summarises the views of the teachers in the pilot towards technology in the classroom after they had implemented the pilot with their students.

- *Technology is a tool with a purpose.* One teacher stated, "[It's] just another tool in the box of tools you've got to teach kids with."
- Several of the teachers qualified their statements by saying that technology is a tool to deepen learning and understanding.
- *It allows exploration and discovery.* One teacher said that it was "a learning tool to help students discover and learn".

- *It allows more open-ended and realistic investigations.* The technology can deal with real expressions in a range of contexts so that students can do realistic problems, not just ones with "nice" whole number answers.
- *It can scaffold learning.* The students can focus on the mathematical concepts rather than just on the manipulative details, moving between multiple representations of concepts, for example, algebraic, graphical, or tabular ways of expressing a problem. Technology can remove lower-level computational errors, allowing higher-level concepts to be explored. Kutzler (2003) describes this pattern of alternation between higher-level and lower-level tasks and says that by using CAS technology to perform the lower-level tasks, students can still make decisions at the higher level.
- *It's not about technology, it's about enhanced teaching and learning.* Comments such as the following all emphasise that it is about appropriate pedagogy more than about the technology itself:

  It's not about technology but pedagogy.

  [It's] the way you teach with it.

  Use it if [and when] it works.

A number of teachers mentioned some possible drawbacks that technology may have. One observed that some students "go blank with technology". Another was hoping that "it would not undermine important basic skills in mathematics". Used in a "black-box" way, this is certainly a threat. Students need to recognise when to use the technology and when not to. The issues are the same as those that surround the use of arithmetical or scientific calculators. One teacher had observed students in a common test using the CAS to confirm their answers to questions about simplifying trivial algebraic expressions that they could easily have worked out in their heads. This may lead to a dependency on technology rather than sensibly choosing when it is appropriate to use it.

Half the teachers reported that they had changed their views on technology as a result of the CAS pilot. Before the pilot began, three of these teachers had been negative towards technology or at least had reservations about it:

> [I was] resistant to it before ... I used to believe that pencil and paper were most important.
>
> [I] used to be anti, and I didn't want to do it.
>
> [I was] initially somewhat reserved and anxious about [my] ability to teach with it.

The other three who changed their views towards technology use in the classroom had become more positive about its use as a result of using it in the pilot.

Students had also observed that their teachers were positive towards the technology. Quotes included:

> Our teacher goes home and plays around on it and finds new stuff.
>
> He knows we find it more fun and more challenging.
>
> Ours is over the moon.
>
> I think he's more excited about it [than us].

Parents had mixed views on the use of calculators in mathematics. Some parents or caregivers were positive, and some were negative. Some saw the CAS calculators as mysterious, while others were just not interested in using them.

Future issues

What issues will need to be addressed and what support will need to be given to teachers to enable the effective and sustained use of CAS technology? A number of these aspects were identified in the project:

1. *Get parents on board.* Parents displayed a range of different attitudes towards the project. Schools adopted a range of strategies to try to redress the negative attitudes of some parents. The key is, of course, quality communication. Several schools had made it the major focus of their initial meet-the-teacher evening or parent interviews, while others had sent written information home to parents. Some teachers who had seen an initial resistance from parents saw it diminish with time. One teacher said, "[They] are really positive now."
2. *Get teachers on board.* This needs to include not only mathematics teachers but also teachers of other subject areas where the calculators have potential value.
3. *School leadership.* This was a strong feature in each of the pilot schools. There was active support in all the schools from the mathematics head of department. One school reported that the principal had been actively involved in ensuring the school was involved with the project. Strong school-based professional leadership is essential for fully effective implementation in a school. Without this support, individual teachers or even a pair of teachers who had undergone PD could be marginalised.
4. *System leadership.* Both the Ministry of Education (MoE) and the New Zealand Qualifications Authority (NZQA) need to have an ongoing role in the promotion of the values that underpin this project. For the MoE, the call from the PD co-ordinator was "to get it out there and to promote it".

   There was praise for the leadership already being demonstrated. A teacher said that they "applauded the MoE and NZQA for getting stuck into technology before it's everywhere". Significant leadership from NZQA will also be needed as they plan for NCEA assessments.
5. *Resources.* The teachers in the pilot saw a need for an expanding set of resources for teaching, but more particularly for assessment.
6. *Professional development.* Over half the pilot teachers believed that to sustain the project, there would need to be continuing PD. This would ideally be led by trained facilitators with time dedicated to the training.

Conclusion

One thing is certain. Technological advances are going to play a continuing role in our lives. These changes will clearly impact upon the way mathematics is performed in the real world. The debate then has to be: what is the best way of helping students prepare to be critical mathematical thinkers and problem solvers in the twenty-first century?

The major thrust of the CAS Pilot Project was all about quality teaching and learning and focused on an exploratory, self-discovery approach to gaining mathematical understanding. The teachers believed that this pedagogy had not only allowed the students to improve their mathematical understandings and skills, it also had enhanced their own professional skills. Classroom teaching had clearly changed as a result of the training given to the teachers in the project. There was some resistance to the project from a number of parents/caregivers and students, mainly because of concerns that it might undermine mathematical skills; some teachers in the pilot had initially shared these views. Effective assessment models will inform this debate. Assessments, both school-based and high-stakes, need to reflect the values of exploration and of understanding as well as acquiring mathematical skills.

The PD emphasised this pedagogy, as well as giving teachers sufficient confidence and knowledge in how to use the technology. Without quality PD, it will be hard to communicate this approach to using CAS in the classroom to the wider mathematics education fraternity. New Zealand has a world-class model now for professional development, namely that developed for the Numeracy Development Projects (NDP). One of the schools in the pilot was debating whether to become involved with the CAS pilot in 2005 or to begin the NDP instead. Ironically, both projects share many common beliefs. The PD co-ordinator said, "[Assessment] is more in line with *numeracy*, where students are exploring and gaining understanding."

Some of these common beliefs can be seen in *The Number Framework* (Ministry of Education, 2004).

- Numeracy talks about a "dynamic and evolutionary approach to mathematics" (frontispiece), and CAS talks about exploration and discovery.
- Numeracy values "children's learning and thinking strategies" (p. 1) and values strategies other than the algorithmic. The CAS pilot also values multiple representations and understanding, rather than an algorithmic approach (see Piez and Voxman, 1997).
- Numeracy values "professional development systems that change teaching practice; and effective facilitation" (p. 1). This has been a major focus of the CAS pilot.

Because of these strong similarities, we believe that it would be fruitful to explore synergies between the CAS pilot and the NDP.

Acknowledgments

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References

Black, P., & Wiliam, D. (1998). *Inside the black box: Raising standards through classroom assessment.* London: nferNelson.

Cedillo, T., & Kieran, C. (2003). Initiating students into algebra with symbol-manipulating calculators. In J. Fey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds), *Computer algebra systems in secondary school mathematics education*, (pp. 219–239). Reston, VA: The National Council of Teachers of Mathematics, Inc.

Harlen, W. (2005). Teachers' summative practices and assessment for learning – tensions and synergies. *The Curriculum Journal*, 16 (2), 207–223.

Heid, K. (2003). Theories for thinking about the use of CAS in teaching and learning mathematics. In J. Fey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds), *Computer algebra systems in secondary school mathematics education* (pp. 33–51). Reston, VA: The National Council of Teachers of Mathematics.

Heid, K., & Edwards, M. (2001). Computer algebra systems: Revolution or retrofit for today's mathematics classrooms? *Theory into Practice*, 40 (2), 128–136.

Kagan, S. (1994). *Cooperative learning.* San Clemente, CA: Kagan Cooperative Learning.

Kendal, M., Stacey, K., & Pierce, R. (2005). The influence of a computer algebra environment on teachers' practice. In D. Guin, K. Ruthven, & L. Trouche (Eds), *The didactical challenge of symbolic calculators* (7th ed., pp. 83–112). New York: Springer Science & Business Media.

Kutzler, B. (2003). CAS as pedagogical tools for teaching and learning mathematics. In J. Fey, A. Cuoco, C. Kieran, L. McMullin, & R. Zbiek (Eds), *Computer algebra systems in secondary school mathematics education* (pp. 53–71). Reston, VA: The National Council of Teachers of Mathematics.

Ministry of Education (1992). *Mathematics in the New Zealand curriculum.* Wellington: Learning Media.

Ministry of Education (2004). *Book 1: The Number Framework.* Wellington: Ministry of Education.

Neill, A. & Maguire, T (2006, in press). *An evaluation of the CAS Pilot Project.* Wellington: Ministry of Education.

Noguera, N. (2001). A description of tenth grade algebra students' attitudes and cognitive development when learning algebra using CAS. *The International Journal of Computer Algebra in Mathematics Education, 8* (4), 257–270.

Pierce, R., & Stacey, K. (2002, July). Monitoring effective use of computer algebra systems. *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia,* University of Auckland.

Piez, C., & Voxman, M. (1997). Multiple representations – using different perspectives to form a clearer picture. *The Mathematics Teacher, 90* (2), 164–166.

Appendix A

EVALUATION OF SECONDARY NUMERACY PILOT PROJECT 2005

Questionnaire for Teachers

1. How would you rate your **mathematical knowledge**?

1	2	3	4	5
Strong				Weak

2. How would you rate your **pedagogical knowledge**?

1	2	3	4	5
Strong				Weak

3. How would you rate your **knowledge of how students learn mathematics**?

1	2	3	4	5
Strong				Weak

4. What should the **key elements** be of a year 9 mathematics programme? Please explain.

5. How do you think year 9 mathematics should be **taught**? Please explain.

6. What mathematical knowledge do you think year 8 students should **ideally** bring to year 9? Please explain.

7. What are your **greatest needs** as a teacher of mathematics? Please explain.

Questionnaire for Teachers – *continued*

8. In mathematics department meetings at your school, estimate the **percentage** of time spent on the following:

   Administration ____________________

   Assessment and reporting ____________________

   Professional and curriculum support ____________________

   Other (please specify) ____________________

9. Are there any other comments you would like to make about the project?

BACKGROUND INFORMATION

Qualifications

Highest mathematics qualification

Years of teaching experience (including this year)