Findings from the New Zealand Numeracy Development Projects 2005

Contracted researchers

F. Ell, J. Higgins, K. C. Irwin, G. Thomas, T. Trinick, J. Young-Loveridge

Associated researchers

M. Britt, R. Carman, N. Hāwera, R. Isaacson, S. Sharma, B. Stevenson, A. Tagg, M. Taylor, S. Tait-McCutcheon, M. Wakefield, J. Ward, J. Woodward, D. Yates

Foreword

D. Holton

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Findings from the New Zealand Numeracy Development Projects 2005

Foreword

This is the second compendium of papers concerning research into various aspects of the New Zealand Numeracy Development Projects (NDP). The work has been undertaken by researchers who are all involved in some way or other in the local mathematics education scene. They are all either university academics or educational consultants. The 12 papers in this compendium span the areas of student achievement, professional practice, and sustainability as they relate to the NDP.

Student Achievement

Young-Loveridge's paper, "Patterns of Performance and Progress on the Numeracy Development Project: Looking Back from 2005" (p. 6), continues the work of assessing the overall performance of students by looking at the New Zealand-wide data reported by teachers (see also *Findings from the Numeracy Development Project 2004*).

Overall, students' results have improved steadily over the years of the NDP. For example, students are now progressing further relative to their initial performance than they did earlier in the life of the NDP. While there are still gaps between various ethnic groups, these appear to be reducing. In fact, if these differences are considered with respect to other studies, they are seen to be relatively small.

However, Young-Loveridge warns that the homogeneity of the data may be due to the fact that the average decile level of schools now is higher than it was at the start of the NDP. Further, students may be aided by the fact that the assessment is oral rather than written.

The paper by Thomas and Tagg ("Numeracy Development Project Longitudinal Study: Patterns of Achievement", p. 22) extends the work reported in *Findings from the New Zealand Numeracy Development Project 2004*. The study involves schools that first participated in the project at least two years ago.

In the first section of the study, 26 schools were involved. Students from four year levels were given especially designed tests: years 4 and 8 were given a balanced selection of questions from the Third International Mathematics and Science Study (TIMSS) 1995 and 2001 National Education Monitoring Project (NEMP) assessments; year 5 students were given items from the 2003 Trends in International Mathematics and Science Study (TIMSS) assessment; and year 6 students received items from Assessment Tools for Teaching and Learning (asTTle). In general, the students performed considerably better on NDP related questions, did less well on number problems not emphasised in the NDP, and did best of all on non-number questions. The authors expressed concerns about students' performance on calculations that were too hard to perform mentally.

In the second part of the study, 20 schools were involved. Students from years 1–8 continued to show the improving achievement reported in Thomas and Tagg's previous paper (ibid). This is confirmed by teachers, 70% of whom stated that at least 70% of their students were attaining their school's numeracy targets.

Te Poutama Tau, which is based on the Number Framework developed for the NDP, is a professional development programme for teachers involved in the teaching of numeracy in Māori-medium settings. In "An Evaluation of Te Poutama Tau 2005" (p. 34), Trinick and Stevenson look at the progress that was made in 2005. This follows on from similar pieces of research that were undertaken in 2003 and 2004 and uses the national database on which teachers record the stage levels of students at the start and end of the year. The purpose of the research was to determine the overall progress of students and to identify areas where students had performed well or poorly over the three years of the project.

There was a small general improvement over the three years for most of the numerical knowledge, but there were large gains in the areas of multiplication, proportions, number identification, and fractions, with a smaller improvement in grouping and place value knowledge. One of the positive aspects of the results from 2005 is that the number of students making no stage gain is declining. In particular, there was a very large drop in the number of students making no stage gain in multiplication and fractions, from 70% to 40%.

The aims for subsequent years are:

- to improve the outcomes for students who currently are making no stage gains
- to maintain an emphasis on grouping and place value
- to maintain a focus on the important areas of multiplication, fractions, decimals, and proportions – essential skills in higher mathematics as well as in everyday life
- to determine how te reo Māori linguistic structures help or hinder the development of the various areas of pāngarau (Māori-medium mathematics).

Irwin and Britt's paper ("Algebraic Thinking in the Numeracy Project: Year Two of a Three-year Study", p. 46) continues work that was started in 2004 and reported in *Findings from the New Zealand Numeracy Development Project 2004.* The same test that was given to students in 2004 was given to those who took part in 2005. The questions on these tests all involved increasingly difficult compensation problems in each of the four arithmetic operations. Each of the four test sections contained (in this order) whole numbers, decimal fractions, whole numbers and a literal symbol, decimals and a literal symbol, and just literal symbols in an algebraic identity.

At the middle stage of this study, the results show that the year 9 students who had been involved in the NDP in intermediate school were significantly better at algebraic thinking than they were in year 8. The students from the intermediate school that had the best performances in the 2004 study were also the top performers in year 9 and had the highest correlation between successes at these two levels. Students from other schools did not show equivalent progress or correlation. There were significant differences between schools.

"What do you think that maths is all about?" is the question that Young-Loveridge, Taylor, Sharma, and Hāwera asked a number of children in "Students' Perspectives on the Nature of Mathematics" (p. 55). There seems to be a reasonable amount of literature on the topic from an adult viewpoint, but few children's opinions have been sought. Young-Loveridge et al. interviewed 459 year 2–6 students from schools that had taken part in the NDP and from schools that hadn't and gathered a large number of interesting responses.

Nearly half of the students' responses covered items of mathematical content, mainly on number; the learning process was mentioned by over 30% (though the percentage seemed to decline as the students got older); well over 20% thought it would be useful at some later stage in their lives; and a small percentage each talked about the value of maths for thinking, solving problems, enjoyment, and current usefulness. Perhaps surprisingly, just under 25% could not think of anything to say on the subject at all. There seemed to be little consistent difference between students who had been involved in the NDP and those who had not.

Professional Practice

"Modelling Books and Student Discussion in Mathematics" (p. 65), the paper by Higgins with Wakefield and Isaacson, takes the using of manipulatives a stage further. Instead of simply letting the students work with some material to introduce or solidify a mathematical idea, Higgins et al. propose the use of a modelling book (also know as a recording book) to be used in tandem with the activity. In this large book that is used by a teacher with a group of students, the teacher and students write down mathematical ideas that have been stimulated by the equipment. This written work reinforces the concepts of the model and provides a stepping stone between the activity and the imaging of the mathematics. In addition, it provides a record of the work that enables the group to look back on what they have achieved. Teacher and student quotes reinforce the value and enjoyment of the modelling book. Although this work was undertaken with a single Māori class, there is no obvious reason why the modelling book shouldn't be of value in all classrooms.

"Contextually Responsive Facilitation" (p. 72), a paper by Higgins and Tait-McCutcheon with Carman and Yates, explores a contextually responsive model of facilitation. This model focuses on the concepts and strategies of the programme rather than attending to the minutiae of the guidelines. Higgins et al.'s research investigated how the use of co-teaching and co-generative dialogue produced facilitation that was responsive to a teacher's context. Data included classroom observations and interviews with students and teachers in schools in two different regions of the country.

The research suggests that this approach produces both a collective capacity and commitment among the teachers and a feeling that they can make a difference for their professional teaching community. But perhaps most importantly, an environment is developed in which the individual builds new teaching habits and transforms their professional practice. In this way, it is hoped the effect will be sustained well after the facilitator withdraws from the school.

The work of Irwin and Woodward (reported in "Advancing Pasifika Students' Mathematical Thinking" p. 80) follows on from initial research undertaken in 2004 and reported on in *Findings from the Numeracy Development Project 2004*. In that previous study, the class was observed only once and the aim of the 2004 research was to see how the students' mathematical language developed over time, especially when the teacher was not involved in the conversation. Again in 2005, the year 5 and 6 class consisted mainly of students from a Pasifika background. The class was in a decile 1 school, and its membership varied over the time of the five observations.

The teacher, a New Zealand European, continued to help her students make progress that was greater than the average for Pasifika students nationwide. She used language that is known to facilitate children's thinking and encouraged them to use similar language. Although the students used similar language when the teacher was present, they used only selected aspects of this language when the teacher was not working with their group. They continued to emphasise method rather than answers but did not solve problems co-operatively. The literature suggests that solving mathematical problems co-operatively enhances mathematical success. The authors recommend that it would be useful to ensure that this takes place whenever students are working together to solve problems.

Thomas, Tagg, and Ward have taken a different and equally important approach in "Numeracy Assessment: How Reliable are Teachers' Judgments?" (p. 91). At least two aspects of the NDP rely heavily on teachers' diagnostic assessments of students. One of these is that the Numeracy Project Assessment diagnostic tool (NumPA) and the Global Strategy Stage (GloSS) assessment, along with a teacher's own diagnostic questions, are the basis of a teacher's knowledge of their students' mathematical ability and provide the starting point for their classroom programme. The other is that the initial and final assessments for the year are used as evidence for the success of the NDP. However, the quality of the teachers' assessment decisions on which the data is based had never been examined. It is of vital importance, therefore, to know how reliable teachers' judgments are.

Thomas et al. compared teachers' judgments against those of educators experienced in strategy stage assessments. This comparison was made by both "live" assessments of actual students and by assessing scripted situations. There was a high level of agreement between teachers and experts in the work with students. Where there were differences, the teachers' ratings were lower in two-thirds of the cases. One reason for this may be that teachers are grouping their children for instructional, rather than reporting, purposes. In the scripted situations, reliability was good but not quite as high. This may be due in part to the limited information available in the written scenarios. Another possible reason is that the scenarios represented all stages of the Framework, whereas the teachers involved teach at specific year levels that focus on specific stages of the Framework. However, the results of this work are encouraging in that teachers' reliability appears to be very good.

In "Te Poutama Tau: A Case Study of Two Schools" (p. 103), Trinick studied two kura kaupapa Māori that had achieved significant gains during 2004 to try to determine the factors that foster student performance. In interviews and through a questionnaire, principals and teachers were asked socio/cultural questions about the school and its community and the links between the two, what experience and qualifications staff had, especially regarding pāngarau, and staff reflections on Te Poutama Tau. This research is similar to that reported last year.

Trinick found that both schools had in common the desire of their communities to revitalise Māori language, knowledge, and culture. It was also suggested that the positive relationship between all the professionals in the schools was critical in producing student outcomes. Further, teachers in both kura collaborated regularly and built up an environment where they could share ideas and gain support. These had all enabled positive changes to be made by both staff and students in their attitude towards pāngarau.

Sustainability

The study on the sustainability of the NDP commissioned by the Ministry of Education in 2005 concentrated on the 2005 lead teacher initiative. The questionnaire used for this study by Thomas and Ward ("Sustaining the Numeracy Project: The Lead Teacher Initiative 2005", p. 115) was sent to lead teachers, teachers, and principals of the schools involved in the initiative as well as to the facilitators of those schools.

The researchers found that the participants believed that the lead teacher initiative had been effective in providing lead teachers with both increased confidence to guide the development of numeracy in their schools and increased knowledge of mathematics.

It was found that nearly all teachers continued to use NDP practices within their teaching programmes. The most common of these were strategy stage grouping of students and the employment of resources and activities. Fundamental to sustaining numeracy in schools appears to be ongoing facilitator support, leadership by lead teachers, and the support of the principal. Barriers to sustainability identified by participants included the challenge of new staff that lack numeracy training and a lack of time to plan, teach, and assess numeracy. A view expressed by some facilitators was that schools need to take increased responsibility for teachers' ongoing professional learning if numeracy is to be successfully sustained.

Five teachers from two schools (including the two lead teachers) took part in Ell and Irwin's study of sustainability of the NDP ("Sustained Numeracy Project Practices in Two Schools", p. 129). The data was obtained through audiotaped interviews and collected documentation.

The literature suggests that internalisation by teachers is the basis for innovation and change and that this leads ultimately to sustainability through school-wide change. Ell and Irwin's study found that all but one of their teachers showed evidence of this internalisation in both their interviews and their practice. The remaining teacher was still holding change at arm's length by her vocabulary in the interviews and by her noting that the project does not align with her practice. However, the authors point out that sustainability involves more than maintaining the system aspects of the NDP; sustained depth of insight into students' progress is also involved, and determining whether this exists requires a deeper study than was possible in the time available to them.

Conclusion

Some aspects of the research reported in this compendium gave cause for concern. Specifically, the performance on arithmetical tasks that cannot be performed mentally and the differences that exist between the performances of ethnic groups were highlighted by researchers. In the former case, teachers may have felt that such calculations were not as important as other aspects of the NDP. Consequently, more emphasis will need to be placed on this in future years. And although there are continuing differences between ethnic groups' results, the magnitude of these differences appears to be declining and seems to be smaller than in comparable studies elsewhere.

However, there is much reported here about the NDP that is positive and shows that the NDP is raising the standard of New Zealand children's mathematical performance. Consequently, it would appear that the NDP continues to play a major role in the education of the nation's youth.

Professor Derek Holton
Department of Mathematics and Statistics, University of Otago

Patterns of Performance and Progress on the Numeracy Development Project: Looking Back from 2005

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

This paper reports on the analysis of 2005 data from the Numeracy Development Projects (NDP). Students' performance improved from the beginning of the year to the end. However, performance on the multiplication and division domain was a little disappointing, with only about a third of year 8 students reaching stage 7, advanced multiplicative thinking, by the end of the year. Likewise, performance on proportion and ratio was lower than hoped, with only about a tenth of year 8 students reaching stage 8, advanced proportional reasoning. Analysis of the patterns of performance and progress over time showed that performance improved from 2002 to 2005. Students also made greater progress relative to their initial stage in recent years compared with earlier years. The gaps between European and Pasifika students appeared to reduce fairly steadily over time. Among students who began the project at lower Framework stages, Pasifika students often made greater progress than those from other ethnic groups. These improvements coincided with changes in the composition of the cohort over time, most notably a reduction in the percentage of students from low-decile schools and an increase in the percentage of students from medium- and high-decile schools. Hence, it is difficult to conclude with any confidence that the Numeracy Projects are primarily responsible for the improvements. Although the gaps in achievement between European and Māori/Pasifika students continued to exist, when these differences are put beside those found in other large-scale studies, it is clear that NDP differences are much smaller (a quarter of a standard deviation versus three-quarters to a whole standard deviation). The use of an individual, orally presented assessment tool with an emphasis on explaining the strategies used to get answers, rather than a written test on which the number of correct answers is simply totalled, may help to explain the positive outcomes for NDP students.

Background

The New Zealand Numeracy Development Projects (NDP), like other reforms in mathematics education world worldwide (for example, Bobis et al., 2005; British Columbia Ministry of Education, 2003; Commonwealth of Australia, 2000; Department for Education and Employment, 1999; National Council of Teachers of Mathematics, 2000; New South Wales Department of Education and Training, 2001), came about as a result of concern about the quality of mathematics teaching. This concern was sparked by the results from the Third International Mathematics and Science Study (TIMSS), which showed that the mathematics achievement of students in many western nations was below international averages (for New Zealand's results, see Garden, 1996, 1997). Much of the rhetoric is about the need to produce confident life-long learners who are better able to cope with the demands of the twenty-first century.

The first Numeracy Development Projects began approximately six years ago. They are part of a wave of educational reform worldwide aimed at improving the mathematics teaching and learning of students at the primary and secondary levels. This paper reports on the results for the NDP for 2005.

Method

Participants

Data from approximately 52 000 year 0–8 students who were assessed at the beginning and end of the year 2005 were included in this analysis (see Appendix A for the composition of the sample).

Just under a third (29.9%) of the cohort were in years 0–3 (Early Numeracy Project: ENP), a little under half (46.4%) were in years 4–6 (Advanced Numeracy Project: ANP), and almost a quarter (23.7%) were in years 7–8 (Intermediate Numeracy Project: INP). More than half of the students (63.3%) were Pakeha/European, and about a fifth (19.5%) were Māori. The remainder consisted of 7.3% Pasifika, 5.4% Asian, and 4.4% of other ethnicities. Only about one sixth (18.1%) of the students were from low-decile schools, nearly half (44.7%) were from medium-decile schools, and more than a third (37.2%) were from high-decile schools. The gender composition was virtually identical. It was interesting to note that, compared to previous years, there were more European students and fewer Pasifika students. In 2005, there were fewer Māori students than in the initial years of the project (2002 and 2003). Other changes in the composition of the cohort included fewer students from low-decile schools and more from high-decile schools, an increase in the average decile ranking for all decile bands – low, medium, and high – and fewer students at years 0–3 and more at years 4–6 and 7–8.

Procedure

Students were interviewed individually by their teachers at the beginning and end of the year, using the diagnostic interview (Numeracy Project Assessment: NumPA). The data was sent to a secure website. Only students with two sets of data (initial and final) were included in the analysis for this report.

Results & Discussion

Patterns of Performance

The first part of this paper examines students' performance at the beginning and end of the year and as a function of grouping variables such as age (reflected in year group), ethnicity, socio-economic status (reflected in school decile band), and gender.

Differential performance as a function of year group

At the end of the year, many students were at a higher stage of the Number Framework than they had been at the start of the project (see Appendix B). Comparison of 2005 results with those from 2004 show similar percentages of students reached equivalent stages on the Framework (see Young-Loveridge, 2005). For example, on the addition and subtraction domain, just over 80% of year 2 students in 2005 were at stage 3, counting from one by imaging, or higher, compared with 79% of year 2 students in 2004. Just over 95% of year 4 students in 2005 were at stage 4, advanced counting, or higher, and more than 65% were at stage 5, early additive part–whole thinking, or higher, compared with 95.5% and 63% of year 4 students in 2004 at stage 4 and stage 5 respectively. Almost 88% of year 6 students were at stage 5, early additive part–whole thinking, or higher, compared with almost 83% in 2004. Almost 59% of year 8 students in 2005 were at stage 6, advanced additive part–whole, compared with just over 55% in 2004. It is important to note here that most of the students were in their first year of the project and that even better results may be expected once the NDP has been established in a school over several years (see Thomas and Tagg, p. 22 in this compendium, on the results of the Longitudinal Study).

On the multiplication and division domain, the percentage of year 8 students at stage 7, advanced multiplicative part–whole thinking, at the end of the year was 36%, similar to the results found for students in 2004 (34%). Combining stages 7 and 8 on the proportion and ratio domain yielded almost identical percentages (36% in 2005 and 33% in 2004). These results are very consistent in showing that only about one-third of the year 8 students reached stage 7 by the end of the year.

The newly introduced stage 7 in the addition and subtraction domain (referred to as stage 7 A/S to distinguish it from stage 7 on other domains) involved addition and subtraction with fractions and decimals ($2 - [\frac{3}{4} + \frac{7}{8}]$ and 5.3 – 2.89). These tasks seemed considerably harder than the multiplication and division tasks used to assess multiplicative thinking in the multiplication and division and proportion and ratio domains. Only 5% of year 8 students were initially at stage 7 A/S, and by the end of the year this had increased to only 19%. Comparable figures for stage 7 M/D were about 15% and 35% for initial and final data respectively. This discrepancy suggests that addition and subtraction with fractional numbers, particularly when the task involves decimals with place-holding zeros, is more difficult than multiplication and division with whole numbers and may be between stage 7 and stage 8 (see later section on inter-relationships between multiplicative thinking and other domains). The fact that more students were successful on addition and subtraction with fractional number than reached stage 8, advanced proportional part–whole thinking, indicates that it is easier to add and subtract with fractional numbers than to work with proportion and ratio.

The newly introduced stage 8, advanced proportional part–whole, in the multiplication and division domain (involving division with decimals) seemed to be of comparable difficulty to the proportion and ratio tasks used to assess proportional reasoning in the proportion and ratio domain. Only 2.2% of year 8 students were successful on these tasks initially, but this had increased to 10.8% by the end of the year. The corresponding figures for the proportion and ratio domain were 2.4% (initially) and 8.4% (finally). These figures were similar to those found in 2004 (2.8% initially and 9.3% finally). Hence by the end of the year, fewer than 10% of the students about to enter secondary schools were at stage 8.

Differential performance as a function of gender, ethnicity, and decile

Appendix C shows the percentages of students at each Framework stage for each domain as a function of gender, ethnicity, and decile. As in previous years, more boys than girls reached the upper stages of the Framework. For example, on addition and subtraction, the percentages of boys reaching stage 6 or higher were 14.7% initially and 30.5% finally, compared with 8.4% of girls initially and 22.4% finally. Similar gender differences were evident on the other domains as well. On multiplication and division, 6.7% of boys reached stage 7 or higher initially, compared with 2.9% of girls. The corresponding percentages at the end of the year were 16.9% and 10.9%. On proportion and ratio, 0.8% of boys reached stage 8, compared with 0.3% of girls. The corresponding percentages at the end of the year were 15.8% and 10.6%.

As Appendix C shows, more European students reached the upper stages of the Framework than Māori or Pasifika students and Māori students did better than Pasifika students. However, Asian students outperformed all other ethnic groups. This pattern is consistent with those found in other large-scale studies of mathematics achievement in which ethnicity has been a variable of interest, for example, TIMSS (see Garden, 1996, 1997), the Programme for International Assessment (PISA), and the National Education Monitoring Project (NEMP).

Appendix C also shows that students at low-decile schools did less well than those at medium- and high-decile schools. Although students at high-decile schools did better than those at medium-decile schools, the differences between students in the high- and medium-decile bands tended to be smaller than those between students at medium- and low-decile schools.

Appendix D shows the percentages of boys and girls at each Framework stage as a function of ethnicity. Each of the main ethnic groups showed a similar advantage for boys over girls at the upper stages of the Framework, and this pattern was consistent across all three operational domains:

addition and subtraction, multiplication and division, and proportion and ratio. Figure 1 shows the patterns for boys and girls from each main ethnic group on the domain of multiplication and division.

It is clear from Figure 1 that more boys than girls reach stage 7, advanced multiplicative part–whole. Figure 1 also shows that Asian and European students did better than Māori and Pasifika students. This can be seen in the greater percentages of students at stage 7 and the smaller percentages at stages 2–3 and 4, as well as by the fact that more of them were assessed on multiplication and division (shown by the height of the shaded bars; the blank space above the shaded bar represents the proportion who were *not* assessed on multiplication and division). It is interesting to note that more Māori boys than Māori girls reached the upper stages of the Framework. This pattern is opposite to that found for large-scale international comparisons such as TIMSS, where paper-and-pencil tests were used to assess mathematics achievement (see Garden, 1996, 1997). The TIMSS results showed that Māori girls tended to outperform Māori boys. It seems likely that the nature of the assessment is a crucial factor in determining these patterns. The diagnostic interview used in the NDP to assess students' mathematical proficiency (NumPA) involves the assessment of students individually by their own teachers, with tasks presented orally. Moreover, the emphasis is on the nature of the strategies used rather than simply whether or not the answer given was correct. By presenting tasks orally and expecting students to respond orally and to explain their thinking and reasoning, NumPA effectively minimises the literacy requirements and allows students to access the mathematics and demonstrate their mathematical proficiency unimpeded by literacy barriers. Of course, it is also possible that teachers unwittingly help certain students in the individual interview situation; that might help to explain the different patterns found for TIMSS and the NDP. However, evidence from Thomas, Tagg, & Ward (see p. 91 of this compendium) on the high level of agreement between the judgments of classroom teachers and those of independent researchers support the reliability of the individual interview data gathered with the NDP.

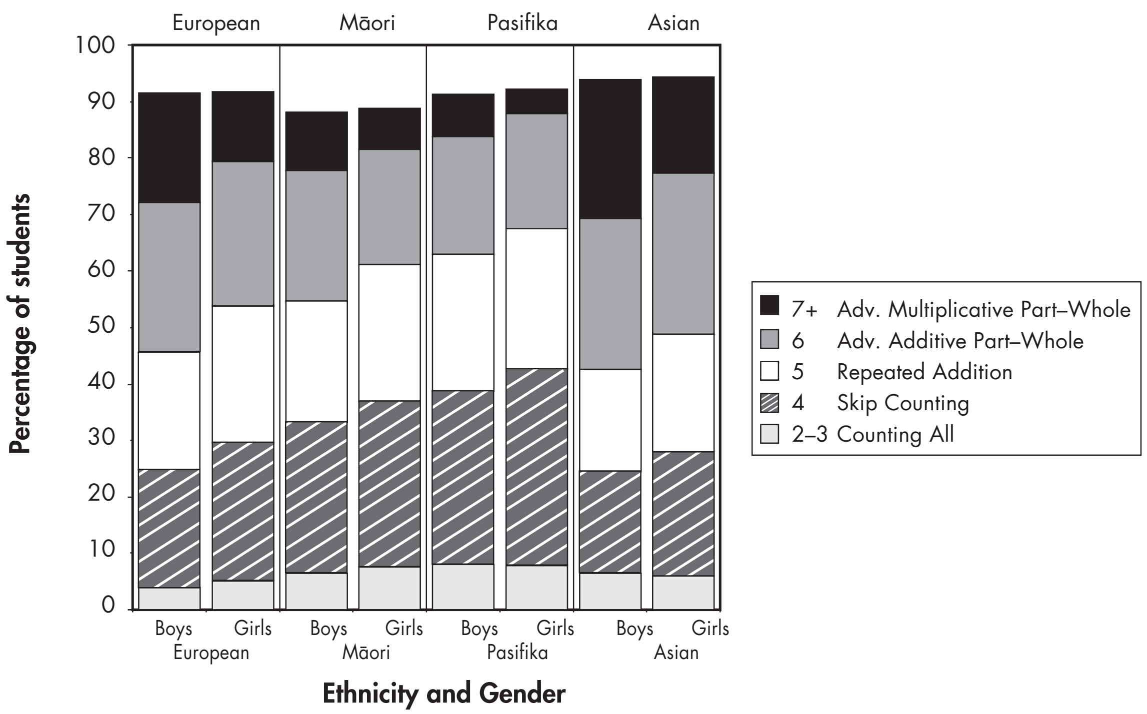

Figure 1. Percentages of students at each Framework stage on multiplication and division at the end of the year as a function of ethnicity and gender

Differential performance as a function of ethnicity: 2002–2005

It was interesting to look back at the patterns over time from 2002 to 2005 (see Table 1). The average Framework stage was calculated for each of the main ethnic groups – European, Māori, and Pasifika. At the beginning of 2003, the average Framework stage dipped. This was possibly because the criteria for crediting students with stage 6, advanced additive part–whole, was made stricter in 2003. Previously, students had been credited as being at stage 6 if they were "able to use a broad *range* of mental strategies to solve addition or subtraction problems with whole numbers". Although two different problems were given in 2002 (394 + 79 = □ and 403 – 97 = □), the instructions did not specify that students had to solve *both* problems correctly (see Ministry of Education, 2002). The instructions in 2003 stated that the students needed to be "able to use at least *two different* mental strategies to solve addition or subtraction problems with multi-digit numbers" (see Ministry of Education, 2003). As these were the only problems presented at that level, students needed to get both of them correct.

Table 1
Average Framework Stage on Addition and Subtraction for Year 0–8 Students 2002–2005

Year	Project Status	European	Māori	Pasifika	Asian
2002	Initial	4.05	3.78	3.50	4.30
	Final	4.61	4.33	4.06	4.79
2003	Initial	3.81	3.54	3.26	3.99
	Final	4.46	4.17	3.84	4.61
2004	Initial	3.93	3.77	3.61	4.06
	Final	4.57	4.38	4.23	4.76
2005	Initial	4.15	3.82	3.81	4.29
	Final	4.74	4.45	4.40	4.89

At the beginning of 2004, the average Framework stage was higher than it had been in 2003. This coincided with an increase in the proportion of students from high-decile schools (25.1% to 32.3%) and a corresponding decrease in the proportion of students from low-decile schools (37.6% to 28.9%). It is interesting to note that the differences between the three ethnic groups were smaller in 2004 than those during 2002 and 2003 (see Figure 1). At the beginning of 2005, the average Framework stage was again higher than at the start of the previous year. Once again, the proportion of students from high-decile schools had increased (32.3% to 37.2%), while those from low-decile schools had decreased (28.9% to 18.1%). It seems likely that the changes in the composition of the cohort might explain the increase in the average Framework stage in 2005.

Effect sizes

It is clear from Figure 1 that there were differences in the average Framework stage for students from different ethnic groups. The effect sizes for these differences were calculated to enable comparisons of effect sizes to be made. Effect sizes were calculated by dividing the average difference between two groups by the standard deviation for the two groups combined. Table 2 presents the average effect sizes for the comparisons between European and Māori and between European and Pasifika over the period 2002–2005.

According to Cohen's classification (see Fan, 2001), an effect size of 0.2 is considered "small" (a difference of less than a quarter of a standard deviation), those of 0.5 are thought to be "medium" (a difference of half a standard deviation), and those of 0.8 are considered "large" (a difference of more than three-quarters of a standard deviation). Hence, the effect sizes for the ethnicity comparisons are quite modest, particularly those for the European–Māori comparison (see later section in the paper on putting effect sizes into perspective).

Table 2
Average Effect Sizes for Ethnicity Comparisons (European vs Māori and Pasifika) 2002–2005

Year	European–Māori		European–Pasifika	
	Initial	Final	Initial	Final
2002	0.19	0.22	0.37	0.43
2003	0.17	0.22	0.35	0.46
2004	0.11	0.15	0.21	0.27
2005	0.22	0.22	0.23	0.26

Figure 2 shows the effect sizes for ethnicity differences over time. The pattern of effect sizes in Figure 2 shows a gradual reduction in the differences over time, apart from a slight rise for the European–Pasifika comparison at the end of the year, followed by a reduction in the magnitude of the differences in 2004 (both before and after the project). In 2005, the European–Pasifika differences appear to level out, while those for European–Māori comparisons increase. One possible explanation for the convergence of European-Māori and European-Pasifika differences in 2005 is that the proportion of year 7–8 students increased for European (20% to 23%) but decreased for Māori (28% to 24%), compared with the previous year. That is likely to have increased the difference in average Framework stage between European and Māori and hence to have increased the effect size. In 2004 (when the effect size for the European-Māori comparison was at its smallest), the proportion of year 7–8 Māori was at its highest.

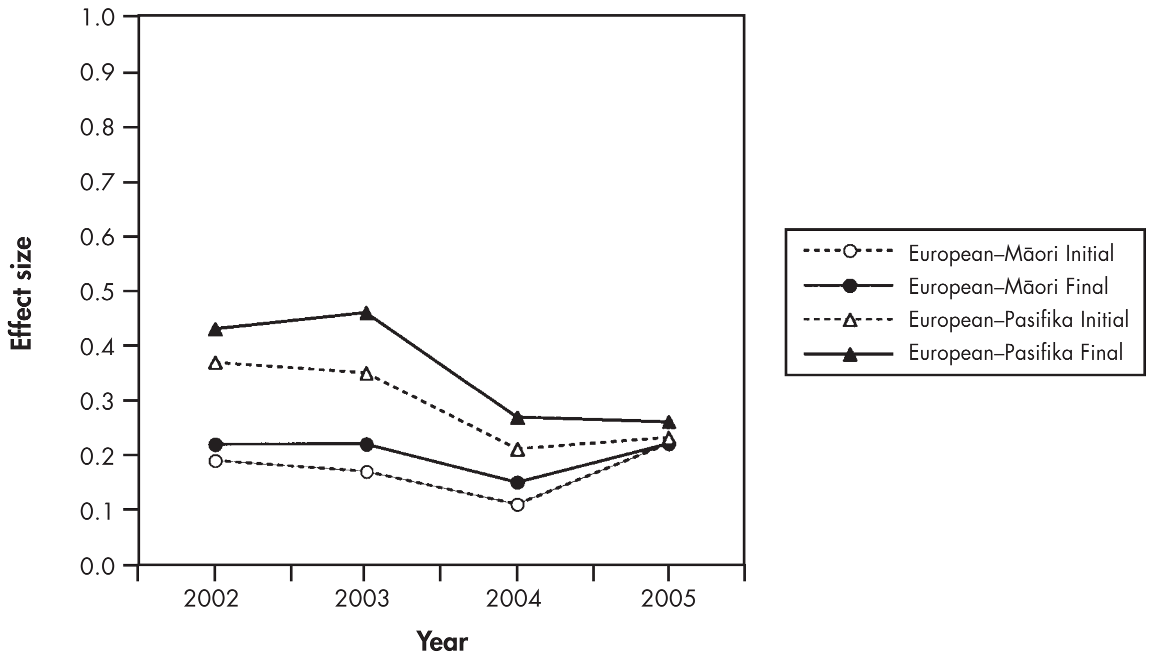

Figure 2. Effect sizes for differences in average Framework stage at the beginning (Initial) and end (Final) of the project over 2002–2005 as a function of ethnicity (European–Māori and European–Pasifika)

It is important to interpret cautiously the data that uses average Framework stage because of the problems already identified with the stages on the Framework not constituting an interval scale (because the steps at the lower end of the Framework are smaller than those at the upper end). A later section of this paper, which looks at patterns of progress with respect to identical starting points, provides a more reliable measure of students' performance and progress.

Patterns of Progress

Patterns of progress were examined by looking at the proportions of students who moved to a higher Framework stage relative to particular starting points. Appendix E shows the percentages of students at each initial stage who moved to a higher Framework stage. Separate results are shown for European, Māori, and Pasifika, and results are presented for 2002 to 2005. Figures 3 to 6 show the graphs presenting the patterns of progress for European, Māori, and Pasifika students over 2002 to 2005. Students who started at stage 0, emergent, or stage 1, one-to-one counting, showed the greatest progress, with about 80% of students moving to a higher Framework stage.

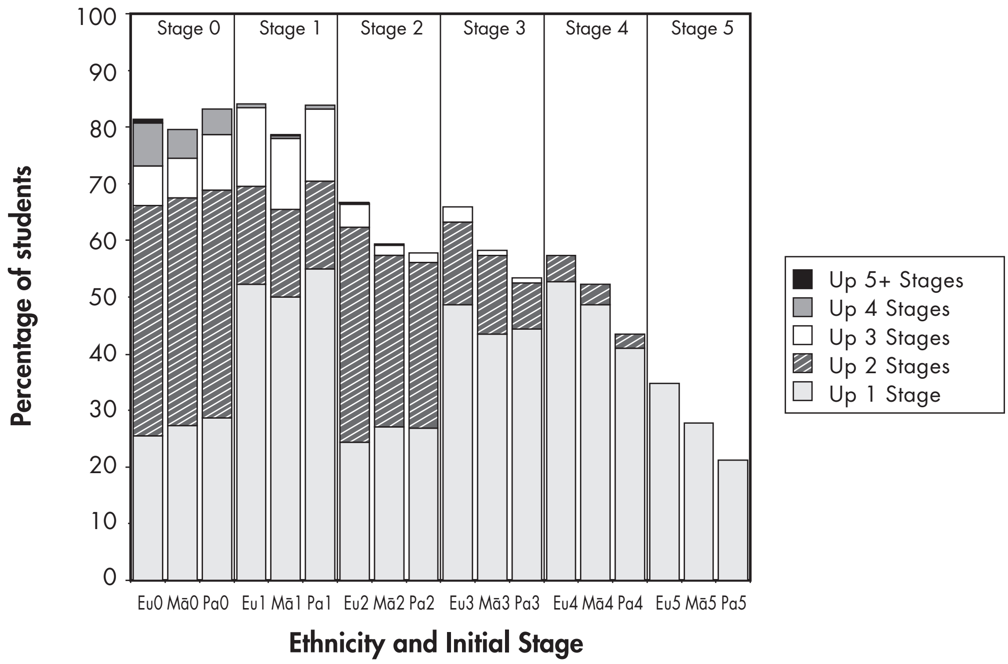

Figure 3. Percentage of students who progressed to a higher Framework stage on addition and subtraction as a function of ethnicity and initial stage (2002)

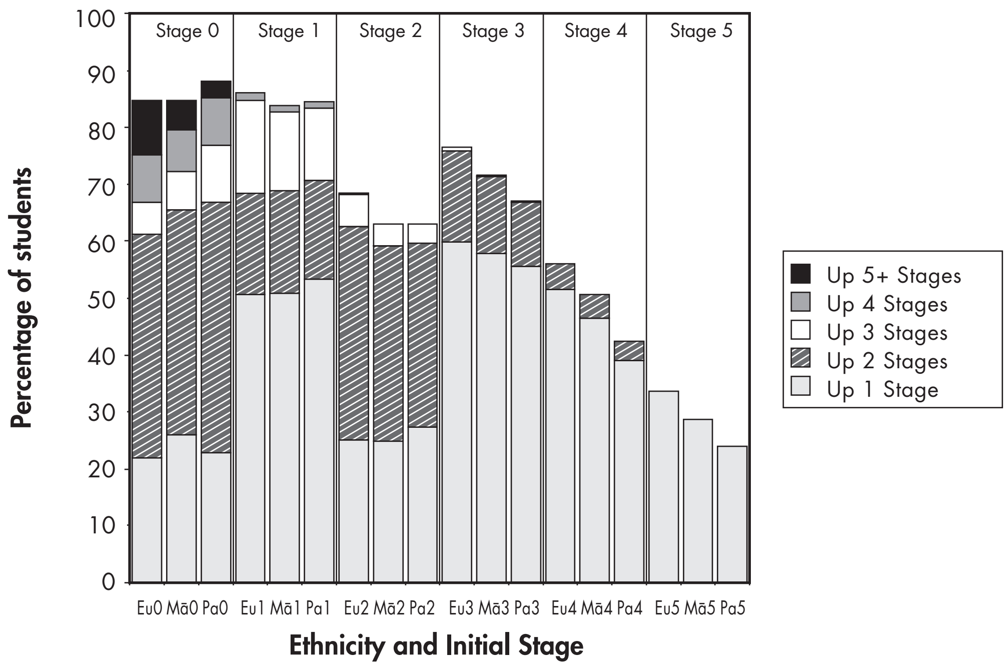

Figure 4. Percentage of students who progressed to a higher Framework stage on addition and subtraction as a function of ethnicity and initial stage (2003)

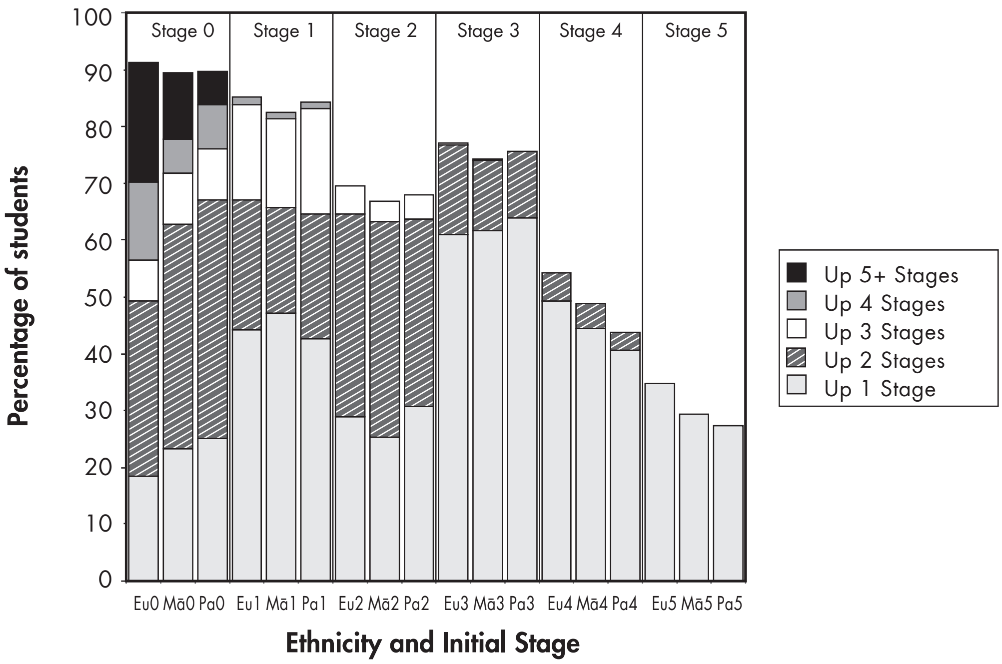

Figure 5. Percentage of students who progressed to a higher Framework stage on addition and subtraction as a function of ethnicity and initial stage (2004)

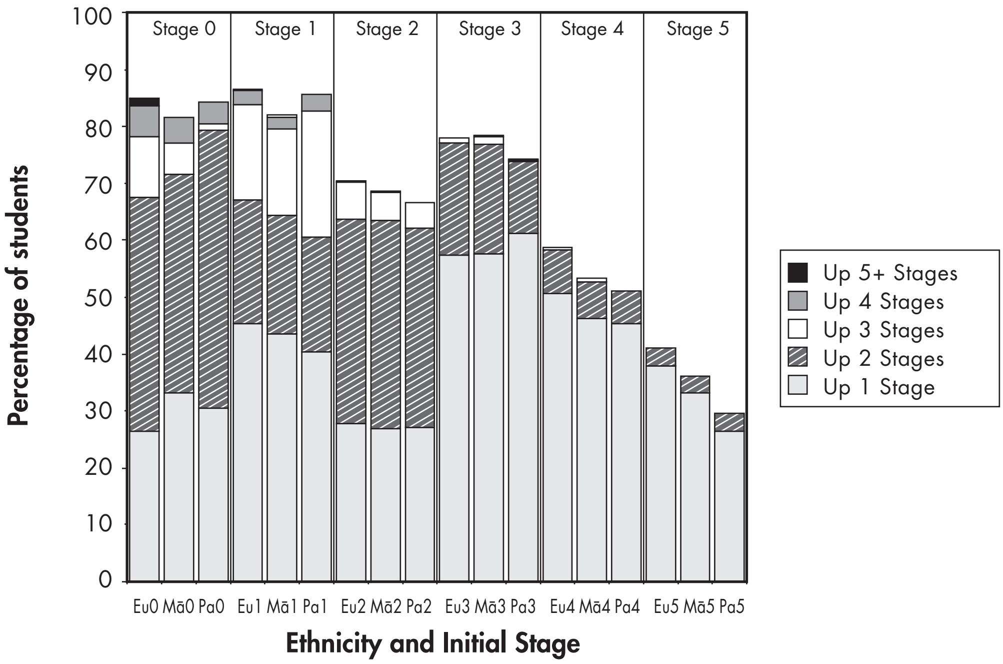

Figure 6. Percentage of students who progressed to a higher Framework stage on addition and subtraction as a function of ethnicity and initial stage (2005)

In some cases, Pasifika students made greater progress than European or Māori students. Approximately two-third of students who began at stage 2, counting from one with materials, or stage 3, counting from one with imaging, moved to a higher Framework stage. Progress was better for those who started at stage 3 than for those who started at stage 2, despite the fact that stage 3 students could progress only three stages at the most, whereas those at stage 2 could potentially improve four stages. This suggests that, once students understand how to use counting to work out the total when two collections are joined, they make rapid progress through at least stages 2 and 3. In the earlier years of the project (2002 & 2003), European students made substantially better progress than Māori and Pasifika students. However, in more recent years (2004 and 2005), the differences in progress between the three ethnic groups have substantially reduced.

At stage 4, advanced counting, and stage 5, early additive part–whole, European students tended to make greater progress than Māori students, who in turn made more progress than Pasifika students. However, there is a steady pattern of improvement over time for both Māori and Pasifika students. For example, for Māori students who began the project at stage 5, the percentages moving to a higher Framework stage (that is, stage 6, advanced additive part–whole) have increased from 27.8% in 2002 to 36.2% in 2005. The corresponding figures for Pasifika students were 21.3% in 2002 to 29.7% in 2005. Patterns of progress for European students have tended to remain fairly stable (34.7%, 33.7%, 34.8% for 2002, 2003, and 2004 respectively), apart from 2005, when there was a sudden increase to 41.1%. Similarly, for those students who started at stage 4, the proportion going up two stages to stage 6 has increased steadily over the years between 2002 and 2005. For example, the percentages of Māori students at stage 4 who went up to stage 6 has improved from 3.7% in 2002 to 4.2%, 4.6%, and 7.0% in 2003, 2004, and 2005 respectively. The corresponding percentages for Pasifika were 2.5%, 3.2%, 3.3%, and 5.9%. The percentages of Pasifika students at stage 3 who went up to a part–whole stage (either stage 5, early additive part–whole, or stage 6, advanced additive part–whole) increased from 9.0% in 2002 to 11.4%, 11.8%, and 13.0% in 2003, 2004, and 2005 respectively. Although these changes are quite small, they could be interpreted as indicating that the project is becoming increasingly effective in enhancing the mathematical proficiency of Māori and Pasifika students. Alternatively, improvements for these two groups could be explained by the greater proportions of Māori and Pasifika students in high-decile schools in more recent years.

Appendix F shows the final Framework stage achieved as a function of initial stage for European, Māori, and Pasifika students and for students in high-decile and low-decile schools. This table allows comparison for a particular group across time (horizontal) and comparison between groups who started at particular stages in a particular year (vertical).

It is important to note here that it is difficult to examine patterns of progress on the multiplication and division and proportion and ratio domains because so many students were not given tasks from these domains, particularly at the time of the initial assessment. Hence, with no initial data, there is no baseline from which to examine progress. This applies to all students assessed on Form A of NumPA, which does not include multiplication and division or proportion and ratio tasks.

Narrowing the achievement gap

In order to investigate the extent to which the NDP narrowed the gap in mathematics achievement between European students and Māori and Pasifika groups, effect sizes were calculated for the differences between European and Māori. European and Pasifika, and students in high-decile and low-decile schools for 2002 to 2005. Because of the problems with the Framework stages not constituting an interval scale, separate effect sizes were calculated for students who began the projects at each initial stage. The median effect size was then used as an indicator of the pattern overall (see Appendix G). Table 3 presents the median effect sizes for each comparison between 2002 and 2005. Analysis shows that the median effect size for differences between European and Māori students reduced from 0.14 in 2002 to 0.09 in 2005. The corresponding values for Pasifika students reduced from 0.26 in 2002 to 0.17 in 2005. Differences between students in the high-decile and low-decile bands reduced from 0.21 in 2002 to 0.13 in 2005.

The effect-size analysis was limited to the addition and subtraction domain because that is the only strategy domain on which there is complete data. The domains of multiplication and division and proportion and ratio are problematic for two reasons: not all students were given the opportunity to do tasks from these other two domains and some students were only given a chance to do tasks from these two domains at the end of the year but not the beginning, so there is no data about their initial stage. Hence, this data is not included in this paper.

Table 3
Median Effect Sizes for Comparisons of Progress on Addition and Subtraction for Students Who Started at Identical Framework Stages (European vs Māori and Pasifika, high-decile vs low-decile) 2002–2005

Year	European–Māori	European–Pasifika	High–Low decile
2002	0.14	0.26	0.21
2003	0.13	0.21	0.22
2004	0.10	0.16	0.14
2005	0.09	0.17	0.13

The changes in median effect size for the three comparisons on addition and subtraction are shown in Figure 7.

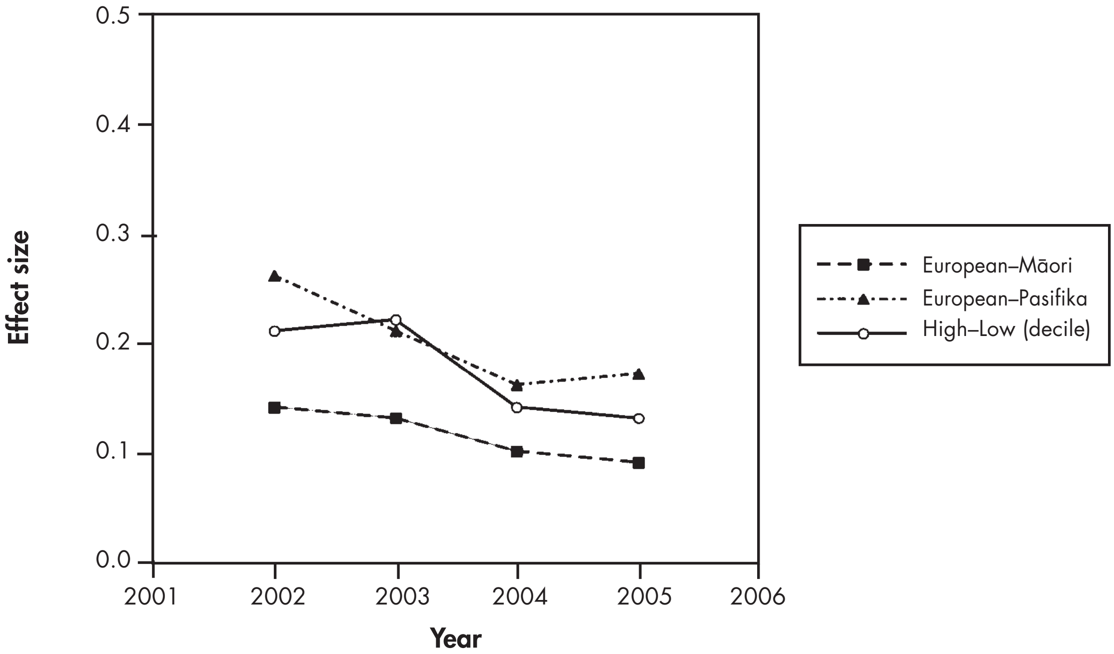

Figure 7. Changes in median effect size for differences in progress on addition and subtraction as a function of ethnicity (European–Māori and European–Pasifika) and decile (high–low)

Figure 7 shows that the median effect size for differences between European and Māori, European and Pasifika, and students from high-decile and low-decile schools reduced over the years between 2002 and 2005, although there is some levelling out for the European–Pasifika comparison. It is difficult to know whether this reduction in effect size is the result of changes in the composition of the cohort, with a greater percentage of students coming from higher decile schools in later years, or whether this reflects the improved effectiveness of the NDP initiative itself.

Interrelationships between multiplicative thinking and other domains

Despite the best efforts of numeracy facilitators, the percentage of students who reached stage 7, advanced multiplicative part–whole, was disappointing (a third of year 8 students). An analysis of the interrelationships between various domains was completed using final interview data (in order to maximise the reliability of the assessment data), and the data was aggregated across the years 2002 to 2005. Across those four years of the project, more than 36 000 year 0–8 students reached stage 7 by the end of the year. The analysis was completed for each of these stage 7 groups (referred to as stage 7 M/D to distinguish them from students who reached stage 7 on other domains), to see how these students performed on other key domains of the Number Framework compared with those who hadn't yet reached stage 7 (see Appendix H). The data was then aggregated by averaging the percentages across the four years 2002–2005 (see Appendix H and Appendix I). Marked differences between stage 7 M/D students and those below stage 7 on the multiplication and division domain were found on proportion and ratio, fractions, and basic facts. As Appendix H shows, 71% of stage 7 M/D students had reached at least stage 7 on proportion and ratio, compared with only 14% of stage 6 M/D and 2% of stage 5 M/D students. Stage 7 on proportion and ratio can be achieved by working out that, if 12 is $\frac{2}{3}$ of a number, the number must be 18. In order to do this task successfully, students need to understand how to divide the 12 in half to work out how much one-third is, then multiply this number by 3 to work out what three-thirds are altogether.

A similar difference was found on fractions, where 53% of stage 7 M/D students were able to recognise equivalent fractions ($\frac{2}{3}$ and $\frac{6}{9}$) while ordering fractions with unlike numerators and denominators. Only 9% of stage 6 M/D students and only 1% of stage 5 M/D students could do this. On basic facts, almost 68% of stage 7 M/D students could recall division facts and some could also recall common factors and multiples, compared to only 20% of stage 6 M/D students and 4% of stage 5 M/D students.

Appendix I shows the percentage of students at each of the upper levels on the proportion and ratio domain who were at particular levels on other domains. Only 65% of students who were at stage 7, advanced multiplicative, on proportion and ratio were at stage 7 on multiplication and division, suggesting that the fraction problem ("If 12 is $\frac{2}{3}$ of a number, what is the number?") was an easier task than the multiplication and division problems used to assess stage 7 on the multiplication and division domain. This is borne out by the fact the percentages of students at stage 7 P/R were lower on fractions, place value, and basic facts than those found for students at stage 7 M/D.

The results suggest a close connection between advanced multiplicative thinking and fractions and division. It may be that teachers have focused on multiplication without also building a strong understanding of division. This could explain why the percentage of students reaching stage 7, advanced multiplicative, is only about 34%. Working with fractions requires a reasonable understanding of division also, and this may be the reason that fraction knowledge and fraction strategies (as seen at stage 7 on the proportion and ratio domain) seem to be related to multiplication and division. Further work with division may help students with their understanding of fractions. It is also possible that further work with fractions, particularly encouraging students to *understand* fractions rather than engage in mindless manipulation of digits, might also foster the development of multiplicative reasoning.

Introduction of new stage 7 A/S and stage 8 M/D tasks in 2005

Appendix J shows the performance of students who are at stage 7 at the end of the year on other domains of the Number Framework that were assessed at the same time. It is interesting to compare the students at stage 7 A/S with those at stage 7+ M/D and stage 7+ P/R. The largest number of students (6612) was at stage 7 or higher on multiplication and division (with whole numbers), and the smallest number (2742) was at stage 7 on addition and subtraction (addition and subtraction with decimals and fractions). The reason for the smaller number at stage 7 on addition and subtraction may have been that many students were not familiar with decimals. There were almost as many students at stage 7 or higher on proportion and ratio (6250) as at stage 7 or higher on multiplication and division (6612).

Of the students who were assessed as being at stage 7, whether it was on addition and subtraction, multiplication and division, or proportion and ratio, virtually all were at stage 5 or above on all of the other strategy domains. More than 90% of them were at stage 6 or higher on all of the other strategy domains. It is interesting to consider what might have kept those students at stage 6 rather than being rated at stage 7. On the proportion and ratio tasks, the method used to solve $\frac{3}{4}$ of 28 was crucial in determining whether the student was at stage 6 or stage 7. It was the use of addition and multiplication (for example, finding $\frac{1}{4}$ of 28 by going up in 4s from 20, as in "$\frac{1}{4}$ of 20 is 5, so $\frac{1}{4}$ of 24 is 6, so $\frac{1}{4}$ of 28 is 7", then multiplying by 3) rather than division and multiplication ("divide 28 by 4 then multiply that number by 3") that led to a stage 6 judgment. Included in this group were students who used successive halving to find the quarter ("$\frac{1}{2}$ of 28 is 14, and $\frac{1}{2}$ of 14 is 7"), and then multiplied by 3. To be at stage 7, students also needed to solve the problem that involved using the information that $\frac{2}{3}$ of a number is 12 to work out what the number is. On the multiplication and division tasks, 25.3% of the stage 7 P/R students were rated at stage 6 rather than stage 7. This would have been because they were not able to use at least two different advanced mental strategies to solve the 24 x 6 task and the 72 ÷ 4 task. It would be very interesting to know which of the two tasks the students had the most problems with. There are part–whole strategies that can be used to solve the multiplication task that are very similar to the written algorithm for multiplication (for example, first multiply 6 by 4, then multiply 6 by 20 and add the two products). The division task is not as closely connected to the "long division" algorithm (for example, build up to 72 in parts such as 4 x 10 and 4 x 8, use successive halving as in "$\frac{1}{2}$ of 72 is 36 and $\frac{1}{2}$ of 36 is 18", or a compensation strategy like "4 x 20 is 80; 80 minus 2 x 4 is 72").

Almost 88% of stage 7 A/S students were at stage 7 or higher on multiplication and division (87.6% could do multiplication and division with whole numbers, and 40.2% could do so with decimals), whereas 80% of them were at stage 7 or higher on proportion and ratio (80.2% worked out what a number was if $\frac{2}{3}$ of it was 12, and 27.9% could solve the two ratio problems – wool and mittens and the percentage of boys in Ana's class). It was interesting to note that 68% of the 6612 students at stage 7 or higher on multiplication and division were at stage 7 or higher on proportion and ratio (n = 4489). This corresponded almost exactly to the 72% of the 6250 students at stage 7 or higher on proportion and ratio who were at stage 7 or higher on multiplication and division (n = 4488). These figures indicate how complicated the relationships are between the various tasks, levels, and domains.

More of the stage 7 A/S students were at stage 7 on fractions (73.9%) compared with the stage 7+ M/D students (53.1%) and stage 7+ P/R students (52.0%). This reflects the fact that the stage 7 A/S tasks were the hardest and the stage 7+ M/D tasks were the easiest. Stage 7 A/S students were also better at basic facts (80.7% were at stage 7) compared to the stage 7+ M/D students

(66.4%) and the stage 7+ P/R students (66.5%). Hence, it was possible to order the tasks used to assess stage 7 by difficulty level. The easiest task seemed to be the multiplication and division tasks with whole numbers (24 x 6 = □ and 72 ÷ 4 = □). However, it may be that the size of the numbers also had an impact on the difficulty level of the task, but there were not enough different tasks to allow comparisons to be made so that that issue could be explored. The fractions task used to assess stage 7 on proportion and ratio ("If 12 is $\frac{2}{3}$ of a number, what is the number?") was of medium difficulty. The hardest tasks by far appeared to be the addition and subtraction tasks with fractional number ($2 - (\frac{3}{4} + \frac{7}{8}) = \square$ and 5.3 – 2.89 = □). It is difficult to know whether it was the fractions task or the decimals task that was the harder of the two, but it seems likely that decimals presented the greatest difficulty.

Appendix K shows the performance of students on stage 8 M/D and stage 8 P/R on other domains on the Number Framework assessed at the same time. It was possible to compare the performance of students at stage 8 M/D with those at stage 8 P/R. The two ratio tasks (wool and mittens, 10:15 = □:6, and the percentage of boys in Ana's class, 21:35 = □:100) were slightly harder (n = 1008) than division with decimals and whole numbers (2.4 ÷ 0.15 = □ and 26 ÷ 8 = □) (n = 1444). Similar proportions of students at stage 8 M/D were at stage 7 on addition and subtraction compared with those at stage 8 P/R (76.1% vs 75.7%). Their performance on fractions was also similar (85.1% of stage 8 M/D and 90.0% of stage 8 P/R were at stage 7+). Likewise, similar proportions were at stage 7+ on basic facts (88.6% and 89.9% of stage 8 M/D and stage 8 P/R respectively).

It was interesting to look at how the students rated at stage 7 on the various operational strategy domains did on fractional number – one of the knowledge domains. Virtually all of the stage 7 students were able to order unit fractions, so it seems they did understand the idea that the larger the denominator, the smaller the fraction. A notable proportion of those at stage 7+ M/D (20.5%) and stage 7+ P/R (18.0%) were unable to coordinate numerators and denominators (as in recognising that $\frac{8}{6}$ is the same as $1\frac{2}{6}$ or $1\frac{1}{3}$), whereas only 8.5% of stage 7 A/S students were in that category. A quarter of stage 7+ M/D and P/R students were able to coordinate numerators and denominators but did not recognise the equivalence of $\frac{2}{3}$ and $\frac{6}{9}$ when ordering fractions with different denominators. Only 16.1% of stage 7 A/S students were in that category.

The differences in performance on identical stages on different domains raise some important issues about what it means to be a particular Framework stage on a particular domain. It seems likely also that the difficulty level of the tasks is affected by the size of the numbers as well as the kind of operation and that caution is needed about drawing firm conclusions about Framework stages on the basis of a student's performance on just one or two tasks.

The difficulties with fractions have been well documented in the literature (for example, Behr et al., 1983; Charalambous & Pitta-Pantazi, 2005; Verschaffel, Greer, & Torbeyns, 2006). According to these writers, fractions are very difficult to teach and to learn because the concept of fractions consists of several sub-constructs and understanding fractions requires an understanding of each of the sub-constructs as well as the ways in which the sub-constructs are connected. Underpinning each of the four sub-constructs of fraction understanding is the idea of part–whole comparison or partitioning. Unlike partitioning for addition and subtraction, which can be of unequal parts, partitioning for multiplication, division, and fractions must be of equal-sized parts. According to Behr et al. (p. 93), many student difficulties in algebra can be traced back to an incomplete understanding of earlier fraction ideas.

The sub-constructs include *ratio* (the idea of relative magnitude, necessary for understanding ideas about proportion and equivalence, as in $\frac{3}{4}$ is the same as $\frac{6}{8}$), *operator* (necessary for multiplication of fractions, as in $\frac{3}{4}$ of 10 metres), *quotient* (necessary for problem-solving, as in $\frac{1}{4}$ of 20 means the division of 20 by 4), and *measure* (necessary for addition of fractions, as in $\frac{3}{4}$ is the same as $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$). Another way of looking at fractions is to see them as both a process (for example, $\frac{3}{4}$ involves division of 3 by 4) and a product ($\frac{3}{4}$ is the result of dividing 3 by 4) (see Verschaffel et al., 2006). Students can easily confuse fractions as *numbers* (as in $\frac{3}{4}$ of a metre) and fractions as *operations* (as in $\frac{3}{4}$ of 10 metres). Fractions appear in two places in the Number Framework: as part of the operational strategy domain of proportion and ratio as well as in the knowledge domain of fractional number. A lot more work is needed to investigate how the various aspects of fractional number fit within the Number Framework. More work also needs to be done on ways of helping students understand fractional number better.

Putting effect sizes into perspective

The NDP was initially designed to raise mathematics achievement for all students. The various projects seem to have been fairly successful at doing this, despite some concerns about how far on the Framework students are progressing on domains such as multiplication and division and proportion and ratio. However, with such a large amount of data, the opportunity to look at patterns of performance and progress as a function of variables such as ethnicity and school decile is too good to let pass. These analyses have shown that, although all students made progress, the gaps in the achievement of European and Asian students compared with Māori and Pasifika students continued to exist. In general, the achievement gaps between groups seem to have reduced slightly over the years, but this may have been due to changes in the composition of the sample. It is important to see these differences in the wider perspective. When the effect sizes for these differences are compared with corresponding differences found on other large-scale studies of mathematics achievement, it becomes clear that the effect sizes for the differences on the NDP are substantially smaller than those found in the other studies. For example, on the TIMSS study, effect sizes are about three-quarters of a standard deviation for the European–Māori comparison and about one standard deviation for the European–Pasifika comparison. Based on Cohen's classification (see Fan, 2001), these are "large" effect sizes (that is, about 0.8 or more), whereas those on the NDP are mostly about 0.2, which is considered "small" on Cohen's classification. The effect sizes for the PISA study are smaller than those on TIMSS (0.38 and 0.53), but this study differs in an important way from the others. The PISA study looked at students aged between 15 years 3 months and 16 years 2 months. There is ample evidence from educational statistics that many Māori and some Pasifika students have left school by the age of 15 years, or even earlier. Hence, the comparison does not include a full cohort of students. It is often those students who are not succeeding at secondary school who decide to leave early. Hence, the PISA results do not include the full range of mathematics achievement levels, and this will inevitably have somewhat reduced the magnitude of effect sizes. Table 4 shows the effect sizes for each study/group, and indicates that the disparities in achievement between ethnic groups are not as great as was previously thought.

Table 4
Effect Sizes for the Comparison of European vs Māori and European vs Pasifika on TIMSS, PISA, and the NDP

		Comparison	
		European–Māori	European–Pasifika
TIMSS	Yr 5 1994	0.73	0.95
TIMSS	Yr 5 1998	0.65	0.97
TIMSS	Yr 9 1994	0.71	1.15
TIMSS	Yr 9 1998	0.66	0.96
PISA	15 yrs 2000	0.38	0.53
NDP	Initial 2002	0.19	0.37
NDP	Initial 2003	0.17	0.35
NDP	Initial 2004	0.11	0.21
NDP	Initial 2005	0.22	0.23

Conclusions

As the amount of data collected in the course of the NDP grows, many more questions are raised about the nature of the Number Framework and the interrelationships among the domains. Not only does the Number Framework provide teachers with a valuable structure to help organise their assessment and teaching, it is also a valid area of enquiry in its own right. It is hoped that, as the emphasis of the NDP shifts away from the primary level towards the intermediate and secondary levels, this data-gathering process continues to provide researchers with valuable information that can help throw further light on the complexities of the Framework itself.

The data indicate that there are important issues to investigate further with respect to multiplicative thinking and understanding of fractional numbers. There are now three different sets of tasks to explore multiplicative thinking: addition and subtraction, multiplication and division, and proportion and ratio tasks. Students' performance varied according to which of these was being assessed. There is clearly an important link between understanding division and fractional number, and this link needs to be explored further.

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Numeracy Development Project Longitudinal Study: Patterns of Achievement

Gill Thomas
Maths Technology Ltd
<gill@nzmaths.co.nz>

Andrew Tagg
Maths Technology Ltd
<andrew@nzmaths.co.nz>

The central aim of the Numeracy Development Projects (NDP) is to improve student performance in mathematics through improving the professional capability of teachers. The aim of the NDP Longitudinal Study is to investigate the longer term impact of the NDP on student achievement in number and in mathematics more generally. This paper reports on the mathematics achievement of students in 26 schools that participated in the NDP prior to 2004. Not surprisingly, the findings suggest that the project has had the strongest impact on the students' performance on number items that are directly related to the NDP. The achievement of students as measured against the Number Framework indicates that, at most year levels, strategy level attainment has increased over time.

Background

From their outset, the Numeracy Development Projects (NDP) have been conceived as a dynamic, evidence-based initiative (Ministry of Education, 2006). The Numeracy Projects have evolved through a series of phases that can be summarised as the formulation phase (pre-2000), the initiation phase (2000–2001), the implementation phase (2002–2006), and the planned sustainability or maintenance phase (post–2007). The focus on the professional capability of mathematics teachers "provided the order and logic for the formulation phase" (Higgins, Parsons, & Hyland, 2003, p. 162). In the formulation phase, the Ministry of Education identified the key issues facing mathematics achievement as:

> developing teachers' pedagogical content knowledge; improving teaching quality and confidence; providing resources to support mathematics teaching and learning; making research information more readily available and accessible to teachers; and emphasising the importance of mathematics education prior to school entry. (Higgins et al., 2003, p. 162)

The initiation phase began in 2000 with a national pilot of Count Me In Too for years 1–3 (Thomas & Ward, 2001), and a year 4–6 exploratory study (Higgins, 2001). The findings from these two projects informed the development of the NDP, with the introduction in 2001 of the Early Numeracy Project, the Advanced Numeracy Project, and the Exploratory Study (years 7–10).

The major implementation of the NDP has occurred since 2002, with over 17 000 teachers and 460 000 students estimated to have participated by the end of 2005 (Parsons, 2005). The projects are nationally co-ordinated, with their facilitation contracted to the six major teacher education providers in New Zealand. The Ministry of Education, through a series of evaluations, monitors the quality of the implementation and, by the end of 2005, has published a total of 25 reports and papers on the various components of the NDP.[1]

While the focus from 2002 to the present has been on the widespread implementation of the NDP, a number of strategies were established in 2004 to address issues of sustainability. These included increased support for lead teachers, providing training opportunities for teachers new to schools that have already participated in the project, and providing online resources to all schools and teachers.

[1] See www.nzmaths.co.nz/Numeracy/References/reports.aspx for the evaluation reports and papers published by the Ministry of Education.

By 2007, almost all teachers of students in years 1–8 will have had the opportunity to participate in the NDP and the focus of the NDP facilitation contracts will shift to consolidation and maintenance (Parsons, 2005). In this phase, the challenge will be to maintain the strategic focus, quality, and momentum of the NDP while shifting the ownership for ongoing development to the school level.

The NDP Longitudinal Study, which began in 2002, has focused on examining the impact of the projects on the mathematics achievement of students in the years following the implementation of the NDP in the schools involved in the study. The information obtained from the longitudinal schools on the achievement of their students as measured against the Number Framework has helped inform expectations of achievement and progress over time (Thomas & Tagg, 2004, 2005a; Thomas, Tagg, & Ward, 2003). The performance of students in the longitudinal schools on items from the Trends in Mathematics and Science Study (TIMSS) suggests that the NDP has had a positive impact on mathematics achievement at years 4 and 5 and to a lesser extent at year 8 (Thomas & Tagg, 2004, 2005b).

The 2005 Longitudinal Study further investigated the mathematics performance of selected year groups of students and continued to track the achievement of all students as measured against the Number Framework. This paper first reports on the performance of year 4, 5, 6, and 8 students on items from TIMSS, the National Education Monitoring Project (NEMP), and Assessment Tools for Teaching and Learning (asTTle). The second section of the paper reports on the performance on the Number Framework of students at all year levels in 20 of the 26 schools participating in the 2005 Longitudinal Study.

Mathematics Achievement

Sample and Methodology

The aim of the mathematics achievement component of the NDP Longitudinal Study is to investigate the longer term impact of the NDP on student achievement in mathematics more generally.

The Longitudinal Study began in 2002 with the participation of 20 schools that first implemented the NDP in either 2000 or 2001. Five of the original 20 schools withdrew from the Longitudinal Study at the start of 2004, but the total number of schools in the Longitudinal Study was increased to 31 through the inclusion of 16 schools that first participated in the NDP in 2002. Sixteen of these 31 schools continued to participate in 2005, and 10 new schools were added to the sample. Each year, new schools are randomly selected from a list of schools that completed NDP training in the previous years. The list is stratified by decile to ensure that the sample in the Longitudinal Study closely approximates the national sample and has similar numbers of students in years 1–8. Four of the schools added in 2005 were intermediate schools, to increase the numbers of year 7 and 8 students involved.

Table 1 shows the breakdown of students by year level and decile band. The low-decile band includes decile 1–3 schools, the medium-decile band includes decile 4–7 schools, and the high-decile band includes decile 8–10 schools. As illustrated by Table 1, there is a disproportionate proportion of year 8 students from medium-decile schools in the sample because one of the intermediate schools selected had over 1 000 students.

Table 1
Analysis of Students by Year Level and School Decile Band

Year	Low	Medium	High	Total
4	251	381	310	942
5	270	377	320	967
6	299	405	313	1017
8	298	739	157	1194
Total	1118	1902	1100	4120

Tests for each of the four year levels were developed from items that had previously been used to assess the mathematics achievement of New Zealand students. There were four sources for these items: TIMSS 1995, TIMSS 2003, the 2001 NEMP mathematics assessment, and asTTle. Items from TIMSS 1995 and the NEMP assessment were used for the tests for years 4 and 8. The year 5 test was comprised of items solely from TIMSS 2003, and the only source of items for the year 6 test was asTTle. Table 2 shows the source of the 24 items contained in each test. The items were selected according to two criteria. Firstly, items were selected to ensure all strands of the New Zealand mathematics curriculum were covered (see Table 3). Secondly, the items were selected so that the average score in each test would be 50%, based on the percentage of New Zealand students answering each item correctly in the source assessments. The tests were piloted in a school that was not participating in the Longitudinal Study to check that they took approximately 40 minutes to administer.

Table 2
Source of Items in the Tests

Source	Year 4	Year 5	Year 6	Year 8
TIMSS 1995	14			16
TIMSS 2003		24		
NEMP 2001	10			8
asTTle			24	
	24	24	24	24

Table 3
Analysis of Strand of Items in the Tests

TIMSS Content Category	Year 4	Year 5	Year 6	Year 8
Algebra	4	3	1	4
Geometry	4	4	4	4
Measurement	4	5	2	4
Number	8	8	10	8
Statistics	4	4	7	4
	24	24	24	24

Test scripts were sent to each of the participating schools in July. The classroom teachers administered the tests, following instructions adapted from those used with TIMSS. The tests were sent back to the researchers for marking during August. Once the scripts had been marked, a report was compiled for each of the participating schools. This report included details on the item responses of each student and their overall test score. The schools' average performance by item and overall test was compared with the performance of same-age peers in the assessments from which the items were obtained.

Results

All reporting of results in this section is based on the average percentage of items answered correctly by students in the stated sub-groups. Longitudinal results refer to the 2005 Longitudinal Study unless stated otherwise. For each item, the 95% confidence limits for the difference in mean proportion between the longitudinal sample and the source sample were calculated. This is the criteria used to define significant differences in the results reported below.

Table 4 shows the performance of longitudinal students on the source assessments over the last three years. The first time a year 6 test was administered was in 2005, so there is no comparative data for this year level. The year 4 longitudinal students have performed consistently significantly better overall than the New Zealand students in the source assessments. The year 5 students had stronger performances in 2003 and 2004 than in 2005, while the year 8 students have continued to perform at a similar level to the year 8 students from the source assessments.

Table 4
Average Score by Year Level and Year of Testing

	Year 4	Year 5	Year 6	Year 8
Longitudinal 2003	56%*	59%*	N/A	52%
Longitudinal 2004	56%*	58%*	N/A	53%
Longitudinal 2005	57%*	54%*	47%	51%
Source average	50%	50%	50%	50%

* $p < 0.05$

Low-decile students in years 5, 6, and 8 performed not only lower than the medium- and high-decile schools, but also slightly lower than the 50% average achieved by New Zealand students in the source assessment (see Table 5). In year 4, the low-decile students scored lower than medium- and high-decile students but similarly to students in the source assessments. There is no decile information available on the source assessments, so no comparisons can be made by decile. The differences between high- and medium-decile students were smaller than those between low- and medium-decile students at all year levels.

Table 5
Average Score by Year and Decile Level

	Year 4	Year 5	Year 6	Year 8
Low decile (1–3)	46%*	39%*	36%*	37%*
Medium decile (4–7)	58%*	58%*	49%*	56%
High decile (8–10)	65%*	62%*	54%*	56%

* $p < 0.05$

Table 6 shows the numbers of number-related items on which longitudinal students performed significantly better or significantly worse than students in the source assessments. In each instance, the numbers in brackets represent the numbers of items where the difference was greater than 10%. The results show that, at all year levels, the longitudinal students performed either better or similarly on 34 of the 38 NDP-related items and either similarly or worse on all 13 number items not directly related to the NDP. The only exception is in year 5, where students performed similarly on the one non-NDP number item. Number items classified as non-NDP included calculations presented in vertical form, calculations involving large numbers, and expressions involving inequalities. Longitudinal students performed either better or similarly on 39 of the 45 non-number (Other) related items. Students at all year levels performed significantly better on 23 out of the 38 NDP-related items and significantly worse on only four items.

Table 6
Performance on NDP, Non-NDP-related Number Items, and Other Items in Relation to Source Assessments

	Item type								
Year	NDP-related			Non-NDP (Number)			Other		
	Declined	Similar	Improved	Declined	Similar	Improved	Declined	Similar	Improved
4		2	7 (4)	3 (3)				5	7 (4)
5	1	6	4 (1)		1		2	7	3 (1)
6	2 (2)	2	6 (3)	2 (2)			4 (4)	4	4 (4)
8	1	1	6 (1)	7 (4)				3	6 (3)
Total	4	11	23	12	1	0	6	19	20

* $p < 0.05$

Performance Highlights

As reported in Table 6, longitudinal students performed significantly better on the majority of items related to NDP topics. These items included both number knowledge items and items involving operating with numbers.

In the year 4 test, the item with the largest improvement was question 12 (Figure 1), in which students were asked to identify the largest of four numbers. Of the longitudinal students, 79% answered correctly, compared with 45% of New Zealand students in TIMSS 1995 (NZ TIMSS 1995). It should be noted that the longitudinal students' performance is similar to that of students internationally in TIMSS 1995 (76%).

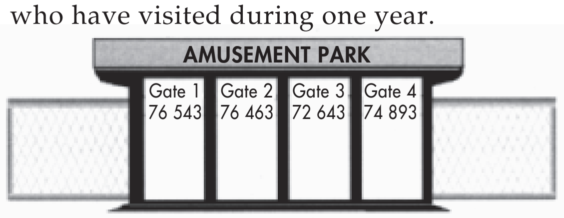
12. Which of these is the largest number?
    A. 2735
    B. 2537
    C. 2573
    D. 2753

	Percentage
Longitudinal	79
NZ TIMSS 1995	45
TIMSS 1995	76

Figure 1. Item 12

Similarly, question 13a in the year 6 test (Figure 2) required students to identify the smallest of four numbers, in this case, in the context of people passing through amusement park gates. Of the longitudinal students, 83% answered correctly, compared with 76% of New Zealand students in the asTTle norms.

13. These are the gateways to an amusement park. Each gate shows the number of people who have visited during one year.



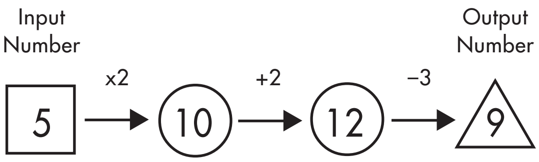
	Percentage
Longitudinal	83
asTTle	76

a. Which gate did the least number of people use? ___________

Figure 2. Item 13

Question 20 in the year 5 test, illustrated in Figure 3, was a number machine from which students were asked to identify the result of a series of operations on a number. Of the longitudinal students, 53% identified the correct response, compared with 45% of New Zealand students and 50% of international students in TIMSS 2003.

20. A number machine takes a number and operates on it. When the Input Number is 5, the Output Number is 9, as shown below.



	Percentage
Longitudinal	53
NZ TIMSS 2003	45
TIMSS 2003	50

When the Input Number is 7, which of these is the Output Number?

A. 11
B. 13
C. 14
D. 25

Figure 3. Item 20

Item 18 in the year 6 test (Figure 4) asked students to identify the temperature 9° hotter than negative 5°. Of the longitudinal students, 52% did so correctly, compared with 29% of students in the asTTle norms. It is encouraging to note that longitudinal students are able to operate with negative numbers in this context.

18. The temperature was –5°. It rose 9°.

What is the temperature now? ___________ °

	Percentage
Longitudinal	52
asTTle	29

Figure 4. Item 18

Longitudinal students performed particularly strongly on items related to fractions, an area in which students in New Zealand had performed poorly in TIMSS 1995 (Garden, 1997). Question 7 in the year 4 test asked the students to write a fraction that was larger than two-sevenths. Of the longitudinal students, 63% did so correctly, compared with 38% of New Zealand students and 41% of international students in TIMSS 1995. In 2004, 58% of year 4 longitudinal students answered this question correctly. Question 9 in the year 5 test required the students to find one-third of 600, with 44% of longitudinal students able to do so, compared with 34% of New Zealand and 49% of international students in TIMSS 2003. Question 6 in the year 8 test asked students to identify the largest of four fractions ($\frac{5}{6}$, $\frac{5}{7}$, $\frac{5}{8}$, $\frac{5}{9}$). Of the longitudinal students, 73% were able to do so, compared with 56% of students in NEMP 2001.

Performance Concerns

Items on which longitudinal students performed poorly included those too difficult to calculate mentally. It is a cause for concern that many students, particularly at the higher year levels, have no strategy for dealing with large or more complex calculations.

As shown in Figure 5, the numbers presented in question 18 in the year 4 test are too large and, with three columns adding to greater than 10, too complicated to be readily added using a mental strategy. Written forms are not introduced until the higher stages of the Number Framework, so typically from stage 6, few year 4 students would be expected to answer this question correctly. This result is consistent with the findings from the 2004 Longitudinal Study, in which 38% of year 4 longitudinal students correctly answered the same question (Thomas & Tagg, 2005b).

18. Add

$$\begin{array}{r} 6971 \\ +5291 \\ \hline \end{array}$$

A. 11 162
B. 12 162
C. 12 262
D. 1 211 162

	Percentage
Longitudinal	34
NZ TIMSS 1995	47
TIMSS 1995	67

Figure 5. Item 18

Question 15 in the year 6 test asked students to give the answer to the number sentence $5024.6 - 2975.8 = \square$. Of the longitudinal students, 15% answered correctly, compared with 39% of students in the asTTle norms. The numbers of digits in this sum are again too large to be solved mentally.

Three items in the year 8 test (Figures 6–8) required students to carry out number operations where the numbers were too large to be easily calculated mentally. While written forms do not appear until the later stages of the Number Framework, it would be hoped that by year 8, students would have been introduced to techniques for calculating sums of this type. The difference is made more significant by the fact that in the two items from TIMSS 1995, the New Zealand performance was already significantly lower than the international average.

2. Subtract 6000
−2369

A. 4369
B. 3742
C. 3631
D. 3531

	Percentage
Longitudinal	57
NZ TIMSS 1995	69
TIMSS 1995	86

Figure 6. Item 2

5. 3074
x 6

Answer _________

	Percentage
Longitudinal	49
NEMP 2001	65

Figure 7. Item 5

22. Subtract: 2.201 – 0.753 =
A. 1.448
B. 1.458
C. 1.548
D. 1.558

	Percentage
Longitudinal	44
NZ TIMSS 1995	56
TIMSS 1995	74

Figure 8. Item 22

Another area in which longitudinal students performed poorly in relation to students in the source assessments was in problems involving inequalities.

It was surprising that only 45% of year 4 longitudinal students were able to correctly identify that 442 is greater than 436 (Figure 9), as number sequence is a key focus of the NDP and 79% of year 4 students from the NDP 2003 were able to order numbers up to 1000 (Higgins, 2004). It can only be assumed that the students were unfamiliar with the symbols for greater than and less than.

21. Which statement is true?
A. 442 > 436
B. 352 > 759
C. 518 > 819
D. 883 < 794

	Percentage
Longitudinal	45
NEMP 2001	56

Figure 9. Item 21

In item 20 of the year 8 test (Figure 10), students were asked to find an inequality equivalent to one given. Of the longitudinal students, 28% were able to do so, compared with 42% of New Zealand students and 36% of international students in TIMSS 1995.

20. $\frac{x}{2} < 7$ is equivalent to

A. $x < \frac{7}{2}$

B. $x < 5$

C. $x < 14$

D. $x > 5$

E. $x > 14$

	Percentage
Longitudinal	28
NZ TIMSS 1995	42
TIMSS 1995	36

Figure 10. Item 20

Achievement on the Number Framework

Sample and Methodology

This component of the Longitudinal Study investigates the longer-term impact of the NDP on student achievement in number. More specifically, it examines the stages that year 1–8 students achieve on the strategy domains of the Number Framework over time. This information helps inform benchmarks or targets set by schools to monitor the achievement of their students.

The 26 schools participating in the 2005 Longitudinal Study were asked to enter the strategy stage results of their students on the online database by the end of November 2005. Not all schools were able to do this, so the deadline was extended and schools were given the option of providing their results to the researchers as an Excel spreadsheet. Twenty of the 26 participating schools provided the requested information for inclusion in the analysis reported in this section.

Table 7 shows a breakdown of students by year level and school decile band. The largest anomaly is at year 8, where there were no student results from high-decile schools. This is due to the only high-decile intermediate school in the sample failing to submit results.

Table 7
Analysis of Students by Year and School Decile Band

Year	Low	Medium	High	Total
1	319	347	346	1012
2	251	227	277	755
3	217	241	292	750
4	281	221	325	827
5	252	192	300	744
6	296	240	278	814
7	92	745	39	876
8	358	690		1048
Total	2066	2903	1857	6826

Some schools do not collect information on all three strategy domains, so a decision was made to report on strategy results by Global Strategy Stage (GloSS). A student's GloSS is usually determined by using the GloSS assessment forms[2] but for the purposes of this study, the GloSS result for each student was determined by taking the highest strategy stage reported for that student across the three strategy domains. The GloSS results of students from schools that participated in the NDP in 2005 were calculated in the same way.

Four hundred questionnaires were sent to all teachers in the 2005 longitudinal schools, with an estimated[3] 60% return rate. The questionnaire included items on numeracy assessment, student achievement, and numeracy targets.

Teachers' Perspectives on Numeracy Achievement

The large majority (94%) of teachers reported tracking the numeracy strategy stages of students in their class, with 70% indicating that their school had developed targets for student achievement related to the NDP. Table 11 summarises the percentage of students that the teachers believed were "on track" to achieve the school numeracy targets. As shown by Table 8, 70% of teachers believe that at least 70% of the students in their class were on track to achieve the school numeracy targets.

Table 8
Percentage of Students Perceived by Teachers as Achieving Numeracy Targets

Percentage of students achieving targets	Number of teachers	Percentage of teachers
Less than 50	11	9
50–59	11	9
60–69	15	12
70–79	23	19
80–89	26	22
90–100	35	29
Total	122	100

Student Numeracy Achievement

Figure 11 shows the GloSS result of longitudinal students over the four years (2002–2005) of the Longitudinal Study. As a point of comparison, results from the schools that first participated in the NDP in 2005 are included. It is pleasing to note that the general trend in years 2, 4, and 6 is one of improving achievement in the longitudinal schools, and that, at those three year levels, the students in the 2005 Longitudinal Study performed better than the students in the NDP in 2005.

The achievement of students in the longitudinal schools on the Number Framework provides evidence for describing the levels of performance that can reasonably be expected from students in schools in the years following their participation in the NDP. As illustrated by Figure 11, approximately 80% of year 2 longitudinal students reach at least stage 4 (advanced counting), approximately 70% of year 4 students reach at least stage 5 (early additive), and approximately 70% of year 6 students reach at least stage 6 (advanced additive).

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[2] Available from www.nzmaths.co.nz/Numeracy/Other%20material/GLOSS.aspx

[3] It is not possible to calculate an exact response rate because the number of questionnaires distributed was based on an estimated class size of 23 students, ensuring that more than sufficient questionnaires were sent to each school.

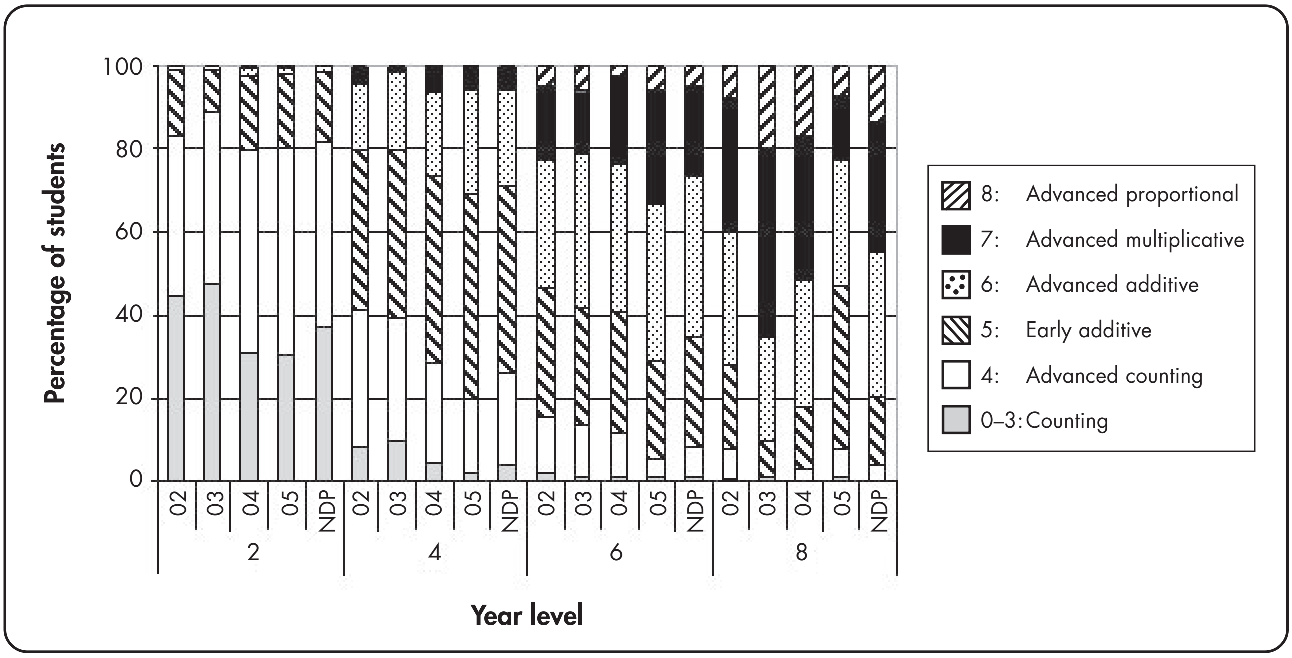

Figure 11. GloSS of Longitudinal Students 2002–2005 and NDP 2005 Students

The results of year 8 students have fluctuated over the years of the Longitudinal Study, with the highest achievement levels being in 2003 and the lowest in 2005. In 2005, the year 8 students in the 2005 NDP outperformed the year 8 longitudinal students. (Note that no strategy data was received for year 8 students in high-decile schools.)

Figure 12 shows the GloSS stage of year 8 students from the 2005 Longitudinal Study, categorised according to the year that their school first participated in the NDP. This figure illustrates the impact that the inclusion in the 2005 Longitudinal Study of one low-decile and one medium-decile intermediate school has had on the results for year 8 longitudinal students. While the sample of schools who first participated in 2003 performed significantly worse than the sample from the 2005 NDP, the sample of schools who first participated in 2000–2002 performed slightly better than the 2005 NDP students.

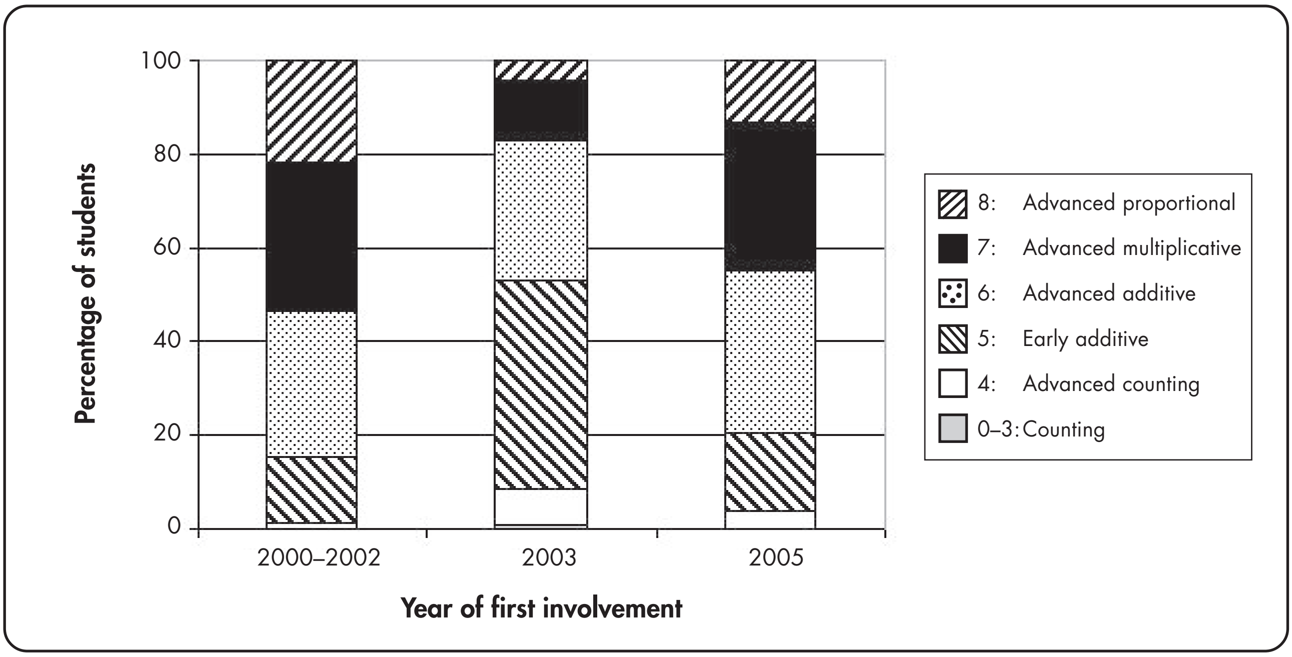

Figure 12. GloSS of year 8 students by first year of school's numeracy PD (no high-decile students in 2005)

Concluding Comment

The performance of students in the longitudinal schools on the mathematics tests is mixed. The year 4 longitudinal students performed on average 7% better than the New Zealand students in the source assessments. The year 5, 6, and 8 longitudinal students performed very similarly to the students in the source assessments, with year 6 longitudinal students rating 3% worse than those students. The year 5 and 8 longitudinal students' average overall test scores were not significantly higher than those of the New Zealand students in the source assessments, although they outperformed them on 8 and 12 of the 24 questions respectively.

The performance of the longitudinal students as measured against the Number Framework continues the trend of previous NDP longitudinal findings of improving achievement over time. The reasonably stable patterns of achievement for years 2, 4, and 6 students provide evidence for establishing benchmarks of numeracy achievement for the strategy domains.

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An Evaluation of Te Poutama Tau 2005

Tony Trinick
The University of Auckland
Faculty of Education
<t.trinick@auckland.ac.nz>

Brendan Stevenson
Massey University
Māori Studies
<b.s.stevenson@massey.ac.nz>

The Te Poutama Tau project is a professional development programme for teachers in Māori-medium education who are teaching numeracy. It is based around the Number Framework developed for New Zealand schools. This paper analyses student data from the Te Poutama Tau project in order to examine students' progress on the Number Framework in 2005. Areas where students performed well and areas where progress has not been as positive are highlighted. The patterns of performance and progress of students involved in the 2005 project are compared with those of 2003 and 2004. The results of this study will inform the future implementation and foci of Te Poutama Tau in Māori-medium schools.

Background

Teachers in Māori-medium mathematics have struggled for a number of years to interpret the learning outcomes of the Marautanga Pāngarau (Māori-medium curriculum statement) (Christensen, 2003; Trinick & Stephenson [*sic*], 2005). Essentially, the outcomes describe how and at what level students must demonstrate mathematical knowledge and skills. However, in a large number of examples, the outcomes do not make explicit the underpinning mathematical knowledge and concepts. The interpretation of the outcomes for planning purposes is thus left to the professional knowledge of the teacher and the associated learning and teaching resources. This is somewhat problematic. As noted in a number of writings, teacher mathematical content knowledge is lacking in a number of areas in Aotearoa (Christensen, 2003; McMurchy–Pilkington, 2004). Combined with the the challenges of the newly developed Māori language mathematics discourse, additional burdens are placed on teachers in Māori-medium education.

A part solution to the challenges faced by teachers in Māori-medium mathematics education has been the use of the Number Framework developed for New Zealand schools (Ministry of Education, 2006) and an associated professional development programme, Te Poutama Tau. Initiated as a pilot in 2002, Te Poutama Tau is a component of a key government initiative aimed at raising student achievement by building teacher capability in the teaching and learning of numeracy.

The Number Framework is divided into two key components – mātauranga (knowledge) and rautaki (strategies). The knowledge section describes for teachers the key items of knowledge that students need to learn. The strategy section describes the mental processes that students use to estimate answers and solve operational problems with numbers. The Framework provides a much more explicit detail of the key concepts and the progressions of learning for students than does the Marautanga Pāngarau. This provides significantly more support for teachers who are not greatly confident in the teaching and learning of mathematics in the medium of Māori.

Resources are also provided by the Ministry of Education in the medium of Māori to support the project and are closely linked to the Number Framework. These resources make explicit the knowledge and strategies that are required and provide examples of good models of linguistic structures for teachers to talk about and to use in teaching the concepts and activities.

Teachers from 27 schools taking part in Te Poutama Tau during 2005 provided data for this paper. Students were assessed individually at the beginning of the programme, using a diagnostic interview, and again at the end of the year.

The aim of this paper is to examine the following questions:

- What overall progress did students make on the Number Framework in 2005?
- In which areas of the Framework did students perform well in 2005 and in which areas did they perform poorly in 2005? Why is this so?
- How do patterns of performance and progress of students involved in the 2005 project compare with the 2003 and 2004 patterns?
- What areas of the Framework have they performed well or poorly in over the three years? Why is this so?

Methodology and Data Analysis

The results for each student, classroom, and school are entered on the national database (www.nzmaths.co.nz). The database shows the progress that students have made on the Number Framework between the initial and the final diagnostic interview. The time between the two interviews is about 20 weeks of teaching. Schools can access their own data on the national database to establish targets for planning and reporting purposes. Teachers can use the data to group students according to ability and use activities that will support students in both strategy and knowledge development. The following summaries of the data were restricted to only those students with both test and re-test results. In 2004, 1295 students completed both the initial and the final diagnostic interview and the te reo Māori proficiency component, and in 2005, there was complete data for 496 students.

Figure 1 reveals that there was some considerable difference in student numbers between the years 2004 and 2005, although there are insufficient numbers in years 8, 9, and 10 to make valid comparisons. The complete numeracy data sets for many other students were available but not used for the analysis because the te reo Māori component had not been entered. The matter has been resolved for 2006.

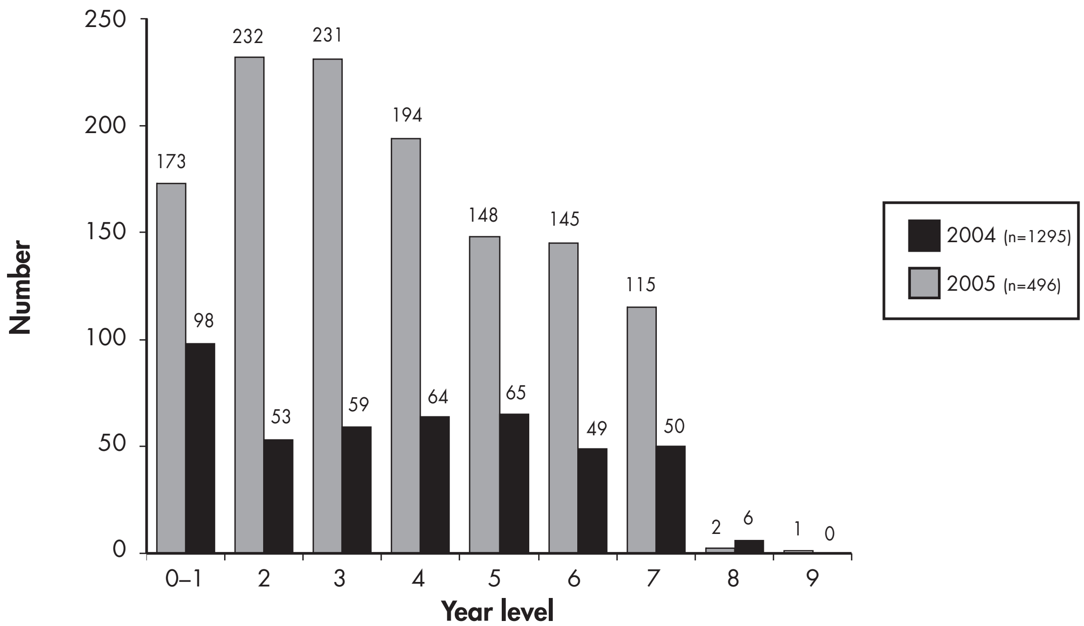


Figure 1. Distribution of Te Poutama Tau students across year levels

Overview of Student Progress 2005

Over the last three years (2003, 2004, and 2005), there was evidence of improved stage gains for addition, forward-number-word-sequence (FNWS), and backward-number-word-sequence (BNWS) knowledge. Despite improved gains made in 2003 and 2004 for decimal knowledge, stage gains were down in 2005. For multiplication, proportions, numeral-identification (NID), fractions, and grouping and place value (GPPV) knowledge, reduced gains from 2003 to 2004 were reversed in 2005, although the 2005 gains were not quite as large as those in 2004.

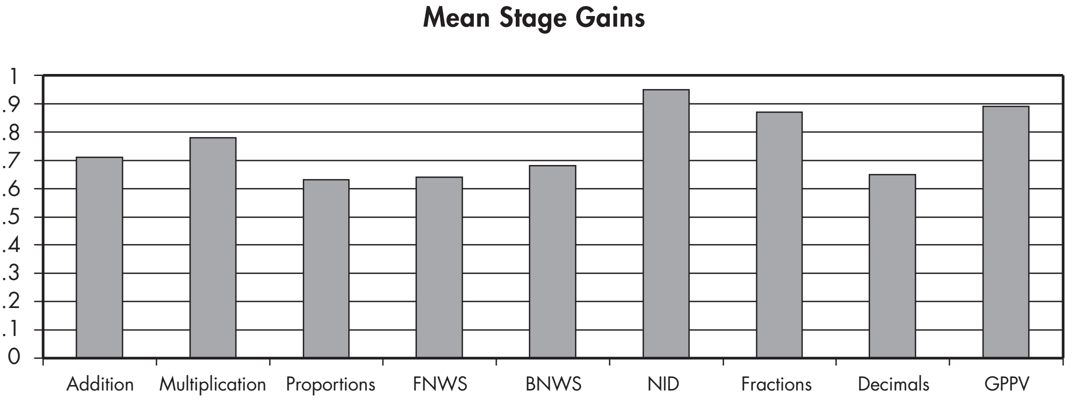


Figure 2. 2005 Mean stage gains across the Number Framework

The positive mean stage gains for numeral identification (NID) occurred mainly in years 1, 2, and 3 (see Figures 3 and 4 NID). This is very much as expected, as previous evaluations have shown (Christensen, 2003). It is also pleasing to see positive mean stage gains continuing in grouping and place value. It has been noted that understanding place value has been a key component of numeracy learning for some time (Resnick, 1983). But with the increased emphasis on part–whole understanding, the ability to manipulate groupings of quantities efficiently is also critical. Many writers have stressed the importance for students of coming to understand the "additive composition of numbers" (Young-Loveridge, 2001). The Te Poutama Tau project has supported this key idea by labelling the split parts of a group the "tauhono-joining numbers" (Ministry of Education, 2006), thus providing a linguistic clue to the learners.

Mean Change Differences Between Girls and Boys in Tests of Significance

The table below assesses whether or not there were significant differences between girls and boys in how well they performed. While none of the results were significant (at the 0.05 level), it appears that girls performed better in the multiplication (F = 3.01, p = 0.084) and decimals (F = 3.006, p = 0.084) interviews and possibly for the proportions (F = 2.254, p = 0.134) diagnostic interview. Given a larger sample, it would be expected that these three interview results would reach significance.

Table 1
Mean Change Differences Between Girls and Boys in Tests of Significance

	Mean Change			
	Tama	Kotiro	F	Sig.
Addition	0.710	0.710	0.000	0.994
Multiplication	0.700	0.860	3.010	0.084
Proportions	0.570	0.690	2.254	0.134
FNWS	0.650	0.630	0.051	0.821
BNWS	0.650	0.700	0.359	0.549
NID	0.940	0.970	0.023	0.880
Fractions	0.870	0.880	0.016	0.901
Decimals	0.590	0.700	3.006	0.084
GPPV	0.870	0.910	0.148	0.700

Student Achievement and Year Level

The graphs in Figure 3 show the variation in the mean gain for each aspect of the Number Framework across the year levels. As with previous years, there was no clear pattern common to all aspects of the Number Framework. The aspects of FNWS, BNWS, and NID showed a "diminishing returns" pattern, where advancement was more difficult for students at successively higher year levels. It is also important to note that numeral identification (Figure 3.6) as a separate data section is only part of diagnostic interview A, so students who proceed beyond tests A to E or U will not register mean stage progress in NID. Figure 3.6 therefore only shows progress for students who were tested using test A. NID continues to be a critical aspect in the upper levels but has been subsumed as part of ordering. In order to count forward or backward or locate numbers, students need to be able to identify numbers.

Multiplication, proportions, and fractions showed an initial stage gain, a fall by the next year level, and a rise slowly thereafter, while stage gains for decimal knowledge were steadily greater at higher year levels. Multiplication and proportions and fractions are introduced further up the framework than addition and subtraction, so it is not surprising that there is no stage gain at years 0 to 1.

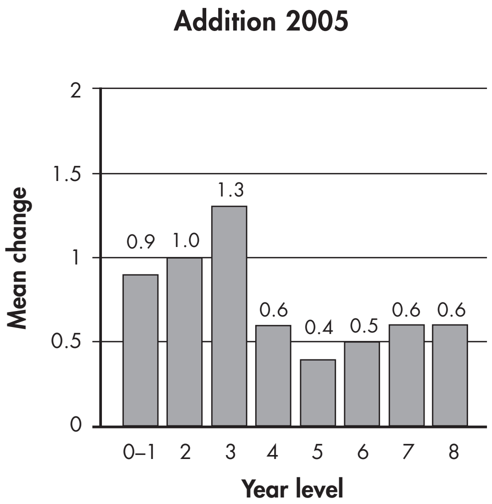


Figure 3.1. Mean stage gain for addition and subtraction

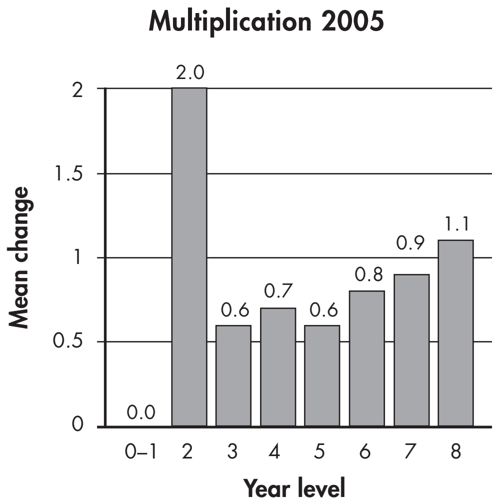


Figure 3.2. Mean stage gain for multiplication and division

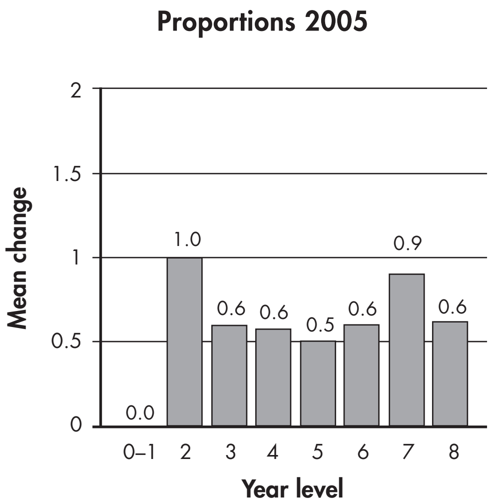


Figure 3.3. Mean stage gain for proportions and year level

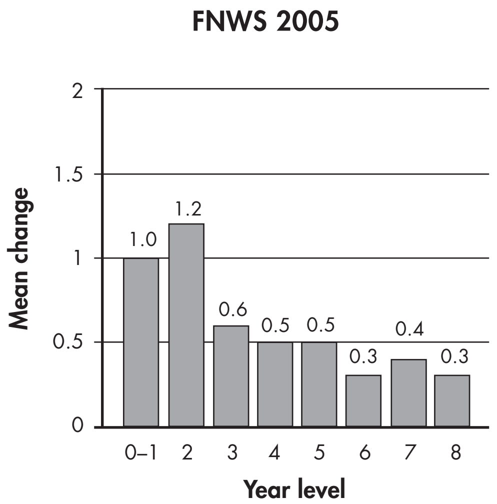


Figure 3.4. Mean stage gain for forward number word sequence

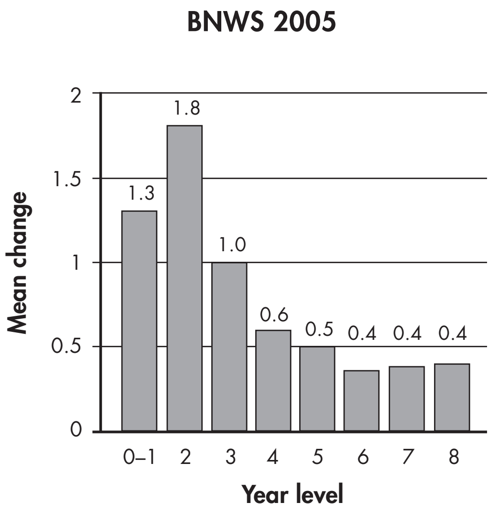


Figure 3.5. Mean stage gain for backward number word sequence

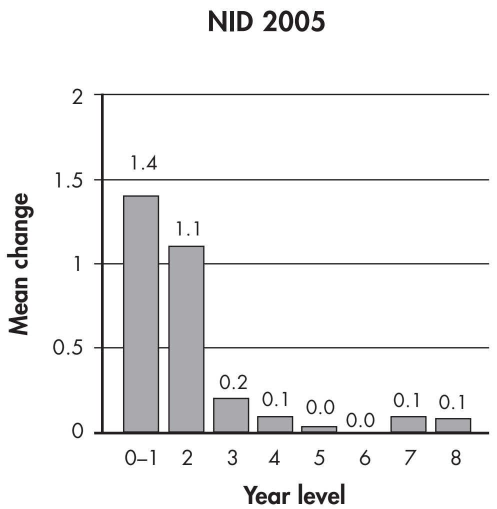


Figure 3.6. Mean stage gain for numeral identification

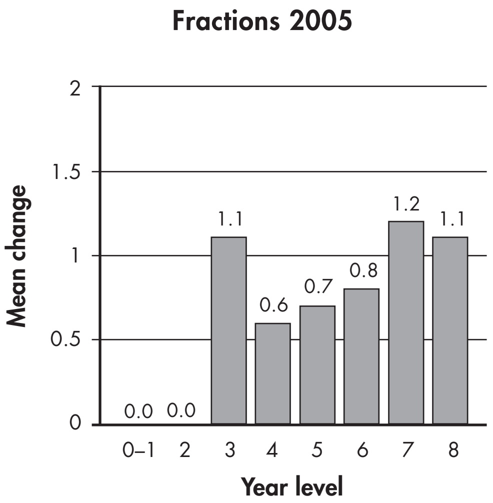


Figure 3.7. Mean stage gain for fractions

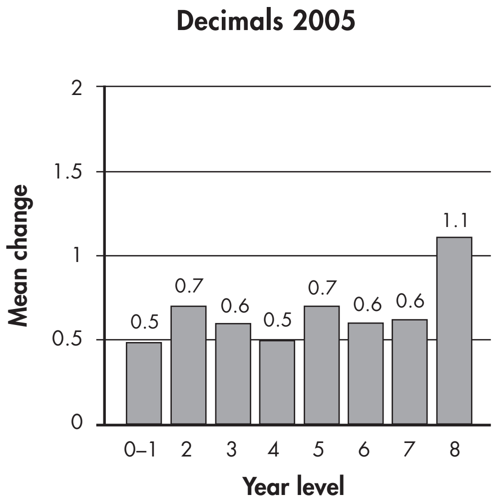


Figure 3.8. Mean stage gain for decimals

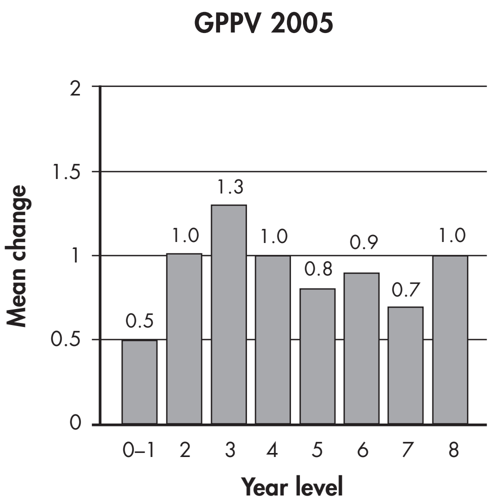
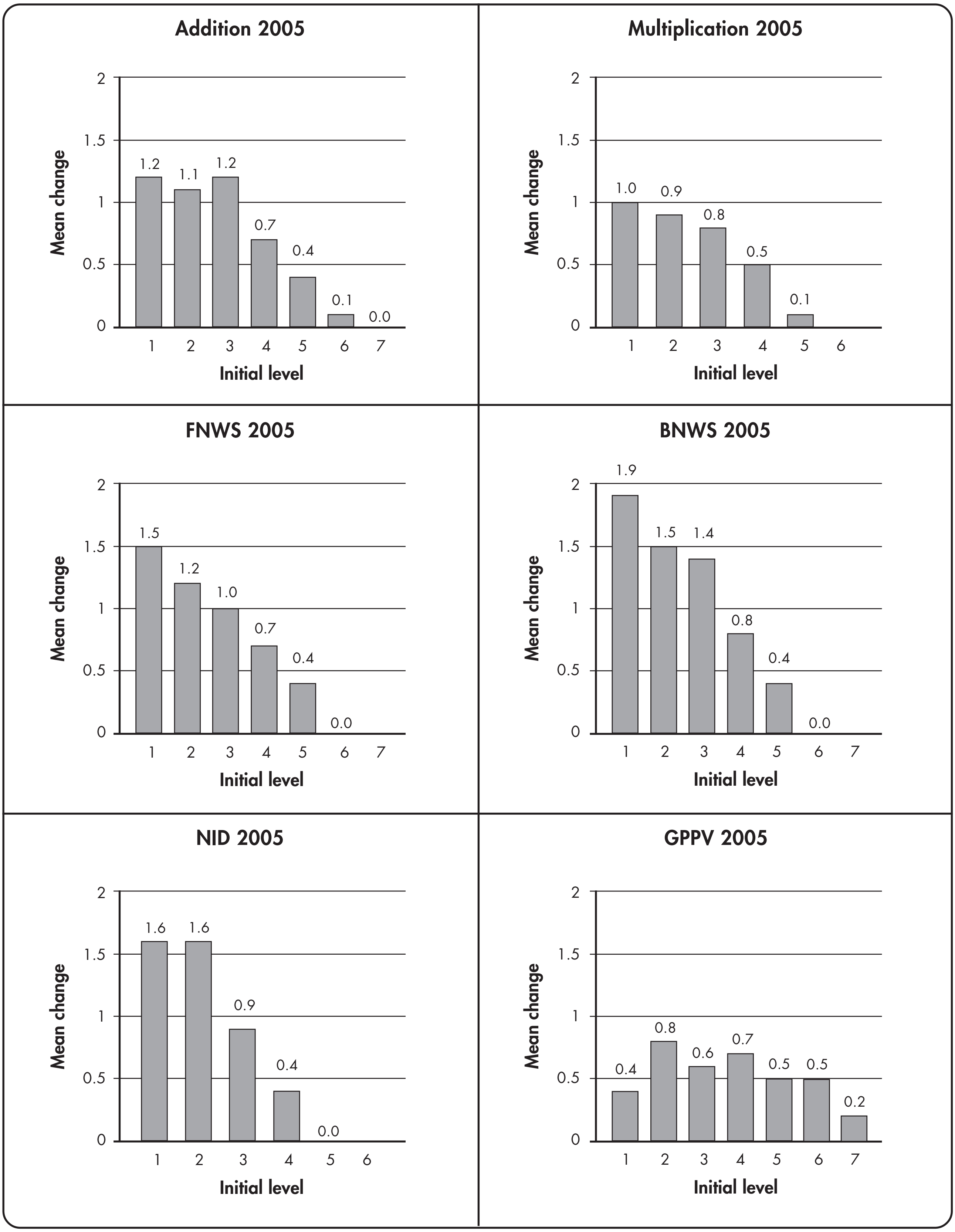


Figure 3.9. Mean stage gains for grouping and place value

Student Achievement and Initial Stage Assessment

The graphs in Figure 4 show how improvement in performance was related to the stage at which students were initially diagnosed. For example, those students who were judged to be at stage 1 for addition when initially tested made a mean stage gain of 1.2 (that is, they reached stage 2 by the second diagnostic interview). Students who were judged to be at stage 6 for addition when initially tested made a mean stage gain of 0.1. The highest stage for a number of the knowledge aspects, that is, FNWS and BNWS, is up to stage 6. Therefore students whose entry stage is stage 6 will show as 0.0 in the graphs. Similarly, the graph for NID only goes up to stage 4.

There was a consistent pattern across all nine aspects of the Number Framework, with improvements in performance being more difficult to achieve with higher initial scores. This can be attributed to higher stages of the Framework being larger and more complex, making it more difficult for students to advance to the next stage. As Christensen (2004) pointed out, this may also "indicate that teachers and facilitators were more effective at the lower levels" (p. 16).


Addition 2005
Mean change
2
1.5
1
0.5
0
1.2
1.1
1.2
0.7
0.4
0.1
0.0
1 2 3 4 5 6 7
Initial level
Multiplication 2005
Mean change
2
1.5
1
0.5
0
1.0
0.9
0.8
0.5
0.1
1 2 3 4 5 6
Initial level
FNWS 2005
Mean change
2
1.5
1
0.5
0
1.5
1.2
1.0
0.7
0.4
0.0
1 2 3 4 5 6 7
Initial level
BNWS 2005
Mean change
2
1.5
1
0.5
0
1.9
1.5
1.4
0.8
0.4
0.0
1 2 3 4 5 6 7
Initial level
NID 2005
Mean change
2
1.5
1
0.5
0
1.6
1.6
0.9
0.4
0.0
1 2 3 4 5 6
Initial level
GPPV 2005
Mean change
2
1.5
1
0.5
0
0.4
0.8
0.6
0.7
0.5
0.5
0.2
1 2 3 4 5 6 7
Initial level

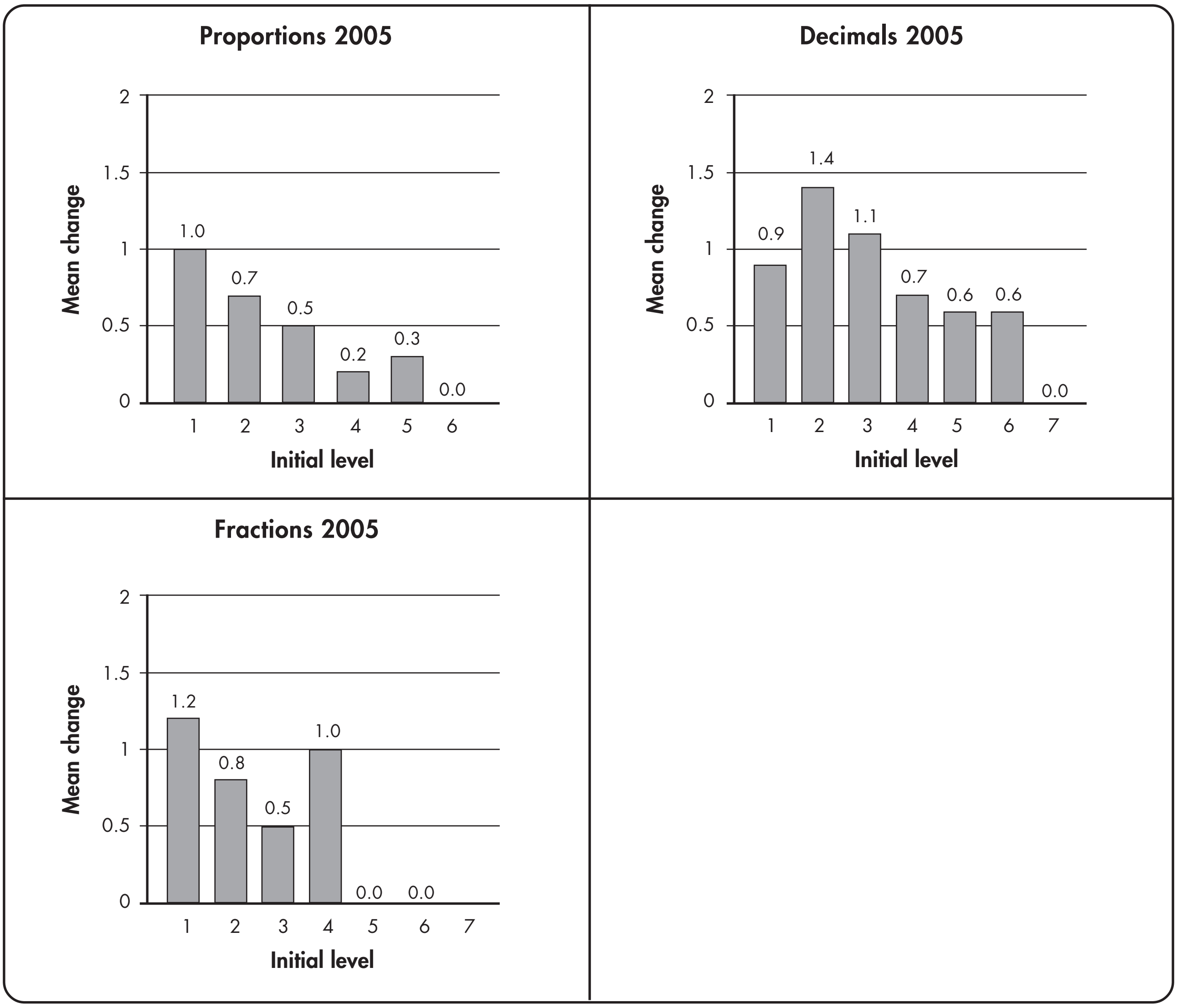


Figure 4. Mean stage gain and initial stage level

Student Achievement and Language Proficiency

There was little difference between 2003 and 2004 in how teachers rated te reo Māori proficiency of the students. However, those participating in 2005 had their language rated as being less proficient than in previous years. The table below shows that only 52% of the students were rated as either “proficient” or “very proficient”, with 17% rated as “not very proficient” or having “poor proficiency”.

Table 2
Language Proficiency of Students

	Language proficiency				
	Percentage				
	Very proficient	Proficient	Reasonably proficient	Not very proficient	Poor proficiency
2005	5	47	31	11	6
2004	13	51	26	8	2
2003	12	48	33	6	1

Longitudinal Patterns of Progress

This section examines how performance has changed from 2004 to 2005 and then longitudinally over the last three years of Te Poutama Tau. Although Te Poutama Tau data was initially collected in 2002, that was very much a developmental year, and so it would not be useful to compare those results with the results from 2003 to 2005.

Table 3 shows final test results for the years 2004 and 2005. There were improvements in mean numerical knowledge at the end of the year for 2005 when compared to 2004 in multiplication, proportions, NID, fractions, decimals, and GPPV. There were decrements in mean performance by the end of the year in addition, FNWS, and BNWS numerical knowledge.

Table 3
Final Test Results 2004–2005

		2004 (n = 1295)			2005 (n = 427)		
	Mean	Initial	Change	Final	Initial	Change	Final
Strategy	Addition	4.1	0.73	4.85	3.7	0.71	4.22
	Multiplication	2.1	0.45	2.58	2.6	0.78	3.16
	Proportions	2.1	0.40	2.41	2.5	0.63	2.92
Knowledge	FNWS	4.7	0.74	5.46	4	0.64	4.55
	BNWS	4.4	0.86	5.27	4	0.68	4.66
	NID	3.0	0.45	3.46	2.9	0.95	3.78
	Fractions	1.9	0.46	2.31	2.0	0.87	2.69
	Decimals	2.6	0.71	3.26	2.8	0.65	3.41
	GPPV	2.5	0.55	3.08	3.0	0.89	3.83

Note: The initial data was rounded to 1 d.p. and the change to 2 d.p. The final was worked out by adding the initial as 2 d.p .to the change (2 d.p.), and then the final was rounded to 2 d.p.

Figure 5 shows how the average for the final result for all tests varies across year levels for 2003, 2004, and 2005. As can be seen in Figure 5, results for 2005 compare favourably with those from 2003 and 2004. Generally, the mean final level for 2005 was marginally improved from that of previous years, with the greatest improvement occurring for years 2 and 3. Results from years 9 and 10 were omitted due to the small numbers in these groups. There was little change across the three years in mean improvement for the majority of numerical knowledge, with marginally more students making no improvement for addition, FNWS, BNWS, and decimals. However, there were large improvements from previous years for multiplication, proportions, NID, and fractions. GPPV showed a slightly smaller improvement from previous years.

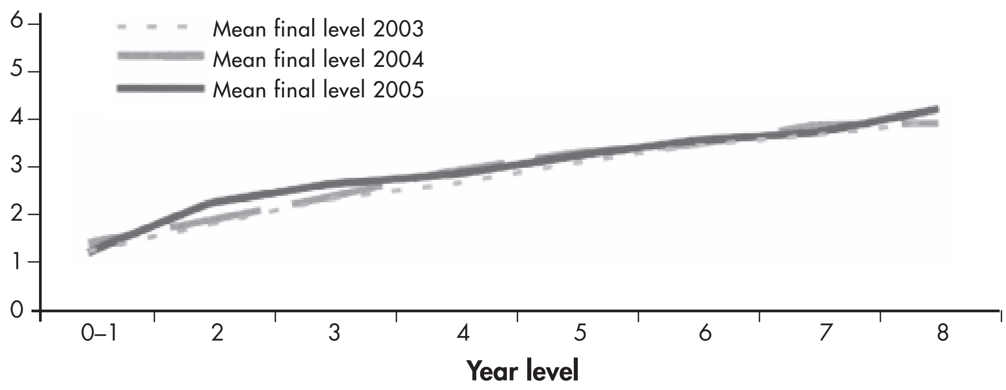


Figure 5. Comparison of students' average mean stage gain across 2003, 2004, and 2005.

Table 4 compares the percentages of students who have not shifted in the years 2003 to 2005. Of concern is the high percentage of students not making any stage gain between the initial and final diagnostic assessment. This issue was noted in the 2002 and 2003 evaluation reports (Christensen, 2003, 2004). One mitigating factor for the data was the larger proportion of students who were older and were generally assessed at a higher stage (Christensen, 2004). It is also acknowledged that significant learning could occur within a stage but show up as minimum mean stage gain.

However, one very positive aspect of the data has been the steadily declining percentage of students who have made no shift between 2003 and 2005. For example, 70% of students made no stage gain in multiplication in 2004. This was significantly reduced to 40% in 2005. This positive trend is also continued for fractions. Many teachers know from their own experiences that fractions are one of the most complex mathematical areas that students will encounter during their school years (Davis, Hunting, & Pearn, 1993). The fractions whānau (ordinary fractions, decimals, percentage) constitute a body of knowledge considered essential not only for higher mathematics but for everyday life skills. In Aotearoa, students' difficulties with fractions have been highlighted in evaluation reports from the Numeracy Development Projects and Te Poutama Tau (Christensen, 2004; Young-Loveridge, 2005).

Table 4
Comparison of Students' Mean Stage Gain across the Aspects of the Number Framework for 2003, 2004, and 2005

	Percentage making stage gain						
Stage gain	0	1	2	3	4	5	6
Addition 2005	**49**	**35**	**12**	**4**	**0.2**		
Addition 2004	48	36	11	4	0.4	0.2	
Addition 2003	42	39	15	3	0.8	0.7	0.2
Multiplication 2005	**40**	**45**	**13**	**3**			
Multiplication 2004	70	18	10	2	0.3		
Multiplication 2003	63	23	10	3	0.7	0.2	
Proportions 2005	**47**	**44**	**8**	**1**			
Proportions 2004	73	16	10	1	0.07		
Proportions 2003	67	18	11	3	0.5	0.2	0.2
FNWS 2005	**52**	**36**	**9**	**3**	**0.2**		
FNWS 2004	49	36	9	4	2	0.2	
FNWS 2003	45	34	13	4	2	1	0.6
BNWS 2005	**49**	**38**	**11**	**3**	**0.3**		
BNWS 2004	44	37	12	4	3	0.3	
BNWS 2003	44	32	13	7	3	0.7	0.7
NID 2005	**42**	**30**	**17**	**10**			
NID 2004	70	20	6	3	0.5	0.2	
NID 2003	68	17	8	4	2	2	
Fractions 2005	**40**	**38**	**17**	**5**	**0.4**		
Fractions 2004	70	19	9	2	0.5	0.08	
Fractions 2003	63	20	13	3	0.7	0.7	
Decimals 2005	**49**	**40**	**11**	**0.8**	**0.2**		
Decimals 2004	47	37	13	2	0.3		
Decimals 2003	91	3	4	1	0.3		
GPPV 2005	**40**	**36**	**18**	**5**	**0.2**		
GPPV 2004	57	33	10	0.9	0.2		
GPPV 2003	43	38	14	4	0.8	0.5	

Recommendations

The following recommendations arise from the research that has been discussed in this report. A stronger emphasis for teacher and numeracy facilitators' professional development in 2006 should be focused on:

- Improving the outcomes for those students who make little or no stage gains (as noted in Table 4).
- Shifting students at stage 4 into stage 5 early additive part–whole. The data suggests many students are making steady progress through the counting stages but are having difficulties transitioning into part–whole stages.
- Maintaining an emphasis on grouping and place value. This concept underpins many of the mathematical concepts associated with numerical thinking, in particular part–whole thinking.
- Maintaining a focus on the higher stages of multiplication, decimals, fractions, and proportions. As noted in the discussion on Table 4, these concepts constitute a body of knowledge considered essential not only for higher mathematics but also for everyday life skills.
- Ensuring all participating schools enter complete initial and final data on the national database.
- Focusing on the relationship between te reo Māori and mathematical thinking. For example, what are the te reo Māori linguistic structures that support or hinder students' ability to learn mathematics? How do students represent mathematical concepts linguistically?

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Algebraic Thinking in the Numeracy Project: Year Two of a Three-year Study

Kathryn C. Irwin
University of Auckland
<k.irwin@auckand.ac.nz>

Murray S. Britt
University of Auckland
<m.britt@auckand.ac.nz>

This three-year study traces the development of algebraic thinking in students who have been in the Numeracy Development Projects. We first assessed a cohort of students in 2004 when this group was in year 8. We assessed the same students again toward the end of 2005 when they were in year 9, the first year of secondary school. There was a significant increase in the scores of these students between years 8 and 9. This compares favourably with different groups of students assessed in 2004, when the year 9 students did less well than the year 8 students. However, there was also a significant difference between schools in progress made. Correlations between test scores of individual students in pairs of schools ranged from 0.06 to 0.71. Correlations on test items including letters ranged from –0.21 to 0.54. The highest correlations came from an intermediate school, whose students had the highest mean scores of all intermediate schools.

Background

In 2004, we embarked on a three-year study of the algebraic thinking of students in schools that were engaged in the Numeracy Development Projects (NDP) (Irwin & Britt, 2005a). For this assessment, we designed a test of algebraic thinking for use over the three years. This 20-item test covers the use of compensation in addition, multiplication, subtraction, and division. For each operation, the test included two numerical items, two that used a letter to express one variable, and a final item that expressed the entire concept using letters. We referred to the items that included letters as literal items as opposed to numerical items.

Results of that study in 2004 showed that some students could demonstrate understanding of the compensation principle in the example by using it in both numerical and literal items. Students in intermediate schools were more successful in demonstrating understanding of this principle than were students in secondary schools. One school had given the test to all their year 7 students as well as to year 8 students. We were surprised to see that these younger students were more successful than the older students.

This report covers the second year of this three-year study. The students who were in year 8 in 2004 were now near the end of their first year of secondary school, year 9. We will test these same students again in 2006, near the end of their year 10. Our intention is to see if these students demonstrate algebraic thinking in all three years, built on a base of what they learned in the NDP. While these students are the focus of our study, we also assessed all of the students in year 9 and year 10 in the participating secondary schools. This provides comparisons with the data in the first year of this study.

The rationale behind this study is that students in the NDP have experience in part–whole thinking that enables them to use algebraic thinking when operating with numbers. We believe that this experience in using algebraic thinking with numbers should give them an advantage when doing algebra in secondary school. We hypothesised that the Intermediate Numeracy Project (INP) was providing intermediate school students with the ability to think algebraically (see: Fujii, 2003; Fujii & Stephens, 2001; Irwin & Britt, 2005b; Kaput & Blanton, 2001; Lee, 2001; Mason, 1996; Steffe, 2001) and that this skill would enable them to succeed in algebra in secondary school.

Method

Students from one intermediate school and four secondary schools were assessed near the end of 2005. It had been our intention to assess only students in the four secondary schools that took students from the intermediate schools assessed in 2004. However, the results from one intermediate school in 2004 were so surprising that we decided to assess them again this year to see if similar results were obtained.

In the report of the 2004 data (Irwin & Britt 2005a), we identified the schools by their decile ranking. The rankings in 2005 (see Table 1) changed from the 2004 rankings due to changes in Government policy. In this 2005 report, we identify the pairs of schools by a number but also, in Table 1, give their decile ranking in 2005 and in 2004.

When these secondary schools were chosen, the nature of the NDP for secondary schools was still being formulated. We did not know if any of these secondary schools would be engaged in the NDP. It is serendipitous that three did participate in the Secondary Numeracy Project (SNP) in 2005. The school that did not participate in the SNP developed their own programme, which concentrated on numeracy and algebra throughout the year.

Participants

The characteristics of the 2005 schools and participants are shown in Table 1.

Table 1
Participants in the 2005 Cohort of the Test of Algebraic Thinking

School	Decile ranking in 2005	Decile ranking in 2004	Year group	NDP involvement	Number of students assessed	Decile of contributing intermediate school in 2004	Number of students assessed in 2004 and 2005
Intermediate 1	2	2	7	INP	80		
Intermediate 1			8	INP	93		67
Secondary 1	4	3	9	SNP	142	2	43
Secondary 1			10	SNP	153		
Secondary 2	4	3	9	SNP	237	3	21
Secondary 2			10	SNP	224		
Secondary 3	6	5	9	Own programme	338	5	69
Secondary 3			10	Own programme	322		
Secondary 4	8	7	9	SNP	260	6	52
Secondary 4			10	SNP	282		

Schools gave the test to all available students in each class. Our primary focus in 2005 was on the students who were assessed in 2004 and 2005, as shown in the last column of Table 1.

All secondary schools except secondary school 4 were participants in this study in 2004. That school had been contacted in 2004, but the head of the mathematics department was leaving and did not want to commit the next head of department to a three-year study. The new head of mathematics was happy to participate.

In addition to assessing these students, we interviewed teachers in the mathematics department of the four secondary schools about the usual ways in which they taught algebra, their knowledge of the NDP, differences they noted between students from NDP schools and those from non-NDP schools, and related topics. We also spoke with the facilitator, principal, and deputy principal of the one intermediate school about possible reasons for the success of their students in 2004.

Materials

The same test was given to all students. This was identical to the test given in 2004. There were five items requiring compensation for the four arithmetic operations: addition, multiplication, subtraction, and division. Two exemplars were provided for each of these sections. For addition, students were to use *Jason's method,* illustrated by 27 + 15 being transformed into 30 + 12 and 34 + 19 being transformed into 33 + 20. The items for the students were similar to: 198 + 57, 25.7 + 9.8, $48 + n = 50 - \square$, $8.9 + k = 9 + \square$, and $a + b = (a + c) + \square$. The first item in each section involved whole numbers, the second item included decimal fractions, the third item involved whole numbers and one literal symbol, and the fourth item included one literal symbol and a decimal fraction. The fifth item required students to complete an algebraic identity with literal symbols only.

Procedure

The teachers administered the test towards the end of term 3 or early in term 4 in normal class time on a day that suited them. Students were instructed to read the section with the two exemplars carefully, to write the answer in the space below each question, and not to use a calculator. Graduate students, who had just completed their pre-service secondary mathematics teacher education programmes, marked the tests under the guidance of the authors. Their marking was checked by the second author and re-marked where necessary. Responses were credited as correct if they followed the structure of the exemplars.

Results

The main focus of our analysis was on the students who had taken this test the previous year, as stated above. However, the performance of all students was analysed to see how overall results compared to that of students in the previous year.

Results for All Students

Table 2 gives the percentage of all year 9 students that were successful on this test in 2005, with the percentage of year 9 students correct in 2004 given in brackets. The 2005 percentages for all items for all year groups is given in Appendix L.

Table 2
Percentage of Year 9 Students Correct on the Test of Algebraic Thinking on Each Item in 2005 (percentage correct in 2004 in brackets)

Operation	Item 1	Item 2	Item 3	Item 4	Item 5
Addition	68 (73)	57 (57)	16 (10)	15 (10)	7 (6)
Multiplication	43 (45)	32 (35)	13 (11)	12 (9)	6 (5)
Subtraction	26 (28)	21 (22)	12 (10)	10 (9)	7 (5)
Division	34 (37)	29 (26)	14 (12)	13 (12)	7 (6)

This table shows that more students were correct on the initial item for each operation in 2004. Literal items (items 3–5) were answered correctly by somewhat higher percentages in 2005. Numerical items continued to be answered correctly by more students than were literal items. Despite the differences between 2004 and 2005, the overall pattern of results was similar for the two years. Compensation in addition continued to be the easiest operation, while compensation in subtraction was the most difficult.

Figure 1 gives a comparison of students' percentages correct on literal items only, both in 2004 and in 2005. These items were judged to give the best indication of understanding of the principles involved, demonstrating algebraic thinking. The irregular pattern of successes in 2004 had been surprising. They led us to speculate that the year 7 students were better prepared in algebraic thinking than were older students. This irregular pattern of success was not repeated in 2005. Note that in the data presented in Figure 1, the 2005 data for year 7 students comes from only one school, the same school as in the year 7 data for 2004. The data for year 8 in 2005 comes from one school, in comparison with four schools in 2004. Year 9 students come from four schools in 2005 (three in 2004), and year 10 data from four schools (two in 2004).

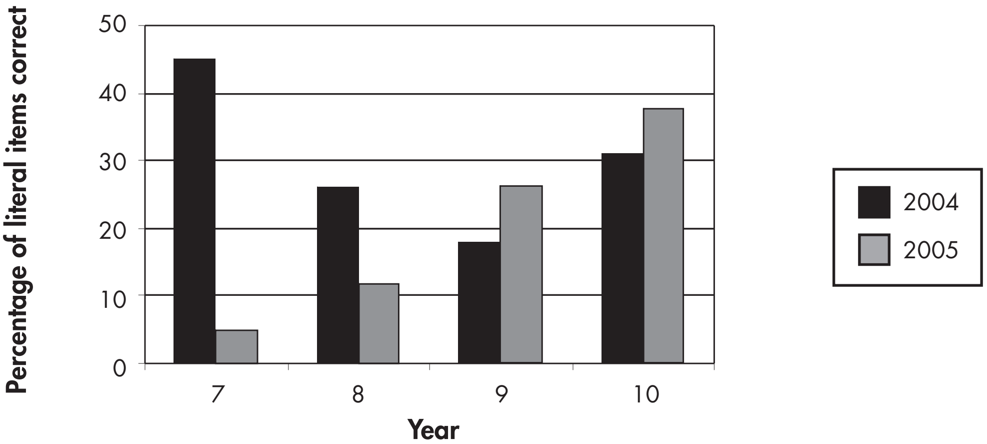

Figure 1. Percentage of students who were correct on some literal items in 2004 and 2005

Clearly, the year 7 cohort in 2005 did much less well than the year 7 cohort in 2004. It is also interesting that the year 9 students did better in 2005 than in 2004, as noted in Table 2. Little can be said about the fact that they did better than the year 8 students because only one school is represented in the year 8 data.

We looked more closely at the results from the one intermediate school assessed this year, comparing it with the results from the same school in 2004. This comparison is shown in Table 3.

Table 3
Results for Year 7 and Year 8 Students from Intermediate School 1 in 2004 and 2005

Year	Number of students in year 7	Year 7 mean	Percentage of students in year 7 correct on at least one literal item	Number of students in year 8	Year 8 mean	Percentage of students in year 8 correct on at least one literal item
2004	98	4.96	45	82	4.55	37
2005	80	2.36	5	93	3.70	12
Same students in 2004 and 2005	67	5.10	43	67	3.75	19

This table shows that while 45% of year 7 students answered literal items correctly in 2004, only 5% of the year 7 students did so in 2005. The same 67 students did better, on average, in year 7 in 2004 than they did a year later. Twenty-one of 29 students who had been able to generalise from numerical to literal items in 2004 did not do so in 2005. We asked the principal, acting principal, and facilitator if they had any idea why the year 7 cohort in particular did well on this test in 2004, but they had no clear idea. They did suggest that teachers had benefited from a workshop on algebra in 2004 and informed us that the school had changed its focus to reading in 2005.

It seems likely that some recent teaching may have been responsible for the good performance of the year 7 group in 2004 but that the effect of this teaching did not stay with the students and was not repeated for the 2005 year 7 cohort.

Results for the Students who Took the Test in 2004 and 2005

As said above, our primary interest was in the students who had been in year 8 in 2004 and had now moved to year 9 in secondary schools. Overall, these students did significantly better in year 9 than in year 8 ($F = 10.286$ [$df = 1, 179$], $p < 0.01$). However, there was also a significant difference between schools ($F = 5.370$ [$df = 3, 179$] $p < 0.01$).[1] Table 4 presents the data on students who had taken this test near the end of year 8 and near the end of year 9.

[1] Because the distribution of scores was skewed to the right, analysis was carried out on the square root of the scores.

Table 4
Results for the Same Students in Year 8 in 2004 and Year 9 in 2005

Pair of intermediate and secondary schools	Number of students assessed twice	Year 8 mean	Percentage of students in year 8 correct on at least one literal item	Year 9 mean	Percentage of students in year 9 correct on at least one literal item	Correlation of total scores in year 8 and year 9 for the same students
1	43	4.45	40	3.38	10	0.06
2	20	4.25	25	6.10	4	0.53
3	69	6.04	22	8.20	50	0.71
4	52	4.71	06	7.27	42	0.54

Table 4 shows that school pair 1 had a low correlation between scores in year 8 and year 9, while pair 3 had a high correlation.

Figure 2 presents the mean scores with standard error of measurement for the four pairs of schools graphically. The pair of schools numbered 3 includes the only secondary school not in the SNP. This school and its contributing intermediate had the highest proportion of successful students.

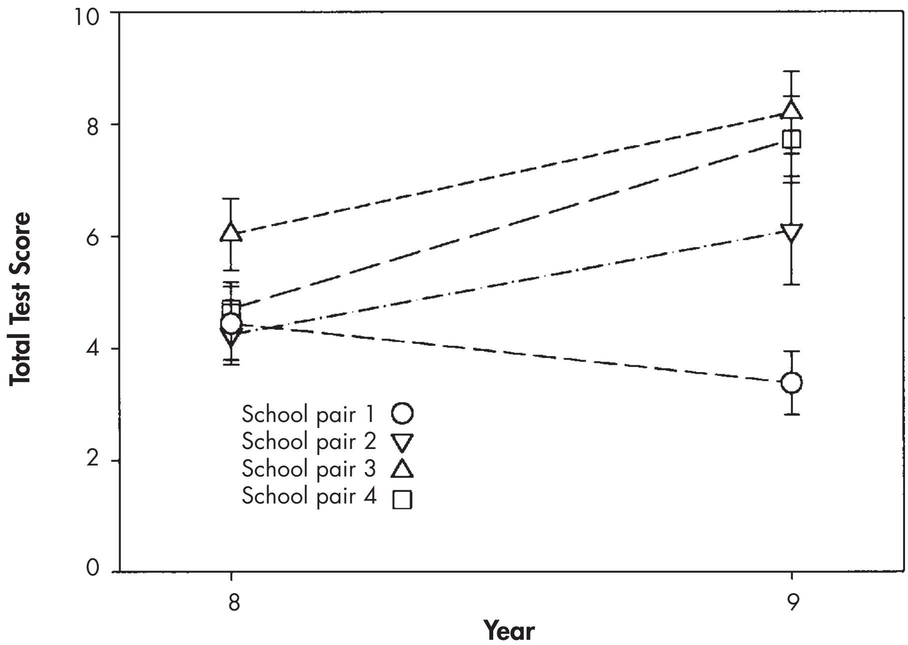

Figure 2. Mean scores and standard errors of the full test for the same students in four pairs of schools

This figure demonstrates that the students from school pair 1 did more poorly in year 9 than in year 8, while the means for students in other pairs of schools increased.

Figure 3 shows the mean totals of the 12 items that involved letters in these four pairs of schools.

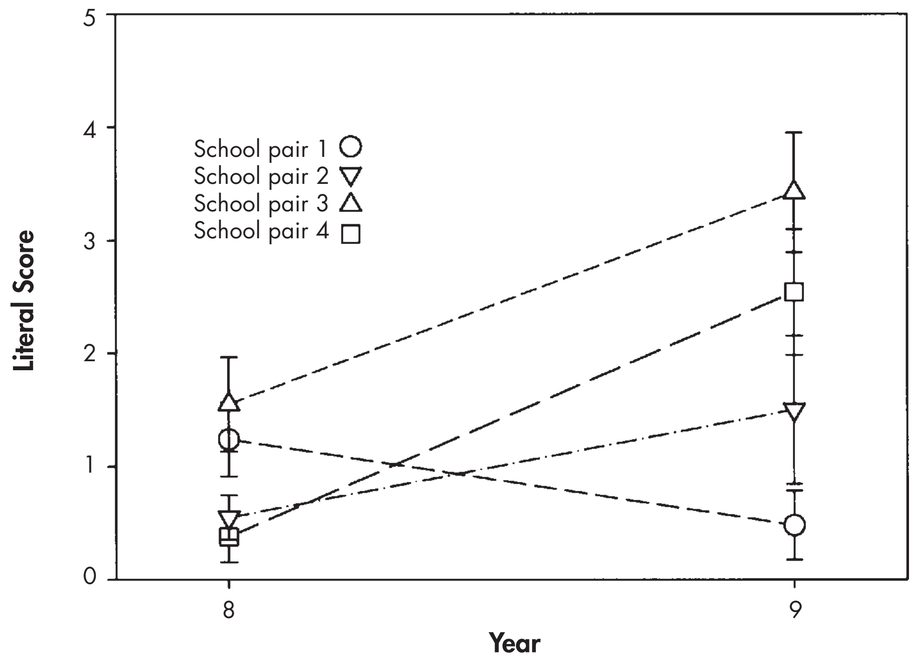

Figure 3. Mean number of items with letters in them and standard errors for the same students in four pairs of schools

This figure shows the superiority of school pair 3 on literal items. On this analysis, as for the analysis of the full scores, pair 1 decreased and the other pairs increased. The correlations for these four pairs of schools ranged between –0.21 for school pair 2 and +0.54 for school pair 3.

In 2005, the majority of year 9 students came from intermediate schools or full primaries that had entered data on the NZ Maths website (www.nzmaths.co.nz). There was no significant difference between those students and those from schools that had not entered data. This may have been because some students came from NDP schools that elected not to enter data or because many schools that do not enter their data on the national website may still have some knowledge of the NDP, possibly through the work of private providers or through reading material on the NZ Maths website.

Interviews with the secondary mathematics teachers were analysed for similarities and differences. Teachers at all schools gave similar responses when asked how they introduced algebra and what were the main difficulties that students had with this topic, although there was a variety of responses within each school. At all four schools, some teachers voluntarily mentioned the link between algebra and number or the NDP. Teachers from two schools did not believe that students from the NDP schools did any better than students from schools that had not been in the NDP (schools 3 and 4). Teachers from the other two schools found that students who had been exposed to the NDP were more open to different ways of thinking, tended to be co-operative learners, were more advanced in part–whole thinking (school 2), were more confident, had better attitudes, and were willing to take risks (school 1). However, comments from teachers did not match well with students' success on this test of algebraic thinking.

Discussion

There are only a few things we can say for certain as a result of these 2005 results. One is that the students who took the test in both 2004 and 2005 did significantly better in their second year. This is in contrast to the pattern provided by different students in years 8 and 9 in 2004. It suggests that many students are carrying forward their understanding of algebraic thinking. The other thing that we can say definitively is that there were significant differences between school pairs. This is shown most markedly by school pair 1, the one cohort of students to do more poorly in 2005 than in 2004. We can only guess at the reason for this. We understand that the intermediate students focused on algebra not long before taking this test in 2004, and we also understand that there were some changes of staffing at the secondary school that could have affected teaching there. There is not enough evidence of the relationship of these factors to the students' success to consider them to be causal.

School pair 3 were the most successful. They also had the highest correlations between student success on the two occasions. Possible reasons for this lie with both the intermediate school and the secondary school. The intermediate school, which has been involved in the NDP for 3 years, is reported to have a particularly skilful numeracy leader. Their students outscored other schools in 2004 and may have carried their algebraic thinking with them into secondary school. The secondary school was not in the SNP but chose to develop their own programme, teaching algebra throughout the year while concentrating on numeracy for the weaker students. It is possible that teachers could be more committed to a teaching programme that they had developed themselves, probably one that built on what they were already doing, than they would be to a different programme developed elsewhere. By teaching algebra for the whole year, their students had more chance to master the concepts in this test.

In this three-year study, we are looking for patterns of performance that might indicate that students who had the NDP in intermediate school carried their understanding of algebraic thinking, used primarily with numbers, with them into secondary school. We believe that this should enable them to have a basic understanding of algebra that would enable them to succeed at this most important topic.

At this point in the three-year study, results are not clear. We cannot say that all intermediate schools give their students the grounding in algebraic thinking that will give them an advantage in secondary school algebra. We have no index of quality control for teaching and learning in the intermediate or the secondary schools, nor is it possible to get one in a study of this sort. Although we cannot say so with confidence, we believe that it is likely that differences in teaching have a lot to do with the unusual results we have shown to date.

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Students' Perspectives on the Nature of Mathematics

Jenny Young-Loveridge
University of Waikato
<jenny.yl@waikato.ac.nz>

Merilyn Taylor
University of Waikato
<meta@waikato.ac.nz>

Sashi Sharma
University of Waikato
<sashi@waikato.ac.nz>

Ngārewa Hāwera
University of Waikato
<ngarewa@waikato.ac.nz>

This paper reports on one small component of a much larger study that explored the perspectives of students towards mathematics learning. Students were asked, "What do you think maths is all about?" Some students responded in terms of mathematical content. Others commented on learning in general or on problem solving in particular. Some students talked about the usefulness of mathematics for everyday life. An overwhelming number of students answered the question by talking about the importance of mathematics for the future, particularly for getting a job. An analysis of students' responses by whether or not they had participated in the Numeracy Development Project showed few differences between the groups.

The nature of mathematics has been the focus of much writing over the last few decades (for example, Begg, 1994, 2005; Dossey, 1992; Fuson, Kalchman, & Bransford, 2005; Ocean, 2005; Presmeg, 2002; Winter, 2001). Dossey (1992) argues that different conceptions of mathematics influence the ways in which society views mathematics. This can influence the teaching of mathematics and communicate subtle messages to children about the nature of mathematics that "affect the way they grow to view mathematics and its role in their world" (p. 42). Similarly, Presmeg (2002) has argued that beliefs about the nature of mathematics either enable or constrain "the bridging process between everyday practices and school mathematics" (p. 295).

Different dichotomies have been used to highlight the contrasting ways in which mathematics is viewed. For example, Dossey (1992) has distinguished between external conceptions of mathematics, held by those who believe that mathematics is a fixed body of knowledge that is presented to students, and internal conceptions that view mathematics as personally constructed, internal knowledge. Begg (1994, 2005) has contrasted mathematical content (knowledge and procedures) with mathematical processes (reasoning, problem solving, communicating, and making connections). Winter (2001) has written about a tension between a mechanistic view of mathematics (as in the development of skills and knowledge) and mathematics as a means towards fostering citizenship and responsibility within society (as in the development of personal, spiritual, moral, social, and cultural dimensions).

A distinction has been made between mathematical activity carried out for its own sake and mathematical activity that is useful for something else (Huckstep, 2000). In order to distinguish between the aims and purposes of mathematics education, Huckstep asks: "What are we trying to do *in* mathematics education?" and "What are we trying to do it *for*?" (p. 8). This particular dichotomy is closely related to the debate about what is mathematics and what is numeracy (Hogan, 2002; Stoessiger, 2002). Definitions of numeracy emphasise the practical or everyday uses of mathematics in contexts such as homes, workplaces, and communities (Stoessiger, 2002). Writers who argue that mathematics is valuable for its own sake often note the beauty and aesthetics of mathematics and the sheer enjoyment of doing it (for example, Holton, 1993; Winter, 2001).

The current mathematics curriculum document for schools in New Zealand also considers the nature of mathematics:

> Mathematics makes use of specific language and skills to model, analyse, and interpret the world ... [It] involves creativity and imagination in the discovery of patterns of shape and number, the perceiving of relationships, the making of models, the interpretation of data, and the communication of emerging ideas and concepts. (Ministry of Education, 1992, p. 7).

The "new" curriculum that will soon replace this 1992 document has a greater emphasis on thinking and defines mathematics as:

> the exploration and use of patterns and relationships in quantities, space, and time ... [a way] of thinking and solving problems ... [that] equips students with effective means for investigating, interpreting, explaining, and making sense of the world in which they live.
>
> Mathematics ... [enables] students [to] develop the ability to think creatively, critically, strategically, and logically. They learn to structure and to organise, to carry out procedures flexibly and accurately, to process and communicate information, and to be positive about intellectual challenge (Ministry of Education, Draft as at June, 2006).

Book 1 from the Numeracy Development Project (NDP) states:

> The Number Framework has been established to help teachers, parents, and students to understand the requirements of the Number strand from *Mathematics from the New Zealand Curriculum.* (Ministry of Education, 2005a, p. 1)

According to NDP Book 3 (inside front cover),

> in the first four years of schooling, the main emphasis should be on the number strand; [but] in the middle and upper primary years of schooling, the emphasis is spread across the strands of the curriculum. (Ministry of Education, 2005b)

This means that mathematical processes, including problem solving, developing logic and reasoning, and communicating mathematical ideas, should be an important part of classroom mathematics programmes from about year 5 onwards.

In recent years, writers have drawn attention to the importance of talking with and listening to students in order to appreciate their unique perspectives (for example, Fielding, Fuller, & Loose, 1999; Rudduck & Flutter, 2000; Young-Loveridge, 2005; Young-Loveridge & Taylor, 2005; Young-Loveridge, Taylor, & Hāwera, 2005). Although children spend a lot of time doing mathematics, we know little about how they view the mathematics they do. A few studies have explored this issue, but most were with students at about the year 5–6 level (Grootenboer, 2003; Howard & Perry, 2005; Masingila, 2002). Howard and Perry held conversational interviews with Aboriginal children living in a remote rural community in New South Wales to explore their beliefs about learning mathematics. Most examples of the children's responses seemed to reflect an external conception of mathematics, with the children positioning themselves as passive recipients of the teacher's wisdom and superior knowledge. According to Howard and Perry, these children did not seem to be aware of their own mathematical competencies, strategies, and problem-solving abilities in mathematics. Instead, they emphasised the importance of watching and listening to the teacher. Grootenboer investigated the views and feelings of New Zealand children on the nature and purpose of mathematics and how they saw themselves as learners of mathematics. The children's responses indicated a rather narrow conception of mathematics, limited mostly to number concepts and arithmetic.

Research on the perspectives of year 5 and 6 students who participated in the NDP has shown that they were fairly similar to those who had not yet participated in the initiative in their ideas about the value of communicating mathematically with others (Young-Loveridge, Taylor, & Hāwera, 2005). There was a somewhat greater difference between the two groups in recognising the value of knowing how others solve mathematical problems, with NDP students valuing this more highly than those who had not yet participated in the initiative. Subsequently, the study was broadened to include students in years 2–4 and years 7–8, as well as those in years 5–6. The present paper reports on the students’ views of what mathematics is.

Method

Participants

The participants in this study were 459 students from years 2–8 (six- to 13-year-olds) attending six primary schools and six intermediate schools in two major urban centres. Half of the schools (three primary and three intermediate) had already been involved in the NDP, and half had not yet been involved. The year 2–4 group were from one of the NDP schools whose year 5–6 students had been interviewed the previous year. The selected schools included substantial numbers of Māori and Pasifika students. This was done so that a better understanding of Māori and Pasifika perspectives could be taken into account in efforts to raise mathematics achievement and narrow the achievement gap.

Table 1
Composition of the Sample as a Function of Ethnicity and Year Group

Ethnicity	Years 2–4	Years 5–6	Years 7–8
European	43.2%	29.0%	23.8%
Māori	35.1%	55.2%	35.1%
Pasifika	8.1%	8.2%	38.1%
Asian	13.5%	6.0%	2.9%
Number of students	37	183	239

Procedure

Schools were asked to nominate students from across a range of mathematics levels. The students were interviewed individually in a quiet place away from the classroom. Students were told initially that the interviewer was interested in finding out more about “how kids learn maths and how their teachers can help them” and “what kids themselves think about learning maths”. The interviews were audio-taped and later transcribed. A content analysis of the tape transcripts was completed to identify common themes and ideas. Coding categories were then constructed for use with each transcript.

This paper focuses on students’ responses to the question “What do you think maths is all about?” If students didn’t respond to this question, they were then asked “If you were going to tell someone about what maths is, what would you say to them?” If that yielded no response, students were then asked to imagine: “What if a spaceship landed on the field, and the people came into your school and wanted to know what is this thing called maths that you kids do, what would you tell them?”

Findings

Students responded to the question about the nature of maths in a number of ways. A notable group of students were unable to give any response at all. Those who did respond seemed to interpret the question in a variety of different ways. Some students appeared to interpret the question in terms of their immediate mathematics learning in the classroom, commenting on aspects of mathematical content or highlighting general learning and thinking processes. Other students interpreted the question in terms of the purpose of mathematics for them in the "here and now" and went on to mention ways that mathematics was useful. Another group interpreted the question with respect to the purpose of mathematics but also considered this in relation to their long-term futures. These students talked about the mathematics they thought was expected at more senior levels of the schooling system, "getting an education", and getting a job. One group of students, who seemed to have really thought about the nature of mathematics, commented on the intrinsic value of learning mathematics, such as for solving problems. They talked about challenges such as "figuring things out". A few students commented on the essence of mathematics as being about "having fun".

Once the coding categories had been determined, a systematic analysis was made of all 459 transcripts, noting which categories were referred to by the students. Cross-tabulation using SPSS (Statistical Package for the Social Sciences) allowed an examination of the frequencies for each category as a function of numeracy project involvement (NDP versus non-NDP). This quantitative analysis is presented here. In the section following, selected excerpts from the transcripts are presented to illustrate each of the coding categories. These excerpts allow the students' voices to be heard and help to bring the quantitative analysis to life.

Quantitative Analysis

Table 2 shows the percentages of students in each aspect of mathematics as a function of year group and involvement in the numeracy project (NDP vs non-NDP). It should be noted that the responses of students were coded in more than one coding category if they referred to more than one aspect of mathematics. Hence, the totals add up to more than 100%.

Table 2
Percentages of Students Who Mentioned Each Category as a Function of Year Group (years 2–4, 5–6, 7–8) and Project Status (NDP vs Non-NDP)

Aspect mentioned	Years 2–4 NDP	Years 5–6 NDP	Years 5–6 non-NDP	Years 7–8 NDP	Years 7–8 non-NDP
Mathematical content	48%	48%	44%	52%	49%
Learning	59%	35%	55%	16%	18%
Thinking	9%	3%	5%	3%	4%
Problem solving	0%	8%	5%	10%	16%
Utility – here and now	3%	12%	6%	4%	3%
Utility – in the future	9%	27%	44%	23%	16%
Enjoyment	6%	9%	3%	1%	3%
Non-responders	12%	19%	5%	17%	12%
Number of students	37	121	62	123	116

Mathematical content was mentioned consistently by students in all year groups. Although there was no breakdown of this overall category, it was noted that the majority of students mentioned number in their responses. Frequent reference to the learning process was made by the younger students (35% to 59% for students in years 2–6), but much less often by the older students (16% to 18% by year 7–8 students). Fewer students mentioned the usefulness of mathematics to them "here and now" than referred to the usefulness of mathematics for their futures. Future uses included later schooling, "getting a good education", and getting a job. Interest in the future was less of an issue for the youngest students (9% of year 2–4 students mentioned it), but by year 5–6, it was much more frequently mentioned (27% and 44% for NDP and non-NDP respectively). Enjoyment of mathematics was mentioned mainly by the youngest students (6% for year 2–4 students) and by year 5–6 students who had participated in NDP (9%). It was interesting to note that only 1% to 3% of the older students (years 7–8) spontaneously referred to enjoying mathematics.

Mathematical Content

One group of students referred to particular aspects of mathematical content in their explanations of what mathematics is about. Many spoke about aspects of number and/or operations. A few mentioned geometry and statistics:

> Maths is not just about numbers; maths is something that you can make really fun, especially with geometry and symmetry, because you can draw shapes and draw characters that you like. (Year 5–6)
>
> I would say maths ... has a lot of different strands like geometry and stuff, where you work with shapes and there's hard sums and easy sums and short cuts and such things. (Year 7–8)

One younger student mentioned patterns and explained how various operations and domains are interconnected:

> Patterns ... because plus is minus and plus is times and times is division and division is fractions and fractions is decimals and decimals is percentages and it goes on and on. (Year 2–4)

Some students' responses reflected the difficulty they experienced in trying to say what mathematics is:

> I know you use maths for everything in normal day life, but I'm not sure what it's about ... I'd just say it's about numbers and working numbers together and taking them away to work out stuff. (Year 7–8)
>
> Maths is like, you write down, you've got all these numbers and you've got all these maths symbols, so you've got numbers from 1 to 10. You have to try and squash them together, so like, for example, 1 plus 9 equals 10. (Year 2–4)
>
> Just memorising numbers, learning how to divide, subtract and stuff, 'cause if we didn't have the numbers then it would be totally different, you wouldn't be able to count things so you wouldn't be able to know how much you'd need for stuff, you'd put the wrong amount, there wouldn't be an amount. (Year 5–6)

Processes

A substantial group of students spoke about processes. These were further subdivided according to whether the focus was on general cognitive processes, such as learning and thinking, or on mathematical processes, such as problem solving specifically.

Learning

Some students commented that the nature of mathematics was about learning. These responses tended to be extremely brief and did not explain how mathematics was akin to learning. Instead, they tended to focus on a justification for being involved with mathematics. The following are just a few examples of responses that referred to learning:

> Just learning, for when you get older. (Year 7–8)
>
> Learning and education and finance, stuff like that. (Year 2–4)
>
> Learning and helping you get brainier. (Year 2–4)

Thinking

Quite a number of students commented that mathematics was about thinking or using their brain:

> Well, it's kind of like a challenge for your brain and stuff. (Year 2–4)
>
> It's about learning, helps kids think. (Year 2–4)
>
> Using your brain and thinking. (Year 2–4)

Problem solving

A small group of students said that mathematics was about problem solving but gave little or no explanation for their views:

> I think maths is about problem solving. (Year 7–8)
>
> It helps you so you can get smarter and when you're in a problem or something. (Year 5–6)
>
> Like, if you had to build a house or something and you have to find out the area and how much more you have to put in to make it bigger. (Year 5–6)

Effort and persistence was mentioned by one student, who responded that mathematics is about:

> trying your best on your work, like don't give up on your work and just do scribble, and just give it a try if it's too hard. (Year 2–4)

The Utility of Mathematics

Quite a large number of students talked about the usefulness of mathematics. This group was further subdivided into those that considered mathematics in terms of its immediate utility (in the "here and now") and those who were more concerned about their long-term futures.

Everyday life here and now

Some students focused on the usefulness of mathematics for their everyday lives, and many of these referred to needing maths to be able to work with money:

> Maths is, like, something you use every day. You need to learn it because it can help you in life, 'cause you use it like every day when you're doing stuff. Like money and stuff, you calculate your money. (Year 7–8)
>
> Trying to learn them for when you're older, for when hard questions come and stuff. Like paying bills and stuff, or loans and stuff. (Year 7–8)
>
> If you need some money out of your wallet, you might be able to use maths and equations, or if you work at a bank or at a dairy, maths would help you out – how much change you get. (Year 2–4)
>
> So when you grow up, instead of, when you go shopping, you know it straight away instead of going like that and using your fingers. (Year 5–6)

Life in the future

Quite a number of students chose to respond to the question about the nature of mathematics by talking about how worthwhile it was for the future. Some students' comments about the future were in relation to higher levels in the school system:

> I think maths is teaching me more so I can move on to the senior school and I can be ready to learn even harder maths questions. (Year 2–4)

> Learning and teaching about maths, like if you go to senior schools, they tell you about maths, and at senior schools, it's more harder and it's better. (Year 2–4)

Students in all age groups commented on the importance of maths for the future in terms of getting a job. Below are the responses of two year 3 and 4 students:

> Learning your maths so you get better when you are older for your job because you need maths. (Year 2–4)

> I think it is about learning new ... like if you want to teach other kids when you are an adult, when you want to be a teacher, you have to learn from your last teacher that taught you. (Year 2–4)

Year 5 and 6 children gave slightly more sophisticated explanations, including comments about handling money:

> Learning so you can handle with money so then you can grow up and get a job – you'll have to know how to sort out money. There's this kid called Mac in my class – he says that maths is stupid and he doesn't need maths to be a mechanic and stuff ... I say that you need maths for every job when you grow up because it has maths. You need maths to sort out the money. (Year 5–6)

One articulate year 6 student gave a response that showed considerable reflection on the importance of mathematics to him personally:

> Well for me ... the main thing in school for me because most jobs you go to, basically every job involves a bit of maths, quite a lot of maths actually, and so by learning maths, sometimes I don't enjoy it, but I know that's like a good thing to learn and so it's sort of like a goal setter for life. If you know it, it just helps you to become more independent in a way 'cause you're not relying on the teacher a lot ... If you don't do maths, you won't really get a good job, so ... it's sort of a thing that sets you up for life really. (Year 5–6)

Not surprisingly, year 7 and 8 students expressed the most sophisticated ideas, commenting on their future roles as adults, jobs, getting on in life, and other activities reflecting independence and autonomy:

> Future jobs, you need to use it a lot. You can't just go through school without maths, you need to know how it works to see, like statistics with graphs and stuff, you need to be able to read them and understand them to see other things. (Year 7–8)

> Figuring out and adding for lifestyle for when you're an adult ... If you're a person at the shop, giving the person back their change, figure out how much they get back. (Year 7–8)

> Maths is in every average day in everything you do, and I think maths is just helping you for the long run. And when you'll need to use it, and it's also a good general knowledge thing, just to know what to do, 'cause it's everywhere, maths. (Year 7–8)

Enjoyment

A small but notable group of students considered the nature of mathematics to be about having fun:

> Having fun and trying to get your numbers and answers right, and just try and learn quicker and easier. (Year 2–4)

> It's actually quite fun. (Year 7–8)

Non-responders

A considerable number of students (between 5% and 19%) appeared to have no view at all about the nature of mathematics. These students said things such as:

> I've never thought about it. (Year 7–8)
>
> I have no idea. (Year 7–8)
>
> I'm not completely sure. (Year 7–8)

Discussion

It was evident from the responses analysed that it was difficult for some students to talk about the nature of mathematics. These findings suggest that many students do mathematics without much thought or opportunity to discuss what it actually might be.

Many students who offered ideas about the nature of mathematics referred to aspects of the number domain. This is consistent with Grootenboer's (2003) finding that children's views of mathematics tended to revolve around number concepts and arithmetic. Like Fuson et al. (2005), we found that many of these students' responses reflected the view that mathematics is about computation. This is not altogether surprising, given that the NDP emphasises number and mental computation, particularly in the early years of school.

We were interested that a large number of students chose to comment on the usefulness of maths. We found, like Masingila (2002), that these students' perceptions of what mathematics is were linked to how they thought they used it. We were intrigued to find that many students talked about the usefulness of maths for their futures. This is consistent with much of the rhetoric about the importance of mathematics for the "knowledge society" (see Commonwealth of Australia, 2000; Ministry of Education, 2001; National Council of Teachers of Mathematics, 2000).

Despite the 1992 curriculum document devoting a strand of the mathematics curriculum to mathematical processes such as problem solving, developing logic and reasoning, and communicating mathematical ideas, we noted that few students talked about the nature of mathematics in this way, although some students spoke about general cognitive processes such as learning and thinking. This may reflect the relatively narrow views of mathematics held by many people, which in turn may impact on children's views. It was interesting to note that NDP students did not differ markedly in this respect from those who had not yet participated in the initiative. Perhaps this reflects the strong focus on the number strand that is a key aspect of the NDP. It might be valuable to remind teachers that mathematical processes become increasingly important once students get through the first four years of school.

Mathematicians such as Holton (1993) have written about the pleasure people get from doing mathematics for its own sake. It was heartening to see that some students saw mathematics as being about having fun, but it seemed to be the younger students who described mathematics as an enjoyable pursuit. It was interesting to note that more of the year 5–6 students who had participated in NDP spontaneously referred to having fun in mathematics than same-aged peers who had not. However, this pattern was not evident for year 7–8 students. The analysis suggests that there is an age-related decline in students' enjoyment of mathematics. Analysis of students' response to other questions may help to throw light on this issue.

We were interested in those students who did not appear to have a view about the nature of mathematics (the non-responders). As Presmeg (2002) has pointed out, beliefs about the nature of mathematics are important because they can either help or hinder the making of links between school mathematics and everyday practices. If children don't have a view about what mathematics is, that may make it difficult for them capitalise on the mathematics they encounter at home and in other out-of-school settings.

Much of the data seems to indicate that children do perceive mathematics in dichotomous ways. Some students considered that mathematics was an external body of "stuff to be learned". Others suggested that they needed to make sense of the mathematics in order to make connections between related mathematical ideas. Many students were aware of the significance of mathematics in society, but others had a more mechanistic view.

Given that mathematics is part of the core curriculum, we think it could be important for teachers to help students engage with ideas about the nature of mathematics. In preparation for such discussions, teachers might benefit from examining their own beliefs about what mathematics is and reflecting on the subtle messages they might convey to students about the nature of mathematics. This is an issue that numeracy facilitators could address as part of their professional development with teachers.

Acknowledgments

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Modelling Books and Student Discussion in Mathematics

Joanna Higgins
Victoria University of Wellington
<joanna.higgins@vuw.ac.nz>

with

Maia Wakefield
Massey University

Robyn Isaacson
Flaxmere School

Ehara taku toa i te toa takitahi,
engari he toa takitini.

My strength is not that of a single warrior but that of many.

Manipulation of materials, commonly referred to as "hands-on", as a strategy for learning mathematics is widely applied in New Zealand primary classrooms. A modelling book can enhance a hands-on approach through linking modelling and discussion of mathematical ideas as promoted through the Numeracy Development Projects (NDP). This approach has been interpreted as "kinaesthetic". One of the potentially most damaging applications of kinaesthetic learning has been to Māori students, among others. Modelling books link modelling and discussion of mathematical ideas, as promoted through the NDP. Modelling books may help teachers to reconceptualise hands-on learning to include discussion of mathematical ideas and provide a means of developing conceptual understanding through the introduction of mathematical abstractions.

Background

Historically, classroom practices in New Zealand elementary mathematics have emphasised the manipulation of objects. The emphasis on "hands-on" can be traced back to the introduction of a child-centred approach around the beginning of the 1970s. Teachers' interpretations of the child-centred approach favoured manipulation of materials (Higgins, 1998). However, discussion of the mathematical ideas represented by the materials has not necessarily been included in their practices.

A matter of greater concern is that, guided by popular literature on learning styles, hands-on has been interpreted as "kinaesthetic". This interpretation does not distinguish between students' pleasure in handling the materials and their understanding of the mathematical principles.

One of the potentially most damaging applications of kinaesthetic learning has been to Māori and Pasifika students (Higgins, 2001). We have been challenged by Bishop, Berryman, Tiakiwai, and Richardson (2003) to examine dominant teacher-centred "monocultural pedagogies developed in New Zealand on the basis of unchallenged metaphors" (p. 23). Furthermore, the treatment of Māori as a homogeneous rather than as a diverse category has led teachers to consider simplistic pedagogical strategies such as "Māori learning styles" (McKinley, Stewart, & Richards, 2004) as kinaesthetic.

An important pedagogical shift promoted through the Numeracy Development Project (NDP) has been to think of learning as participation in a mathematical conversation. Such discussions are frequently situated in a small teacher-led group of about six to ten students. Part of the reason for the emphasis on such a setting is to enable students to individually and collectively manipulate mathematical representations such as a number line or an abacus and afford all members of the group the opportunity to participate in discussion.

The NDP strategy teaching model (Ministry of Education, 2006) emphasises both the manipulation of materials and the explanation by students of their action in terms of the underlying mathematical ideas. If interpretations of kinaesthetic learners equate in practice to restricting students to "modelling with materials" without an emphasis on discussion of mathematical ideas, students may have little opportunity to move beyond modelling to the imaging and abstraction through the number properties phases of the teaching model.

The Modelling Book

This investigation is based on the assumption that manipulation and discussion need to be linked. A pedagogical strategy in common use that links manipulation and discussion is to record the thinking of the teacher-led group. Some teachers use a large scrapbook or hand-made book to record the collective thinking. This book has various labels, such as a "recording" or "modelling" book. For the purposes of this discussion, we will call it a modelling book.

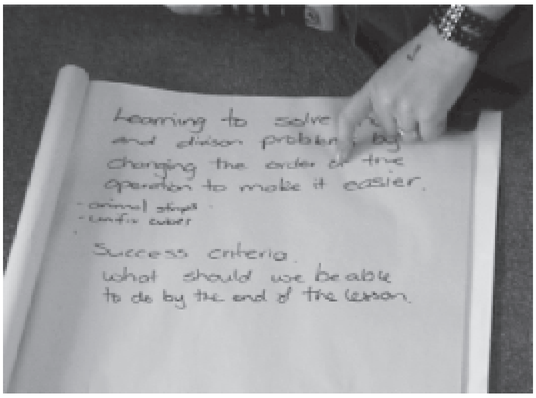

Figure 1. Modelling book

Typical practice is for the modelling book to be either placed in the middle of the floor with the group sitting around it in a semi-circle or on an easel with the students in front. In both cases, as the main recorder, the teacher is adjacent to the book.

The present research investigated the ways in which a modelling book supported interaction and student learning in teacher-led small group work in mathematics through conversations among Māori students and between them and the teacher. Of particular interest was the extent to which the modelling book helped the teacher to bridge hands-on manipulation of materials with mathematical abstractions; and secondly, the usefulness of the modelling book in generating collective responsibility for the learning of the group. The first part of the paper presents theoretical frames for the analysis of the use of the modelling book as a classroom practice. The latter part of the paper examines evidence gathered from classrooms in the light of the extent to which modelling books were helpful learning tools for Māori students in the classroom investigated.

Theoretical frames

Conceptualising Cognition as Physically and Socially Situated

For the purposes of the discussion, the modelling book has been conceptualised as an inscription that facilitates interaction (Roth, Woszczyna, & Smith, 1996). In their study of the affordances and constraints of computers in science education, Roth et al. noted that "the physical presence of the object of talk provides students with a means for coordinating just what they are talking about" (p. 1009). They argue that visual support is more important than manipulation of an object. In this sense, drawing on the earlier work of Erickson (1982), "students coordinate their conversation directly over and about the display which provides an anchor for the conversational topic" (p. 1009). Furthermore, they noted the importance of gestures as an important element of the co-ordination and sense-making and that "participation in discourse is dependent on access to the representational devices" (p. 1008).

This paper is based on the notion of cognition as physically and socially situated. The joining of the physical or material object (in this case, the modelling book) and the social setting (in this case, the small teacher-led group) is an important shift from the traditional notion of cognition as being solely in the heads of learners. While much has been recently written about socially situated cognition, the place of inscriptions in cognition has been less frequently examined. Notable exceptions to this are the work of Cobb (2002), Roth and McGinn (1998), and Sfard (2000), who use inscriptions in the sense that material objects are inscribed with messages.

Findings

The claim is that modelling books can be used as a strategy for engaging students in mathematical discussion by supporting student interaction in a teacher-led small group. This support is given through co-ordinating the conversation and contributing to students' sense-making activities. Furthermore, modelling books may help teachers to reconceptualise hands-on learning to include discussion of mathematical ideas and as a means of developing conceptual understanding through the introduction of mathematical abstractions. Modelling books are a way of addressing Alton-Lee's (2003) challenge for students to "have the relevance of their learning activities made transparent" (p. 90). The rest of this paper will examine the evidence of the affordances and constraints of using modelling books as a pedagogical strategy to support discussion and manipulative use when solving mathematical problems in the teacher-led group. This will be discussed under the dimensions of physical, social, and conceptual aspects of modelling-book use. The illustrations are excerpts from interviews informed by classroom observations in eight English-medium year 7 and 8 classrooms across two regions in New Zealand as part of the evaluation of the NDP in 2005.

Physical Dimensions of Using Modelling Books

The simple physical existence of a modelling book may afford students and teachers a reference point in teacher-led groups. It may act as a memory jog in tracking previous aspects of the discussion. Where records are illegible, their "scrawly/messy" form serves as a constraint by inhibiting the communication of ideas and distracting focus from the mathematical topic at hand. A visual record of the oral discussion may be particularly helpful for those with impaired hearing or with English as another language.

> Writing it down is really important because it's the visual aspect to tune in. (Arthur, teacher)[1]
>
> Cos it [the modelling book] is something to see instead of like doing it in your head and you forget when you go to do another part of it and you can see it on the paper. (Student)

Teachers commented on the usefulness of a modelling book for reminding them not only where a particular group is at but also where they need to go next to develop students' understanding of mathematics. Teachers using the Number Framework (Ministry of Education, 2006) to guide pedagogical decisions typically refer to stages of mathematical thinking (for instance, early additive or advanced multiplicative). The process of applying the Framework to the teaching of specific students is often a complex process involving ongoing analysis of student responses. A modelling book can support this process as a tool that is immediately to hand in the teaching setting (Roth & Tobin, 2001).

> Having those [modelling books] open gives me the focus for that particular group and I look at things in the past. I've never grounded myself [before using modelling books] in terms of keeping to what the kids know and where I could take them. (Trisha, teacher)

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1 Teachers' names have been changed.

Students commented that they found the modelling book helpful as a reminder of material covered, as well as a way of charting the teacher's examples.

> Cos then we know what to do and she gives us good examples, and so next time we do it correctly, and she gives us harder ones, and then we do other things, and then she reminds us later on when we do it again. (Student)
>
> She showed us the pattern that we did, just to remind us. (Student)

The placement of the modelling book, in terms of proximity and ease of access to the teacher and to students, is critical to the extent to which it can act as a reference point in discussion. The book may constrain interactions when it is physically accessible to the teacher but not to the students, such as when it is upside down or out of the students' reach. By contrast, the book affords or supports interaction when both students and teachers are able to physically access it, such as when it is placed on the floor in the centre of the group facing the right way up for students to see.

The physical space on a page, or parts of a page, of a modelling book may be regarded by group participants as "teacher", "student", or shared "teacher/student" space. This can be demonstrated by the ways in which participants physically (for example, through gestures) incorporate the inscriptions in the modelling book into their verbal responses. In the observed teacher-led sessions, teachers generally regarded the book as shared teacher/student space.

> It's where I write, it's where they write. (Esther, teacher)

Students had a similar understanding that the book was a shared tool.

> We can all sit around the table and look at the big maths [modelling] book. Sometimes [when people explain], they have a different way of figuring out from you, so you think, oh yeah, I could have done it like that too. (Student)

Students and teachers also appeared to have shared understanding that a teacher may use parts of a page as "teacher" space. This may be indicated to students through gesture, as in Figure 2,

Figure 2. Teacher space in modelling book indicated through teacher gesture

or as space that is specific to particular students:

> Cos when she does it she does different [examples] for kind of like each person, she does a hard one for like Nancy, cos she knows quite a lot of maths and all that, and she does, like, quite an easy one for us lower kind of average kind of maths people. (Student)

Social Dimensions of Using Modelling Books

Modelling books can fulfil a social function through supporting group interactions. The extent to which a book co-ordinates discussion is evident through reference to it in conversations. As a common focal point, it appears to be useful to both the teacher and to students in providing an anchor for the conversation that can contribute to the development of students' understanding.

> If someone did jump ahead ... that's the same as a quarter or that's the same as two-quarters ... you could see where they were basically at in terms of just looking at what the [student] picture [in the modelling book] looks like. (Trisha, teacher)
>
> If someone else has a different opinion, she just writes down on the side and then we all figure it out. (Student)

A book may become a shared recorded history of previous learning that affords both the teacher and students a means of informing discussion through linking back to previous mathematics sessions.

> The following day, you go back and you've got it right there in front of you, you haven't got everybody's workbook out ... There's times in my classroom when I've had to hunt those books down. (Trisha, teacher)
>
> The kids can go back to it. (Esther, teacher)

Such opportunities may be constrained when the book is merely the teacher's recorded history of previous learning. In this case, it becomes the teacher's choice of whose ideas are recorded and how this is done.

The teacher may specifically use the book as a means of focusing each group as they take their turn with the teacher. The visual record displayed affords both the teacher and students a quick reference point.

> And the main thing I use [the modelling book] for is to be focused ... I'll do three groups today, I'll do two groups tomorrow ... then they come back to me in two days' time ... they'll come back, they'll refocus, we'll talk about it and they can see visually and they connect them, that's how I use it. (Arthur, teacher)
>
> ... one of the kids said, "Whaea, I don't remember how to play that game." I said, "Gee, I wonder if that will be in our workbook – shall I go and find out?" She goes, "Oh yeah," and away she went, she opened it up and then had a look at the samples that we did and they got it. (Trisha, teacher)

Students find a modelling book useful as a reference point for gauging their progress as a group.

> So we know what we have improved on and what we need to improve on. (Student)
>
> She keeps old work in the big scrapbook so we can see if we've improved. Like we started off and we've gone to the end of our maths and seen if we've improved ... sometimes I don't think I've improved. (Student)
>
> [if you didn't have the scrapbook in the middle] we'd miss what we improved and we won't know if we've learnt anything or not. (Student)

Students may become motivated to participate in discussion by seeing their ideas valued through being recorded. From the students' perspective, it becomes a recorded shared history of previous learning while making students' ideas accessible to all other students.

> It's got all our pieces of work that we've finished, and she writes down some ideas from the lessons that we've been taught in maths. (Student)

Furthermore, a modelling book has potential as a collective mechanism for supporting the enactment of groups as an instructional tool such as described by a teacher in a previous study who thought of her class as a waka, or groups within groups (Higgins et al., 2005).

Conceptual Dimensions of Using Modelling Books

Having a way of recording immediately to hand allows teachers to introduce the symbolic form through providing students with a bridge between physical models and mathematical abstractions. The modelling book provides a means of making the links between models and abstractions explicit. However, teachers may be unable to make links between recording (inscriptions) and mathematical abstractions because of limited subject and/or pedagogical content knowledge.

> Yeah, it's important to have the written recording alongside actual hands-on stuff. You can do lots of hands-on stuff, but then you give them a Figure It Out book and perhaps like three times two and they go what does that mean ... So that recording is there. So anything I do or any equation we talk about we write it down. ... So quite often I give them the pen [and ask] "How would you write that down?" (Esther, teacher)

As part of this process, the teacher and students are using literacy to support the language (vocabulary and syntax/structure) of mathematics.

> They say three groups of two ... you write down three groups of two ... and I'll actually put down three times two. So ... they're picking up the language ... (Trisha, teacher)

The act of recording gives students thinking time by slowing down the pace of the lesson through the extra time taken to record the ideas raised in discussion.

> She makes us think instead of her just saying it. (Student)

Students saw the modelling book as a tool for the teacher to track their conceptual understanding.

> She keeps a [modelling] book so she knows, she can find out the answers that we like to say and how we can improve on doing what we've done, making the same answer but making it sound better. (Student)

> She ... tries to make you improve on it so you can find different answers for [the problem] ... She finds other ways for you to answer it so it's easier. (Student)

> Sometimes we don't know that we have the answers and we try and ... we end up finding the answer ... we say it [and] find out that we did know. ... It was helping us learn for something we already knew but didn't know we did know it. ... If we didn't have the book, we'd pretty much never know all the things we did know. (Student)

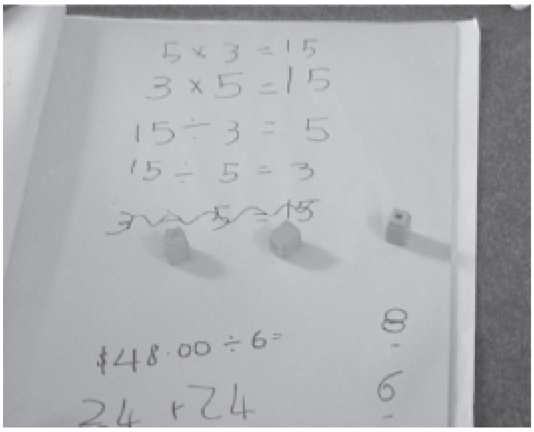

Figure 3. Tracking students' conceptual understanding

Conclusion

This paper began by discussing the problems of using the notion of kinaesthetic learners as a strategy for working with Māori students. As has been argued elsewhere (Higgins, 2001), the manipulation of materials by itself will not necessarily lead to an understanding of mathematical principles. Nor can we assume that students understand the purpose of a pedagogical tool (McDonald et al., 2005).

The modelling book reinforces the complexities of hands-on learning in ways that the notion of kinaesthetic learning tends to gloss over. The aspects of learning made visible through the use of modelling books in mathematics learning may provide important support for diverse learners.

The modelling book gives the students additional information beyond the manipulation of materials and participation in discussion that they can use in building their mathematical understanding. It also supports the development of collective enterprise in solving mathematical problems. In the settings in which the use of modelling books was investigated, Māori students responded well and there is no reason why this approach should not suit all students.

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Contextually Responsive Facilitation

Joanna Higgins
Victoria University of Wellington
<joanna.higgins@vuw.ac.nz>

Sandi L. Tait-McCutcheon
Victoria University of Wellington
<sandi.mccutcheon@vuw.ac.nz>

with

Raewyn Carman
University of Auckland

Donna Yates
University of Auckland

Contextually responsive facilitation enables a facilitator to take account of each teacher's context of practice through making explicit the mathematical concepts and strategies underlying materials or activities. This paper explores co-teaching and co-generative dialogue as a means of developing contextually responsive facilitation and contends that such an approach engages teachers in contextually based participatory learning of new practices. Through the role of co-teacher, a facilitator is able to guide teachers' interpretation of the core principles of the project.

Orientations to Professional Development

Any large-scale intervention is challenged by shifting practice across a wide range of teaching contexts. An example of a large-scale initiative is the Numeracy Development Project (NDP), which is in its sixth year of implementation. It has focused on improving the quality of teaching and learning in mathematics in English- and Māori-medium settings in Aotearoa New Zealand.

Debates about what counts as school change have typically focused on quantitative measures and omit a consideration of changes to teaching and learning that impact on student achievement (Coburn, 2003). Some commentators (Elmore, 1996; McLaughlin & Mitra, 2001) emphasise the importance of teachers understanding the core ideas of an initiative for its longer-term sustainability. Elmore, in particular, is critical that much of what counts as change does not address what happens in classrooms and suggests "the primary problem of scale is understanding the conditions under which people working in schools seek new knowledge and actively use it to change the fundamental processes of schooling" (p. 4).

A major issue in shifting practice is fostering teachers' interpretations of core principles of a project in ways that ensure that both the integrity of the principles and the contextual factors impacting on classroom implementation are privileged. Historically, large-scale interventions in mathematics in New Zealand have failed to have an impact longer-term because the balance between core principles and local contexts of implementation has been lost through adherence to practices without adoption of their core underlying principles (Higgins, 2001; Young-Loveridge, 1997). Specifically, we need to understand how facilitators best enable an evolutionary dynamic, arising from a contextually informed understanding of the principles, to develop in a school so that it leads to new practices being sustained.

The degree of alignment between the model of instruction being promoted for classrooms through a professional development programme and the model of instruction of the professional development itself is critical (Tharp & Gallimore, 1988). The orientation of a facilitator's pedagogy to aspects of practice in the in-class setting can vary from one of design adherence to one of contextual responsiveness (Higgins, 2005). In a design adherence orientation, the facilitator's emphasis is on classroom activity that follows the guidelines of the teacher manual. In a contextually responsive orientation, by contrast, a facilitator emphasises student learning through attention to structural elements of a programme.

Opportunities for teacher learning in each orientation can be examined from the perspective of a transformation of participation theory (Rogoff, Matusov, & White, 1996). From this perspective, a teacher's participation can be said to be shaped by their underlying orientation to professional development. In a design adherence orientation, the participation of a teacher is evaluated by the degree of adherence of the teacher's actions to the design. For instance, when using materials or activities, the teacher and facilitator would focus on the surface features of the activities and judgments about effectiveness would centre on the degree to which the materials actively engage students. By contrast, in a contextually responsive orientation, a facilitator would evaluate a teacher's participation by the degree to which the use of the materials met the mathematical needs of the students as determined by the Number Framework. Similarly, when modelling new practice, a facilitator oriented towards design adherence would emphasise and encourage adherence to the procedural aspects of activities as scripted in the teacher handbook and would discourage variations to these procedures. A facilitator modelling new practice from a contextually responsive orientation would highlight possible variations and encourage discussion of multiple perspectives in working with different groups of students. The Teaching Model and the Number Framework would underpin such discussion.

Contextually, responsive facilitation enables a facilitator to take account of each teacher's context of practice through making the concepts and strategies underlying materials or activities explicit. It does this by drawing on the elements of the Number Framework, the associated diagnostic interview, and the Teaching Model, all of which are structural features or key components of the NDP. Through introducing a framework of ideas, it is suggested, "teachers are able to internalise the changes to their practice and sustain the programme in terms of the context within which they work" (Higgins, 2005, p. 143).

Theoretical Framework

In teacher-centred professional development programmes, it is important to manage the introduction of classroom activities with discussion of the underlying core principles. This study draws on sociocultural perspectives, such as those articulated by Wertsch, del Rio, and Alvarez (1995), who suggest that a facilitator's role is to mediate core principles of a project and their enactment in a classroom setting. Newman, Griffin, and Cole's (1989) work is useful when thinking about the use of examples of classroom practice in feedback sessions between facilitators and teachers. In these sessions, material generated from co-teaching can be appropriated by the facilitator or teacher as a basis for interactions about the core principles of the NDP.

Newman et al.'s work uses Sewell's (1999) notion of culture and follows Tobin's (2005) work, and thus it views teaching as cultural enactment and learning new practices as cultural production, as in the reproduction and transformation of existing forms of culture. Teacher agency is important in this process of learning new practices. Agency can be thought of as the degree to which teachers can interpret and transform core principles of practice as they enact them in a classroom setting.

One way of explaining how structure can be seen as dynamic is by using Sewell's (1992) theory of structure, which suggests that core principles form a dialectic relationship with related resources. Sewell argues that structure is composed simultaneously of virtual schemas and of actual resources. When this is applied to the structural elements or core principles of the NDP, the schemas fall into two groups: one relates to pedagogy, and the second relates to professional learning (Higgins & Parsons, 2005).

Of particular relevance to this paper are schemas relating to professional learning that are classroom-based professional learning and schemas relating to school-based professional community (Higgins & Parsons, 2005). The associated resources include the practices of teachers and facilitators as well as the associated dialogue about practice. These practices are enactments of the schemas or core principles and serve to validate them. As Sewell (1992) explains, “If schemas are to be sustained or reproduced over time – and without sustained reproduction, they can hardly be counted as structural – they must be validated by the accumulation of resources that their enactment engenders” (p. 47).

The Professional Development Components of the NDP

The professional development components of the NDP include workshops with clusters of schools as well as in-depth work in individual teachers’ classrooms. The pedagogy used by facilitators when working with teachers one on one in their classrooms is important in affording teacher agency (see the explanation in the section above) in their adoption of new practices. The way in which a facilitator sets up the in-class professional development setting shapes the opportunities for teachers to participate in, and learn from, the professional development activities.

This research investigated co-teaching and co-generative dialogue (Tobin & Roth, 2005) as a means of developing contextually responsive facilitation that provides teacher agency. In particular, the research looked at how facilitators work with teachers in their own classrooms in ways that incorporate teachers’ context of practice. An important factor in this process was the roles taken by facilitators as co-teachers working alongside the classroom teacher. The impact of this approach enabled facilitators to work with a teacher on embedding core principles of practice in a specific teaching context. Facilitators were able to help teachers to contextualise core principles of the NDP through a strategy of co-teaching and co-generative dialogue.

This paper contends that co-teaching and co-generative dialogue engages teachers in contextually based participatory learning of new practices.

A Contextually Based Participatory Model

Whole-class and strategy group modelling as elements of the NDP professional development can be interpreted as a process of mimesis, where learning occurs through the mimicking or imitating of another. Furthermore, this suggests a level of compliance and the forsaking of current pedagogical knowledge and ability. It could be tacitly suggested that within this dyad, the facilitator takes the “expert” role and the teacher the “apprentice” role.

The demonstration/observation delivery of facilitation can assume a prerequisite for readiness on behalf of the teacher and can limit the extent to which they can access and participate in the learning. By placing the teacher in the spectator role, the person with the most contextual knowledge about how to best teach *these* children in *this* class or group with *these* tools at *this* time may be subjugated to a position where they have few opportunities to participate in the lesson. This implies a design adherence model that may limit opportunities for variations from the lesson plan (Higgins, 2005).

Added to this is the awareness that knowledge acquired out of context is not easily generalised or transferred to unfamiliar situations. While the facilitator and teacher are in some sense co-ordinated, they are still, as Rogoff, Matusov, and White (1996) suggest, compartmentalised in a way that differs from collaboration, in which people's ideas and interests meet (p. 394).

Roth and Tobin (2001) describe how the presence of an evaluator in the observation model changes the situation, and in such staged lessons there is a fundamental shift from the goal of teaching for student learning to demonstrations of effective teaching. Sergiovanni (2004) sees this as teachers show-boating the required behaviours but when no-one is looking, reverting back to what they know and what makes sense to them. Lave and Wenger (1991) suggest that learning does not arise by replicating the performances of others or by acquiring knowledge transmitted in instruction; rather that learning occurs through participation in a community of learners (p. 100).

Lave and Wenger (1991) believe that a condition for the effectiveness of learning is engaging in learning rather than being its object (p. 93). By limiting the actions and involvement of teachers, the construction and re-construction of their conscious and unconscious schemas and practices may be inhibited. Remembered events may not invoke the same accuracy of recall or reliability as events that have been participated in, and it is difficult to contextually build understandings from the sideline.

A more responsive paradigm of facilitator modelling and observing would be to make an impact on practice by participating in it – learning to teach through teaching and through talking about teaching. Roth and Tobin (2002) describe this as co-teaching and co-generative dialogue. Research by Roth and Tobin has shown that co-teaching is a powerful context that provides new opportunities for enhancing learning and for learning to teach. Learning becomes increasingly salient and is grounded within the teacher's own experiences. The teacher polishes their own practice within the active context of others. Lave and Wenger (1991) propose that within the context of a changing shared experience, learning becomes an integral and inseparable aspect of generative social practice and a part of the "lived in" world. Learning involves social participation in a community of practice and is in the relationships between the people.

Co-teaching is a seamless collaboration of relearning to teach together in anticipatory ways in which all teachers are tuned in and focused on collective goals and learning (Roth & Tobin, 2002). Their argument is that individual capacity increases through collective activity and action. A productive learning environment is created through actively teaching and actively learning to teach in ways that afford the learning of their students. It is "at the elbow" support for teachers in their classrooms as they apply new ideas and skills. Teachers, researchers, and, where appropriate, students become co-creators of their learning and environments; they become agents of change and are empowered to act.

Co-generative dialogue is collective remembering and theorising to improve the quality of teaching through co-participation in conversations over shared experiences. According to Roth and Tobin (2005), co-generative dialogue articulates the different kinds of individual and collective experiences and explains them in and through collective interpretation, from which new possibilities for individual and collective actions emerge.

Local theory is constructed through forums of shared respect, rapport, and responsibility. In these forums, attention can be drawn to salient aspects of teaching in which changes can be contemplated (Roth & Tobin, 2002). Through co-generative dialogue, the gap can begin to be closed between educational theory and teaching practice and between the theory of best practice and the practice of best practice. Roth et al. (2002) propose that "regular co-generative dialogues can be forums for building shared responsibility, respect for one another, and the rapport necessary to enact curricula that affords learning" (p. 279).

Lave and Wenger (1991) believe that learning is a situated activity and that the way to understand the learning is through the analytical perspective of legitimate peripheral participation. This is a process by which newcomers become part of a community of practice through speaking about relations, activities, identities, artefacts, and communities of knowledge and practice. Participation is based on situated negotiation and renegotiation of meaning in the world. Understanding and experience are in constant interaction and are mutually constitutive. Learning implies becoming a different person with respect to the possibilities enabled by the systems of relations (activities, tasks, functions, and understandings). The key to legitimate peripheral participation is having access to the community of practice and the information, resources, and opportunities that that membership entails. Eick and Ware (2005) found that situated learning theory, when applied to teacher communities of practice, addressed the inadequacies of a traditional approach where learning to teach occurred mostly out of the context and culture of the classroom.

Two case notes follow: one that examines co-teaching and co-generative dialogue within whole-class knowledge lessons; and the other that examines co-teaching and co-generative dialogue within strategy group lessons. The data was gathered through observation of classroom sequences and co-generative dialogue sessions (a total of 15 hours) and 12 formal face-to-face interviews with both teachers and students that were complemented by informal discussions with teachers, students, principals, and facilitators.

The Best Way to Learn How to Teach Is to Teach

Case Note One: School A

During 2005, the praxis of co-teaching and co-generative dialogue was undertaken with four teachers (two new to the NDP), two teacher aides, and 100 year 0–3 students at a co-educational integrated full primary school with a roll of 220. This school was beginning their fourth year of NDP professional development. The aim was to build pedagogical content knowledge through contextually responsive experiences and dialogues.

The teachers created a mind-map of teaching and learning intentions underpinned by the NDP Number Framework (Ministry of Education, 2005). At weekly meetings, the teachers planned the co-teaching sessions by determining the learning intentions *(we are learning to)* and the success criteria *(we will know we have learned this because).* Student experts were identified and included as co-teachers. The teachers selected the student experts on the basis of the mathematical thinking that they had previously shared in a teacher-led group. The students had no special preparation for their role. Teachers collectively decided on teaching and learning for small groups and individuals.

Each week, one teacher had the responsibility for being the lead co-teacher in a joint session of the four teachers, two teacher aides, and 100 students. This entailed introducing the co-teaching session and organising the tools and resources (equipment/whiteboards and pens).

To support this ongoing professional development, the NDP facilitator participated as a member of this community in the co-teaching and co-generative dialogue sessions, each of which lasted for approximately 30 minutes.

Case Note Two: School B

School B participated in whole-school development over a year with a NDP facilitator. It was the school's first year in the project. The school is a large intermediate in an urban area, with a roll of 680 students. The findings are based on the work in five year 7 and 8 classrooms. The facilitator ran workshops with the staff in four groups (7–8 teachers in each) alongside the work in each teacher's classroom. The facilitator modelled a strategy lesson on the first visit, and each subsequent visit entailed co-teaching a strategy lesson with the teacher and modelling further lessons as required. The facilitator ran 30 minute co-generative dialogues with the group of teachers with whom she had worked that day.

Findings

The goal of co-teaching and co-generative dialogue is to generate praxeology – theory that provides new possibilities and new knowledge for action by the participants. Through co-teaching and co-generative dialogue, the learning potential of all is maximised.

> It's building on – when someone says something you can build that on and others will pick it up and take it further; you aren't thinking "we shouldn't be going down there", you're thinking "well, they can do it". It gives me ideas because everyone has different ideas about doing things and you learn so much from others. (School A, teacher D)
>
> It's an authentic kind of environment. You've got a model right there beside you, expertise, and then you bring it across to yourself and then apply your own touch to it. The intervention is right there. It happens straightaway rather than getting feedback later and trying to put it into practice. You get it right there and you can apply it straight away. (School B, teacher E)

There is a strong feeling of collective capacity and commitment among teachers and students and a compelling belief in the potential of all to make a positive difference for all.

> The ones who know things ... they are just so proud and they are so proud to share their knowledge. (School A, teacher A)
>
> We are all learning together, and so everybody can get to really big numbers like ones that go past ... like even past 100. We all know about even bigger numbers cos we are all doing it together. (School A, student aged 5)
>
> She [the facilitator] is helping our teacher basically learn new ways of teaching. New strategies that we haven't learned before that help us figure out and make it easier to learn. (School B, student aged 11)
>
> The kids' learning is the teacher's learning. And the teacher's learning is the kids' learning, they are both interwoven. (School B, facilitator)

Multiple teachers create a synergistic effect whereby more possibilities are available to the participants. Collective action provides a much greater space for change than would exist if teachers attempted change on their own (Roth & Tobin, 2005). Teacher and student experts are available to all learners and expand the opportunities for all learners.

> I talk to the kids who are stuck. I listen to the teachers and then I talk to the kids, so if they aren't sure, I can help. Like getting past ten [holds out hands and wiggles fingers], I can help them get past ten. (School A, student aged 7)
>
> There's heaps of thinking in the room. When I get stuck, I can have someone else's thinking like Jed, and then if others get stuck they can have some of my thinking. (School A, student aged 5)
>
> I supported her [facilitator] with the children in my class who she was working with and she helped me to seek and ask more questions and we just complemented each other. (School B, teacher B)

By directly experiencing the teaching of others, a relevant context is provided for building new teaching habits and transforming praxis. Understanding of the specific teaching/learning situation is developed and so too is local theory.

> The co-teaching for me as a new entrant teacher, I don't really go into those other regions very often, but it's good for me to see where other children are ... what we are aiming for and where we are going to and it helps me to be more focused about what we are doing. (School A, teacher C)
>
> I think that actually it's not like recipes where somebody's talked to you and told you something and you file it. It happens then and there, so it's like you change your behaviour or you modify your behaviour right then when it happens. (School B, facilitator)
>
> I have content knowledge, but the teachers have content and contextual knowledge. They provide the link for me of how to best teach these kids at this moment. With that knowledge, the lesson is personalised and custom-made. (School A, facilitator).

The teachers saw these opportunities for co-generated theory and co-teaching as tools for their individual and collective professional development.

> What's been really good for me is listening to the way you people are getting excited about maths and it's the way maths has become quite high-profile. Maths is a feature; it's flavouring everything. (School A, teacher B)

Facilitators' and teachers' shared experiences, arising from co-teaching sessions, provided a useful resource for appropriation in co-generative dialogue in the feedback sessions between facilitators and teachers. There was evidence of depth in discussion of the mathematical purposes of activities through the detailed analyses of teacher action and student responses in the shared teaching sessions.

The shared practices generated in co-teaching enabled teachers to contextualise the core principles of the project. Teachers reported having agency through the appropriating resources, in particular as it related to their own classroom setting. This enabled them to interpret the core principles and transform their practices as they adopted the underlying ideas. Facilitators also reported having agency as they extended their interpretations of the core ideas through co-teaching sessions and the associated dialogue with teachers.

> Every time I participated in the co-teaching and co-generative dialogues, I learned more about how students acquire mathematical knowledge and how teachers encourage and scaffold that learning. All the other schools I work in benefit from what I have learned with these teachers. (School A, facilitator)

Conclusion

The analysis of the classroom-based professional development has identified dimensions of practice that have facilitated teacher learning and enabled teachers to adopt and integrate practices into their specific teaching context. The investigation highlights the fact that the facilitator's role as a co-teacher enables them to guide teachers' interpretations of the core principles of the project. Such an approach suggests that shifts in teachers' practice will be sustained after the facilitator visits conclude. This has important implications for ensuring the sustainability of reforms.

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Advancing Pasifika Students' Mathematical Thinking

Kathryn C. Irwin
University of Auckland
<k.irwin@auckland.ac.nz>

Joanne Woodward
University of Auckland
<jwoo088@stat.auckland.ac.nz>

Five sessions of a year 5 and 6 mathematics class were videotaped and the language used by the teacher and by the students when they were not working with the teacher was analysed. The students in the class were primarily from Pasifika backgrounds. We were interested in the relationship between the teacher's language and the language that students used among themselves. The teacher used the language for advancing children's thinking advocated by Fraivillig et al. (1999). She encouraged students to use similar language, but they did so selectively. They routinely gave their method or asked others for their method rather than for answers, and they reiterated the teacher's request that students be given time to think. However, they used little of the exploratory talk needed to explore mathematical ideas together. The students in this class made excellent progress in relation to national NDP achievement norms. We raise the question of whether or not they would have made more progress or more lasting progress had they engaged in exploratory talk among themselves.

Background

The teacher discussed in this report was observed in 2004 (see Irwin & Woodward, 2005). In that report, we documented her use of enquiry language. In contrast to another teacher who had also received Numeracy Development Project (NDP) training, she asked students more open questions, waited longer for them to think and respond, and was interested in the variety of their responses. In her sessions with students, she did less of the talking and emphasised listening more than did the comparison teacher. A few students also used the language of enquiry with one another. In that study, the students were observed on only one occasion. This study was undertaken in order to study the development of her students' language across a school year. We recorded the teacher's language and that of the students on five occasions.

There has been other research on the nature of students' and teachers' participation in the NDP. Higgins (2003) examined the language that teachers and students used while the teacher was working with a group of students. Young-Loveridge, Taylor, and Hāwera (2005) interviewed students, both in the NDP and not in the NDP, about their views on communicating their strategies for solution to peers. Irwin (2004) interviewed year 9 students on their views of their mathematics programme that was based on the NDP. None of these studies looked at the language that students used among themselves when the teacher was not present. This study extends our brief look at the discourse of student groups in 2004 as well as recording the language used by the teacher.

An analysis of the students' discourse when not with the teacher is important because the general practice in the NDP is to have students working in groups. This means that the majority of students are not with the teacher during group times. The NDP recommends a teaching model for advancing children's thinking (ACT) (Fraivillig, Murphy, & Fuson, 1999), which is based on a study of classes where students were engaged in collaborative problem solving. The examples from that study were of times when groups are working with the teacher, as are those in Higgins (2003) and in other writers such as Bowers, Cobb, & McClain (1999) and Wood (2001). NDP *Book 3: Getting Started* (Ministry of Education, 2005) recommended grouping students for development of strategies. While one group was with the teacher, the other groups would be working independently. Similarly, knowledge sessions usually involved working in groups. Attention was given to managing independent groups in *Getting Started* (2005), but little was said about the expected nature of students' learning in those groups. The general intention appeared to have been that students would practise knowledge skills or strategies when they were not with the teacher.

The National Council of Teachers of Mathematics (NCTM, 2000) recommends that students discuss their mathematical reasoning with one another, and several other studies such as Fraivillig et al. focus on discussion in a group with the teacher present. Wegerif, Mercer, and Dawes (1999) analysed students' talk when the teacher was not present in an experimental programme for advancing students' reasoning. They distinguish among disputative, cumulative, and exploratory talk, all of which involve different degrees of working constructively with each other's ideas. They show that students in the experimental group used exploratory language, including phrases such as "because", "I think", and "agree", and made greater gains on a reasoning task than students in a control group.

Other writers have found that it is beneficial for students to use exploratory talk with one another when not working with the teacher. Is this happening in a NDP class that is successful on the Numeracy Framework? Is this a goal that the NDP should further?

Method

Participants

The class studied was a combined years 5 and 6 (ages 9–10) in a school classified as decile 1, which indicates the lowest socio-economic level. The membership of the class varied as students came or left the area. At the start of the year, there were 26 students in the class, and at the end of the year, there were 28 students, five of whom had been there only for a term.

Table 1
Students in This Class at the End of 2005 N = 28

Females			Males		
Number of students	Year	Ethnicity	Number of students	Year	Ethnicity
1	5	Cook Islands Māori-Niuean	1	5	Tongan
1	5	Sāmoan	1	5	Māori
1	5	Tongan	2	6	Cook Islands Māori
1 (new)	5	Cook Islands Māori	4	6	Sāmoan
4 (1 new)	6	Cook Islands Māori	1	6	Sāmoan-Tongan
3 (1 new)	6	Sāmoan	2	6	Māori
1	6	Sāmoan-Tongan	1	6	European
1	6	Māori-European			
1 (new)	6	Tongan			
2 (1 new)	6	Māori			

Table 1 shows the year level, sex, and ethnicity of students at the end of the year. Those who had been there only for a term are indicated as "new". This table giving each student is included to show the complex ethnic make-up of this class. This class can be summarised as having, at the end of the year, 6 students in year 5 and 22 students in year 6. Of these 28 students, 21 were Pasifika, 5 were Māori, 1 was European, and 1 was Māori-European. Twenty-three of the students had been in the class for more than one term. These 23 students are the ones whose progress is measured below. The only test of these students' skill in English was made by asking the teacher at the start of the year if any of the students were unable to understand her. She reported that two students could not understand her, and she therefore placed them with other students for help.

The teacher of this class was a New Zealand European in her third year of teaching. She reported that she had trained at Dunedin College of Education and that all of her training there was based on the NDP. Her school was involved in NDP training during her first two years there. She had never taught mathematics any other way than that promoted by the NDP. NDP equipment was in the room and in use. The teacher was not seen to refer to any NDP booklets, but her teaching was compatible with furthering students' NDP stages. She grouped her class by stages and changed the groupings for different topics and as she saw students advancing. She chose to assess all students on the full Numeracy Project Assessment diagnostic tool (NumPA).

Method

We videotaped five class sessions with a hand-held videotape recorder. The teacher selected the class topics in relation to the overall plan for the year. Times for videotaping were not related to the topic taught, although it happened that all videotaped sessions covered number and operations. During each lesson, the camera focused on the teacher for the full-class portions of the lessons, with some panning of the students to record what they were doing. When the class broke into groups, we focused on one group recommended by the teacher. The size of the group and the students in the group changed, but some of the same students were filmed in four of the sessions. The group filmed was usually the second-to-top group by assessed stages.

The content of the lessons videotaped is given in Table 2.

Table 2
Characteristics of the Five Lessons Videotaped

Date of class	Topic	Group and activity videotaped
31 March	Showing multiple ways of writing a number (for example, 21 is the same as 3 x 5 + 6); how to write a subtraction problem	3 girls and 3 boys each contributing ideas to a single question
11 May	Multiplication: deriving unknown tables from 2 times, 5 times, and 10 times tables	1 girl and 2 boys finding products from numbers on thrown dice
16 August	Finding fractional parts of whole numbers; translating improper fractions to mixed numbers and vice versa	6 girls and 4 boys with individual work sheets, talking when they needed help from each other
11 October	Review of mental subtraction, 2-digit from 3-digit numbers	5 girls and 3 boys solving problems mentally and sharing their methods
8 November	Multiplication: 2-digit by 1-digit numbers	2 girls and 3 boys finding products from numbers on thrown dice

Findings

Students' Progress on Numeracy Stages in Comparison to National Norms

Table 3 shows the percentage of students in this class at the end of the year in comparison with the 2004 national percentages for year 6 Pasifika students and the 2004 national average. This is an oversimplification of the nature of this class. We did not observe the teacher while she did these assessments, but when observed in 2004, she was flawless in her presentation of items from memory. It is highly likely that she was consistent and accurate in her assessments.

Table 3
Percentage of Students at Each Strategy Stage of the Numeracy Assessment Profile at the End of 2005 in Comparison with National Percentages for Pasifika Students and for the Total National Sample in 2004. N = 23 (not all totals equal 100% because of rounding)

Stage	Add/sub			Mult/div			Proportional		
	This class	Year 6 Pasifika average	Year 6 national average	This class	Year 6 Pasifika average	Year 6 national average	This class	Year 6 Pasifika average	Year 6 national average
Not assessed				4	3	2	4	3	2
< Stage 4	0	1	1	0	5	2			
Stage 4	22	29	16	35	27	15	39 *	39 *	23 *
Stage 5	43	49	46	9	31	26	13	35	33
Stage 6	35	22	37	39	27	36	30	18	25
Stage 7				13	8	18	13	5	14
Stage 8							0	0	2

*Includes all stages up to and including stage 4

This table shows that, at the end of 2005, students in this class in a decile 1 school had a higher proportion of students at the upper stages of the Framework than the national percentages for all Pasifika students in year 6 and was comparable to the overall national average for these strategies. This is true even though this class had five students in year 5 whose stages were usually lower than those of the year 6 students (the national figures for year 5 are markedly lower than year 6 for some stages). We note that, in this class, low percentages of students were judged to be at stage 5, early additive, for multiplicative reasoning or proportional reasoning, although the majority could use additive or part–whole reasoning for addition. This may have been related to the class emphasis on multiplication, which is essential for both of these strategies. Note that in a class of 23, one student is equivalent to 4.3%.

We compared the students' strategy stages at the end of the year with their stages at the beginning of the year or the end of the previous year. In looking at this information, we note that progress between stages is not equally difficult (see Irwin, 2003; Young-Loveridge, 2004). Table 4 shows the number of stages changed for each strategy. This data shows that this is a class in which students make good progress in NDP stages. Previous reports of progress at the higher stages of the Number Framework showed that about 40% of students at these higher stages progressed to further stages (for example, Irwin, 2004; Irwin & Niederer, 2002; Young-Loveridge, 2004). On these three scales, 60%, 53%, and 58% of the students who were assessed and who were not at the ceiling initially moved to a higher stage. This excellent progress provides an important background for examining the teacher's and the students' language in this class across the year.

Table 4
Number of Students Changing Stages on Each Strategy from the Beginning to the End of 2005 (N = 23)

Stage	Add/Sub	Mult/Div	Proportion
At ceiling initially	3	0	0
Not assessed initially	–	4	4
Decreased	–	1	1
Stayed at the same stage	8	8	7
Gained 1 stage	8	6	7
Gained 2 stages	4	4	4

Teacher's language

The teacher's language was compared to that suggested by the model proposed by Fraivillig, Murphy, and Fuson (1999). These authors identify ways in which teachers orchestrate classroom discourse through eliciting children's solution methods and supporting and extending children's understanding. Combining these three components is shown through allowing additional time for student thinking, assisting students with their narrations, probing for better descriptions or solution methods, asking the students to generate alternative solution methods, using challenging follow-up questions, highlighting and discussing errors, providing assisted practice at the top of students' performance levels, assessing students' thinking on an ongoing basis and adjusting instruction accordingly, and continually adapting classroom discussions to accommodate the students' zones of next development (see Fraivillig et al., 1999, Figure 1).

This teacher carried out all of these activities. Some or all of these activities were noted in each of the five lessons recorded. Especially frequent were eliciting of different solution methods, waiting for and listening to student explanations, asking students to elaborate, accepting effort and different answers, listing all solution methods, pushing students beyond original methods, promoting more efficient solution methods, and encouraging a love of challenge.

Common responses by this teacher were:

> Do you want to share your way, B? (lesson 1)
>
> How do you know it's going to be 8? (lesson 2)
>
> Good to see you thinking and really taking time. (lesson 2)
>
> Who can tell me what they did? (lesson 3)
>
> If S comes up with an answer and none of you agree with him, ask him how he did it and get him to work through how he did it. (lesson 4)
>
> So how are you going to work it out? ... F got a different answer. How did you work your one out? (lesson 5)

She had high expectations for her students and repeatedly told them that they were clever and could do it, accompanied by appropriate praise. This was particularly evident in lesson 3 when the worksheet was difficult for the students, and they told her so. She replied:

> Smile because it's hard. It's hard because you're clever and you can handle it.

A time when the teacher routinely challenged students and asked for different methods was in the "Quick 20" that came at the start of four of the five lessons. An example from lesson 4 of these questions and her responses after students gave their answers appears in Table 5.

Table 5
"Quick 20" Questions Asked in Lesson 4 and the Teacher's Response to the Answers Given by Selected Students

Questions	Teacher's response to students' answers
7 x 3	(nod)
2 x 15	Good. How did you work it out?
14 ÷ 2	No, sorry. (To other student.) Good job.
48 – 17	Yes, how did you work it out? Who else did it that way? Did anyone do it a different way?
6 x 6	Sorry. (Asks another student.)
9 x 7	No, I'm sorry, it is not 54. J?
124 – 25	How did you work it out?
18 x 3	How did you work it out?
3 x 4 x 8	Tell me how you did it. ... Who agrees? Who doesn't agree? Why don't you agree? Oh really? How did you do it? Okay, does everyone have 96 apart from those who added?
10 x 100	1 000 (revoiced)
$\frac{1}{4}$ of 80	How did you work it out? Okay, did anyone do it a different way?
25 is what fraction of 50?	(revoiced)
5 x 2	(Asks a lower stage child.) Not quite. (Asks another student.)
7 + 8	(nod)
19 – 6	Very good
12 is $\frac{1}{3}$ of what number?	No, 4 is a third of 12. (To other student.) How did you work it out?
Name a shape with no corners	There will be different answers, so keep your hand up until we have got your shape. Circle, oval, sphere, and cylinder. (A student offered a spiral.) No, a spiral isn't a shape because it doesn't join up.
How many corners on an octagon?	Nice

After seeing who got how many correct, she said:

> Some of you are making really good progress. Well done, guys.

Students' Discourse When Not with the Teacher

Although students worked in groups and the teacher gave guidelines for how they should work together, the groups observed did not appear to be working to solve problems co-operatively except in the first session, which required multiple answers to one question. They were selective in their adoption of the language used by the teacher. On two of the observations, lesson 2 and lesson 4, the discourse involved one student imitating the teacher's role and language and the others playing the role of students. The methods of discussion that the teacher had asked them to use appeared to be taken as class rules rather than models for discussion or sociomathematical norms (Bowers, Cobb, & McClain, 1999). In lesson 2, a boy adopted the teacher's words, for example, saying "Are you sure?" and scaffolding to help a student derive 7 times 4 by asking her what 7 times 5 was, saying "We've got to give her time to think," as well as scolding another student by saying "You're not supposed to tell." Other students played their role as students, for example M asked, "Can I please say it?" when she knew an answer. In lesson 4, a girl took over leadership in a large group, but as an instructor rather than as discussion leader. She said things such as "Turn your cards (number fans) around", "I said, we're doing that one", "No, you've got to work it out with your team", and writing new problems on the board. These episodes have the feel of children "playing school" and not that of co-operative learning. Where students did help each other, they used a very different type of dialogue. In lesson 3, for example: "What is number 3?" "I know you got it, but how did you get it?" "How did you get that?" This was a difficult activity for the students and one in which they genuinely needed each other's help. The fact that a student asked "But how did you get it?" indicates that he realised that "how" was an essential thing to know.

An initial question behind this research was whether or not students improved in using the language of enquiry with one another over the year. This was difficult to determine because of the different groups and tasks. However, lessons 2 and 5 included the same students and similar tasks. Comparison of these two small-group sessions is given in Table 6. See Table 2 for the tasks.

Table 6
Types of Student Discourse in Lesson 2 and Lesson 5

Lesson	Participants	Explaining answers/methods	Barriers to adopting the teacher's language
2	S, N (boys) M (girl)	S: "Are you sure?" x 3 S attempts to guide M to an answer through teacher's steps. When N gives answer, S tells him, "You're not supposed to tell." S and N: "You've got to give her time to think."	Competition between S and N S and N boast about knowing more tables: • S: "I'm going to win, man." • N: "You're going to lose. I'll get revenge with my bare hands." • M taunting: "I know the answer." x 2 • N: "I won."
5	S, N, and L (boys) M and A (girls)	L proves his answer when others show doubt. S asks M for an answer, so she shows her workings on paper. S asks M: "What are you doing?" and she gives her method. L explains his answer to N by giving method.	Competition between boys and girls: A and N argue about who goes first. • S to L: "Don't cheat." • N awards the boys "points". • A: "Beating you." • M: "We got 3 points." • L: "We're winning, you're losing." • L: "Yeah, we won."

The quality of discussion about mathematical methods was better in lesson 5. It was also the case that the intensity of competition was greater on the second occasion. In both of these groups, students were supposed to be deriving answers from known facts. This never happened when the teacher was not there.

In the framework of disputational, cumulative, and exploratory talk, as defined by Wegerif, Mercer, and Dawes (1999), the students' talk was mostly disputational or cumulative, although they used some of the language of exploratory talk to justify their answers. Instead of reasoning collaboratively, on most occasions the students were working as individuals trying to solve problems. They adopted some of the teacher's language, but what was valued was knowledge and "winning". They valued both the knowledge of facts and of procedures. In the second lesson involving multiplication, knowledge of the tables won out every time over attempts to derive them. In lesson 5, on multiplication of two-digit by one-digit numbers, it was skill with the vertical algorithm that they valued. They viewed the competent mathematics student as one who knew the answer and how to get it rather than as a partner in mathematical reasoning.

Influences on Students' Language

Other factors besides a teacher's language influence the discourse of students. Lindfors (1987) wrote about the nature of children's questions in and out of school. She argues that questions out of school are influenced by curiosity, while their questions in school are largely procedural. She gave several examples, including the out-of-school question, "If everybody in the world keeps drinking water, are we going to run out of water some day?" The same child in school asked, "Do you want us to skip every other line?" and "Do we write the date on this paper?" (Lindfors, 1987, p. 287). The dialogue in these five lessons was usually procedural – if "How did you get that?" is considered a procedural question.

It is possible for teachers to foster the language of curiosity in school, but it is not easy in the face of school traditions. In adopting language, Lindfors points out that children use whatever is salient and interesting to them. Similarly, Cazden writes of the classroom containing "the official world of the teacher's agenda, and the unofficial world of the peer culture" (Cazden, 1988, p. 150). These two worlds appeared to intersect in group work in this class when the teacher was not present.

Cobb et al. (2003) have written about the influence of the school structures on instructional practices. That was evident in this case in that the school had decided to focus on writing, and therefore mathematics had been moved to after lunch. The teacher noted that the students did not seem alert at this time. The students may also have been more influenced by their discussions on the playground at that time.

Discussion

On two of the five occasions when small groups were observed, the activities were primarily for practice; on the three other occasions observed, the teacher appeared to expect students to use exploratory language in small-group work. The students used some of the teacher's exploratory language but often used a different style of discourse. While these students were mostly Pasifika, this might happen in any class.

The NDP advocates a set of behaviours for teachers for advancing students' thinking that is based on a model that furthers co-operative learning in classrooms. Teaching in this model usually involves giving one challenging problem to a group to work on throughout their time in the group. This is quite different from group work, which is primarily for practice. In the lessons discussed, competition and the lack of reflection were influenced by the fact that there were several problems to be solved. Again, this is appropriate for practice but less appropriate for co-operative mathematical enquiry. Despite attempts by the teacher to get group co-operation when she was not present, the students did not adopt that pattern. They did show evidence of other characteristics necessary for success. They appeared to be willing to work hard, despite the fact that mathematics was held after lunch when attention wandered. They were interested in getting right answers and in using techniques that would help them do mental mathematics quickly. They decided for themselves what was the easiest way to do a mathematics problem, and using tidy numbers or decomposing multiplication problems were not easy for them. They knew that it is faster to know your tables and carry out the vertical algorithm mentally, usually from left to right. Most of the students observed seemed to have a good sense of number.

Prior to 2006, NDP resources did not suggest co-operative mathematical reasoning by groups working on their own[1]. However, the 2006 version of *Getting Started* (for example, p. 13) does recommend co-operative learning for groups who are not with the teacher. As indicated above (for example, Wegerif, Mercer, & Dawes, 1999), co-operative reasoning has been found to result in better individual reasoning. Theoretical reasons why students should be encouraged to reason mathematically largely relate to the theory that social reasoning precedes individual reasoning and that language is the tool students use for thinking (Mercer, 2002; Resnick, 1987; Vygotsky, 1978, 1986). An example of a NDP class with a large proportion of Pasifika and Māori students that developed a culture of mathematical exploratory talk is described by Hunter (2005). She indicated that in this class, the students' use of enquiry and argument increased their autonomy and deepened their collective responsibility to engage in mathematical practices. This paper would be useful for the NDP to consider.

Watson and Chick (2001) list factors affecting collaboration in groups. These include the nature of the task, disagreement, doubt, and tenacity of ideas as well as social factors of leadership, social conflict, and egocentrism. Among the effects that they found in groups was that students looked for the simplest way of solving a problem. It was evident that these students were not interested in multiple ways, as in lessons 2, 4, and 5. The simplest method was knowledge of tables or of the vertical algorithm done left to right for subtraction (but by using a method that did not require "borrowing") and right to left with carrying for multiplication. Other "simplest" methods seen were looking at a wall chart for tables and using a calculator. In the sessions we observed, the teacher's attempts to get students to use known facts or tidy numbers for finding an answer did not succeed when she was not with them.

Small groups of students can either practise strategies and knowledge or they can explore novel problems. If both types of activities are intended, it would be useful to make the purpose of an activity clear to the students. If groups are to explore novel problems, this is likely to enhance their ability to reason mathematically when not guided by a teacher. If a teacher uses both purposes, it would be useful to be explicit about the expectations for a particular session.

Many classroom styles and discourses can lead to success for students. This classroom demonstrated one of them. It was a style in which the teacher appeared to set rules for thinking for yourself, being able to describe your strategies, listening to others, and respecting different strategies. She repeatedly told the students that they could do difficult work because they were clever. Results showed that they lived up to her expectations. What we do not know is if these students would have been even more successful if they had engaged in co-operative mathematical reasoning. We also do not know if they will continue to reason at high stages on the Number Framework.

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[1] Note that although the process may have been used in some classes, we have found no written recommendation that this should happen.

The use of co-operative mathematical reasoning in groups is an issue for the NDP as a whole. Does the project want to fully implement the programme for advancing children's thinking advocated by Fraivillig et al. (1999) by having classes adopt sociomathematical norms in which students help each other engage in mathematical reasoning even when not with a teacher? The literature suggests that this would provide them with greater success than they now enjoy.

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Numeracy Assessment: How Reliable are Teachers' Judgments?

Gill Thomas
Maths Technology Ltd
<gill@nzmaths.co.nz>

Andrew Tagg
Maths Technology Ltd
<andrew@nzmaths.co.nz>

Jenny Ward
Maths Technology Ltd
<jenny@nzmaths.co.nz>

Research reports on the Numeracy Development Projects (NDP) have consistently shown that student performance in numeracy improves as a result of participation. The evidence for these reports has been based largely on student achievement data collected by classroom teachers. This paper reports on an investigation into the consistency of teacher judgments of students' strategy stages on the Number Framework. The findings show a high level of agreement between the judgments of classroom teachers and those of independent researchers.

Background

The New Zealand Numeracy Development Projects (NDP) are a large-scale Government initiative in mathematics education aimed at improving students' mathematics ability through the professional development of teachers. Each project has three main components: the Number Framework that describes a progression in strategy stages that students use to solve problems and the key knowledge that they will require to progress; a diagnostic interview that enables teachers to assess students' capabilities according to the Number Framework; and a professional development programme.

Since their inception, the NDP have been informed by research that has examined both the experiences of the participants and the achievement of students within the projects (Ministry of Education, 2005c). Positioned within this body of research is the NDP Longitudinal Study, which has examined the impact of the projects in schools that have been involved over a number of years. One component of the Longitudinal Study in 2005 focused on the teachers' use of numeracy assessment information and the accuracy of their strategy stage assessments. This paper reports on those findings.

The primary and most compelling reason for teachers to use diagnostic assessment information is to improve teaching and learning. After reviewing literature on the consistency of teacher judgments, Bobis (1997) noted:

> Such findings emphasise the necessity of determining the reliability of teacher ratings ... particularly given the pivotal role such ratings play in determining instructional decisions for individual and groups of children to help them advance thorough the stages and levels of the Learning Framework. (Bobis, 1997, p. 4)

The NDP encourage teachers to use the information they obtain about their students from the diagnostic interview to group students for instruction and help determine learning experiences.

> NumPA [the Numeracy Project Assessment diagnostic tool] provides a wealth of diagnostic assessment information about students. There needs to be an appropriate link between this data and the learning experiences you provide for your students through the classroom programme. (Ministry of Education, 2005b, p. 2)

By the end of 2005, approximately 460 000 students had participated in the NDP (Parsons, 2005). Data for the majority of these students has been collected on the national database. This data has been used to describe student achievement within the project, but the reliability of the teacher judgments on which this large data set is based has never been fully examined. The consistency of patterns of student performance and progress over several years has been used as the primary source of evidence of the consistency of the teacher judgments.

> For the third year running, the majority of students improved during their participation in the Advanced Numeracy Project. The patterns of achievement within year groups in 2003 confirm previous years' results. (Higgins, 2004, p. 11)

Reliability is an issue in all educational assessments, but the degree to which it can be considered a problem is related to the function and use of the assessment (Nystrom, 2004). One of the aims of the NDP Longitudinal Study is to collect numeracy data from students in the years following their school's participation in the NDP to help establish benchmarks or expectations for achievement and progress. If schools are using this data to compare the performance of their students with national norms, it is important that the data is reliable.

> ... if the performance of students from different classes, schools or regions are to be compared for any reason in the future, it is imperative that the inter-rater reliability be evaluated. (Bobis, 1997, p. 4)

A study with similarities to the current investigation was undertaken in conjunction with the Count Me in Too project (CMIT) in New South Wales (Bobis, 1997). The key features of CMIT include the Learning Framework in Early Number and a performance-based assessment instrument, the Schedule for Early Number Assessment (SENA). In the Bobis (1997) study, 16 teachers viewed a video recording of five students and were asked to rate the students' performance using the SENA. Quantitative analysis found a high degree of inter-rater reliability between teachers' ratings, with a high correlation between individual teachers' ratings and the mean rating of the whole group.

The present investigation differs from the Bobis (1997) study in that it focused on the observation of teachers assessing their own students. This enabled a more naturalistic examination of teachers' assessment judgments. In addition, the current study also involved analysing the responses of a large number of teachers to written assessment scenarios.

Method

The Longitudinal Study began in 2002 with the participation of 20 schools. Each year, new schools are randomly selected from a list of schools that have recently completed the NDP training. The list is stratified by decile to ensure that the sample in the Longitudinal Study closely approximates the national sample and has similar numbers of students in years 1 to 8. In 2005, a total of 26 schools participated in the Longitudinal Study, including 12 of the original 20 schools. Table 1 summarises the composition of schools participating in the 2005 Longitudinal Study, hereafter referred to as the 2005 longitudinal schools. The low-decile band includes decile 1–3 schools, the medium-decile band includes decile 4–7 schools, and the high-decile band includes decile 8–10 schools.

Table 1
Composition of Schools in the 2005 Longitudinal Study

	Decile Group			
School Type	Low	Medium	High	Total
Years 1–6	4	5	5	14
Years 1–8	4	3	1	8
Years 7–8	1	2	1	4
Total	9	10	7	26

Two methods of data collection were used in the current study: observations and questionnaires. The observations of teachers' assessment interviews took place in two locations, with one southern and one northern city selected for this purpose. A random selection of the 2005 longitudinal schools within these two cities was used to determine the sample of schools for the observations. Letters of invitation were sent to the principals of selected schools describing the purpose of the research, what would be required of participating teachers, and the observation process. All five schools accepted. Up to 10 teachers within each school were invited to participate in the assessment observations. Where the school had fewer than 10 teachers, all were observed. Where there were more than 10 teachers, the lead teacher invited 10 of the teachers to participate in the assessment observations.

In total, 37 teachers were observed assessing the strategy stages of their students. Each teacher individually interviewed one or two students, and a total of 70 students and 156 teacher judgments across the three operational domains (additive, multiplicative, and proportional) were observed. Two researchers, both experienced in the assessment of student strategy stages, undertook the observations.

The teachers were instructed to select a diagnostic assessment technique to assess their students' strategy stage. While the teachers were encouraged to use whatever techniques they employed in their usual classroom practice, NumPA, the Global Strategy Stage (GloSS) assessment, or the teachers' own diagnostic questions were identified as possible methods in the letter of invitation. The teachers were told that the researchers might ask the students extra questions to clarify their strategy stages and that there would be a brief follow-up discussion with the teacher to clarify how their judgments for each student were made. The teacher and students were withdrawn from the classroom for the assessment observations.

Questionnaires were sent to all teachers in the 2005 longitudinal schools. Of the 400 questionnaires distributed, 230 responses were received. It is not possible to calculate an accurate response rate because the number of questionnaires distributed was based on an estimated class size of 23 students, ensuring more than sufficient questionnaires were provided to each school. The questionnaire included assessment scenarios that asked the teachers to identify the students' strategy stage based on the information given. Also included were questions focused on student achievement and the assessment of numeracy.

Findings[1]

The Picture of Numeracy Assessment in Longitudinal Study Classrooms

The large majority of teachers (83%) in the 2005 longitudinal schools have completed the full NDP professional development programme. Of the 17% of teachers who have not completed the full professional development programme, 10% have received no training at all while the others have attended a variety of professional development workshops or covered the numeracy project in their pre-service training. The majority of teachers (80%) had undertaken the NDP professional development programme in their current school.

Almost all teachers (94%) in the 2005 longitudinal schools track the strategy stages of students in their class. Eighty-four percent of teachers reported using NumPA, with over half the teachers (58%) using it once or twice a year to group students. Eleven percent of teachers use NumPA to assess new students when they enter the classroom:

> At the beginning of school year to identify stage and assign maths group in class. (Teacher)
>
> Beginning of year to recheck levels and usually at beginning of Term 4 for school reports. (Teacher)

The large majority of teachers (84%) reported the use of informal ongoing numeracy assessment during class time, with approximately two-thirds of teachers (69%) using informal methods of assessment at least once a week or on an informal basis as required to check student progress or confirm their instructional groupings.

> I regularly observe during group time to check the appropriateness of the group level and the placements in the group. (Teacher)
>
> I do one-to-one checking and conferencing, especially with targeted children. (Teacher)

One-third of the teachers reported use of the GloSS assessment, primarily to check students' strategy stages throughout the year:

> Reporting to parents – a way to show gains. (Teacher)
>
> To show progress through the strands. (Teacher)

The main use reported by teachers for the numeracy assessment information was classroom grouping. A small number of teachers noted that they used the assessment information to report to parents and/or the principal.

> At the beginning of the year, I interview each child and use this for grouping. (Teacher)
>
> I use the children's responses to questions that are part of my teaching to make changes to groups. Where I am not sure, I will target these questions to particular children. I try to make sure that I check all the children in this informal way every couple of weeks. (Teacher)
>
> I make up my own "test" to determine what levels children are at. I use the responses to this written test as information for regrouping if necessary. (Teacher)

Teachers reported a high level of confidence in the assessment judgments they are making, with the large majority of teachers (84%) identifying that they are fairly confident or very confident that they know the strategy stage each student in their class is operating at.

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[1] Where questionnaire responses do not total 100%, this is due to teachers leaving questions unanswered.

Time Spent on Numeracy Assessment

Time is an important commodity for any teacher, and the amount of time taken to complete an assessment has important implications for classroom practice. With the large majority of teachers using a full NumPA assessment at least once a year and nearly a quarter of teachers (24%) conducting NumPA assessments on all students twice a year, the amount of time taken for each assessment is worthy of consideration.

In the observations of the teacher assessment interviews, the average time taken for teachers to assess the strategy stages of a student was 13 minutes. These observations involved the assessment of students' strategy stages and did not include assessment of the knowledge domains. Adding the knowledge domains to the assessment would conservatively increase the time taken to 20 minutes. This equates to over 8 hours of assessment time for a class of 25 students, which in turn creates resourcing issues. Teacher comments support this assertion.

> Testing requires at least two full relief days per teacher. It's very costly. (Teacher)
>
> At times I use the diagnostic test to help teachers with stage placement, as there are time issues when it comes to the teachers testing all the children in their class. (Teaching principal)
>
> We are very fortunate to be given as much release time as necessary to interview each child at the beginning of the year. I believe this is the key to appropriate and effective grouping and instruction. (Teacher)

The observation of the teacher assessment interviews indicated considerable variation between teachers in the time taken to assess students' strategy stages. A comparison of teachers who made assessment judgments in all three domains shows an interesting pattern. Table 2 compares the accuracy of teacher judgments made in all three domains with the time taken to make decisions. The table shows that the teachers taking the most time to make assessment decisions were least likely to be in agreement with the researchers.

Table 2
Time Taken to Assess Three Operational Domains

	Less than 10 minutes	10–20 minutes	More than 20 minutes
Number of judgments	36	45	42
Number in agreement with researchers	30	38	28
Precentage in agreement with researchers	83	84	67

This finding demonstrates that at least some of the teachers were able to make assessment judgments swiftly and accurately, as recommended in the NDP teacher materials.

> As you become more familiar with the items and how to evaluate students' responses, you will become much quicker at administering the NumPA. You will get better at assigning stages for each area of strategy and knowledge from the assessment using the least possible number of questions. (Ministry of Education, 2005a, p. 1)

Reliability of Teacher Judgments in Assessment Interviews

A total of 156 teacher judgments were observed across the additive, multiplicative, and proportional domains. Table 3 summarises these judgments and compares them to those made by the two researchers. The two researchers had 100% agreement in their judgments.

Table 3
Agreement with Teacher Judgments

	Domain			Total
	Additive	Multiplicative	Proportional	
Number of judgments	70	45	41	156
Number of judgments in agreement	62	34	31	127
Percentage of judgments in agreement	89	76	76	81

The majority of teacher judgments (81%) agreed with those made by the researchers. The judgment decisions made in the additive domain showed a higher level of agreement with the researchers (89%) than those made in the multiplicative and proportional domains (76%).

Table 4 shows that in approximately two-thirds of the judgment differences observed, teachers rated students' strategy stages lower than the rating of the researchers. Anecdotally, teachers explained some of these decisions on the basis of consolidating students' understanding at an existing level, describing the student as "not ready" to move up to the next instructional group. In this way, teachers' assessment judgments were closely aligned to their classroom programmes and the instructional groupings of students. These assessment decisions differ from the NumPA instructions that direct teachers to judge students at the highest strategy stage demonstrated within each operational domain:

> enter the highest stage the student demonstrates within each operational domain. (Ministry of Education, 2005a, p. 3)

Table 4
Differences between Teacher and Researcher Judgments

Domain	Additive	Multiplicative	Proportional	Total
Number of differences	8	11	10	29
Number of differences where teachers rated lower	7	6	7	20 (69%)
Number of differences where teachers rated higher	1	5	3	9 (31%)

Most of the judgment differences between the teachers and the researchers occurred in stages 4 and above. Table 5 below shows where these differences occurred in all three operational domains. The shaded regions indicate where the teacher ratings are higher than the researcher ratings.

Table 5
Stage Ratings for Judgment Differences

	Researchers	Teachers	Number of incidences
Additive	Stage 5	Stage 4	6
	Stage 4	Stage 3	1
	Stage 6	Stage 7	1
Multiplicative	Stage 7	Stage 6	1
	Stage 6	Stage 5	3
	Stage 5	Stage 4	1
	Stage 4	Stages 2–3	1
	Stage 6	Stage 7	1
	Stage 4	Stage 5	4
Proportional	Stage 7	Stage 6	1
	Stage 6	Stage 5	3
	Stage 5	Stages 2–4	3
	Stage 4	Stage 5	3

The highest number of disagreements (6) occurred in the additive domain, where teachers rated a student stage 4 while the researchers assigned a rating of stage 5. When asked to clarify their decisions, the teachers said that the students predominantly used counting strategies and that they felt that using one simple partitioning strategy wasn't sufficient to rate them at stage 5 (early additive).

Of the teachers observed, 27 (72%) had taken part in the full NDP professional development programme. Three had received NDP professional development within the pick-ups programme[2], while the remaining seven had received within-school training. Table 6 reports on the variations in judgments between the groups of teachers with different training. The small sample sizes for those teachers without numeracy project training make direct comparison difficult, but there appear to be no significant differences in judgments between these groups of teachers.

Table 6
Reliability of Teacher Judgments and NDP Training

Training received	No. of teachers	No. of judgments	No. of agreements	% agreement
NDP training	27	114	93	82
Within school	7	30	25	83
Pick ups	3	12	9	75
Total	37	156	127	81

2 The pick-ups programme refers to the national NDP workshop programme that teachers new to previously trained schools are invited to attend.

Reliability of Teacher Judgments Using Written Scenarios

Assessment scenarios were included in the questionnaire given to all teachers in the 2005 longitudinal schools. The scenarios were based on examples outlined in the diagnostic interview provided to teachers (Ministry of Education, 2005a). The teachers were instructed to rate the student at the stage of the Number Framework at which they appeared to be operating, based on the information given. Both the number and name for the stages were given with each scenario. A space was provided for the teachers to either comment on their choice of strategy stage or to suggest a question they might ask to further clarify the student's strategy stage.

An expert panel comprising three regional co-ordinators, a national co-ordinator, and three researchers discussed the scenarios and came to a consensus on which ratings could be considered reasonable, based on the limited evidence available. The panel also identified the types of questions that could be used in each scenario to confirm or question strategy judgments. Teacher ratings are compared with the decisions of the expert panel.

Of the teachers who responded to the scenario illustrated in Figure 1, 198 either made a singular stage rating or left the scenario unrated but indicated a further question was required. The numbers reported in Table 7 are based on these responses. There were 15 teachers who chose to identify a range of stages, and these responses have been excluded. Eleven of these 15 teachers identified the student as using a counting strategy (stages 0–3).

> Teacher: Please hold out your hands for me. Here are four counters. Here are another three counters. How many counters have you got altogether?
>
> Student: Four and three.

Figure 1. Scenario One

Table 7
Teacher Judgment of Student Strategy Stage for Scenario One

	Strategy Stage						
	0	1	2	3	4	5–7	No rating
Percentage of teachers without question	17	20	12	2	1	2	
Percentage of teachers indicating question	10	12	10	3	2		9

Shaded regions indicate those responses considered appropriate by the expert panel, giving a total of 78% agreement for scenario one. The panel believed that on the evidence given in scenario one, a rating of zero or one would be reasonable. Questions considered appropriate could clarify the student's counting ability by asking them to either count out a specified number of counters or to count how many counters altogether. The questions identified by teachers reflected both these themes:

> "Can you put them in my hand and point to them as you count?" "Can you carry on counting?" (count with child) Also repeat "Let's see how many we have altogether." (Teacher, left student unrated on scenario one)
>
> Can't determine – ask follow-up question: "How many is that altogether?" From that, you can determine what level they are at. (Teacher, left student unrated on scenario one)

The expert panel also believed it was reasonable to make no judgment on the student's strategy stage if a follow-up question was asked to clarify that the student needed to give the total number of counters in both hands. The responses of a further 9% of teachers can be considered in agreement on this basis. The panel also believed a rating of stage 2 could be considered in agreement if it was indicated that there had been a follow-up question asking about the total number of counters present. This accounts for another 10% of teachers.

Teachers commented that they felt most comfortable rating the scenarios that were in the same range as the strategy stages of the students in their classrooms. It is interesting to note that of the eight teachers who incorrectly rated scenario one at stage 4 or above, seven were middle-school (years 4–6) or senior-school (years 7–8) teachers.

> This [rating scenarios] is quite difficult. Teachers tend to focus on the stages they deal with daily! (Teacher)
>
> As a new entrant teacher, I would not be familiar with all the scenarios – however, I feel confident with testing stages at my level. (Teacher)

Of the teachers who responded to scenario two (Figure 2), 183 made a singular stage rating. The numbers reported in Table 8 are based on these responses. There were 20 teachers who chose to identify a range of stages, and these responses have been excluded. Nineteen of these 20 teachers identified the student as using an advanced strategy (stages 6–8).

Teacher: Ivan has 2.4 kilograms of mince. Each pattie takes 0.15 kilograms of mince. How many patties can Ivan make?

Student: 10 patties would be 1.5 kilograms, so 15 would be 2.25 kilograms, and one more would make 2.4 kilograms exactly. So that's 16 patties.

Figure 2. Scenario Two

Table 8

Teacher Judgment of Student Strategy Stage for Scenario Two

	Strategy Stage					
	3	4	5	6	7	8
Percentage of teachers without question	1	2	3	20	41	19
Percentage of teachers indicating question				2	9	3

Seventy-two percent of the teacher judgments were in agreement with the expert panel rating of stages 7 and 8 as appropriate. Follow-up questions considered appropriate were those of a similar nature to the original question but using different numbers to check whether the student was able to use another strategy. Teacher questions reflected this.

> Ask for another way of working this out. If they can use another strategy, mark student as level 8. (Teacher, rated student stages 7 to 8 on scenario two)

It is interesting to note that of the 27 teachers who did not answer this question, 18 were junior school teachers (years 0–3) and so would be less familiar with the advanced stages of the Number Framework. Also of note is the fact that teachers in the 2005 longitudinal schools may not have been aware of the addition in 2005 of stage 8 to the multiplicative domain of the Framework. Prior to 2005, stage 7 was the highest stage in the multiplicative domain.

Scenario three (Figure 3) received single-stage rating responses from 201 teachers. These results are summarised in Table 9. Of the 17 teachers who chose to identify a range of stages, 14 rated students at stages that included stage 5.

Teacher: There are nine counters under this card and eight counters under this card. How many counters are there altogether?

Student: 17.

Teacher: How did you work that out?

Student: I know that nine plus nine is 18, and it is one less, so that's 17.

Figure 3. Scenario Three

Table 9

Teacher Judgment of Student Strategy Stage for Scenario Three

	Strategy Stage				
	3	4	5	6	7
Percentage of teachers without question	3	14	64	5	1
Percentage of teachers indicating question	1	4	8		

A stage rating of 5 was considered appropriate by the expert panel, and 72% of the teachers were in agreement with this. The panel also believed it would be reasonable for further questions to probe the student's strategy repertoire, looking for evidence of a strategy other than doubles. The questions given by teachers were in accordance with this.

> "Can you think of another way to work this out without using your doubles?" (Teacher, rated student stage 5 on scenario three)
>
> I would ask another similar question with larger numbers and also ask if there was another way the student could solve this problem. (Teacher, rated student stage 5 on scenario three)

There were 194 teachers who gave a single stage rating in response to scenario four (Figure 4). The results presented in Table 10 are based on these responses. Of the 18 teachers who chose to identify more than one strategy stage, 11 of these responses identified students as being at a range of stages that included stage 4.

Teacher: Here is a forest of trees. There are five trees in each row, and there are eight rows. How many trees are there in the forest altogether?

Student: Um, 5, 10, 15, 20, 25, 30, 35, 40.

Teacher: If I planted 15 more trees, how many rows of five would I have then altogether?

Student: 5, 10, 15 [counting on fingers]. That's three.

Figure 4. Scenario Four

Table 10
Teacher Judgment of Student Strategy Stage for Scenario Four

	Strategy Stage				
	2–3	4	5	6	7
Percentage of teachers without question	5	62	16	3	
Percentage of teachers indicating question		9	3	1	1

The expert panel believed a rating of stage 4 was reasonable, and 71% of the teacher judgments were in agreement. The panel considered it reasonable for further questions to clarify that the question is asking for a total number of rows of trees. Teachers who rated the student at stage 4 and indicated a further question was required identified questions with this purpose.

> "So how many rows would I have altogether with the new rows and the rows I already had?" (Teacher, rated student stage 4 on scenario four)

Scenario five (Figure 5) received 184 responses from teachers making a singular stage rating. The numbers reported in Table 11 indicate the judgments made by these teachers. A further 18 teachers who identified a range of stages were excluded from the analysis. Sixteen of these 18 teachers identified students at stages that included stages 7 or 8. Of the 27 teachers who did not respond to this scenario, 25 taught years 1–4.

> Teacher: There are 21 boys and 14 girls in Ana's class. What percentage of Ana's class are boys?
>
> Student: Well if you add them together that's 35, so you can multiply them by three and that's pretty close to the percent. Twenty-one times three is 63, so it must be a little bit less than 63%.

Figure 5. Scenario Five

Table 11
Teacher Judgment of Student Strategy Stage for Scenario Five

	Strategy Stage				
	4	5	6	7	8
Percentage of teachers without question	1	3	18	42	23
Percentage of teachers indicating question		1	1	7	4

The expert panel believed a rating of either stage 7 or stage 8 was appropriate for this scenario, and 76% of teachers were in agreement. The panel agreed that effective further questions would focus on asking the student to calculate an exact answer to the problem. Teachers' follow-up questions reflected this.

> "An excellent answer. But now work out exactly the percent of boys in the class." (Teacher, rated student stage 8 on scenario six)
>
> "That's a good estimation. Can you tell me the exact percentage? Is there another way to work this out?" I would look at other strategies of working this out. (Teacher, unrated student on scenario six)

Table 12 summarises the levels of agreements across the five written scenarios, giving an average of 74%.

Table 12
Teacher Agreement with Expert Panel

	Agreement with expert panel
Scenario one	78%
Scenario two	72%
Scenario three	72%
Scenario four	72%
Scenario five	76%
Average agreement	74%

Concluding Comment

Most teachers in longitudinal classrooms track the strategy stages of students in their class, using a variety of approaches to determine students' strategy stages. In general, teachers report a high level of confidence in the assessment judgments they are making.

There was a high level of agreement between the teacher assessment judgments observed and those made by the researchers. The judgment decisions made in the additive domain showed a higher level of agreement than those made in the multiplicative and proportional domains. In approximately two-thirds of the judgment differences observed, teachers rated students' strategy stages lower than the rating of the researchers, explaining their decisions in terms of consolidating students' understanding at an existing level and aligning their judgments with the instructional groupings of students. The amount of time taken by teachers to make assessment decisions varied. Some teachers were able to make swift and accurate decisions, while those that took the most time tended to be less accurate.

Teacher ratings in the written scenarios were judged to be slightly less reliable than those that were observed, but agreement with the expert panel was still high. The slightly lower rating may be attributed to the limited information available to teachers in the written scenarios.

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Te Poutama Tau: A Case Study of Two Schools

Tony Trinick
The University of Auckland
Faculty of Education
<t.trinick@auckland.ac.nz>

This paper reports on a case study of two schools that have achieved positive outcomes in the Te Poutama Tau project. The outcomes are based on student achievement in the diagnostic assessments and feedback from the numeracy facilitators supporting the project. Te Poutama Tau aims to lift teacher capacity and raise student achievement in numeracy in Māori-medium education. The results of these case studies may help to inform schools, numeracy facilitators, and policy initiatives in order to support the future implementation of Te Poutama Tau and other professional learning experiences in Māori-medium and kura kaupapa Māori.

Background

Te Poutama Tau is a professional development programme focusing on numeracy for teachers in Māori-medium schools and classrooms. It involved approximately 30 schools in 2005. It is a component of a key government initiative aimed at raising student achievement by building teacher capability in the teaching and learning of numeracy. Te Poutama Tau is based upon the Number Framework developed for New Zealand schools. The Framework is divided into two key components – knowledge and strategy. The knowledge section describes the key items of knowledge that students need to learn. The strategy section describes the mental processes that students use to estimate answers and to solve operational problems with numbers. It is important that students make progress in both these sections of the Framework.

This study examines a range of factors that have supported two kura in their achievement of very positive outcomes in the Te Poutama Tau project in 2004. Māori-medium mathematics education is still very much in its infancy, so it is important to identify key factors that promote student achievement.

The few studies to date in Māori-medium education highlight a range of factors that support and encourage student achievement. Studies by Hohepa (1993), Smith (1999), Bishop & Glynn (1999), and Bishop, Berryman, & Richardson (2001) note that culture plays a key role and that effective teachers create caring relationships and structured, positive, and co-operative teaching and learning environments. Other studies, particularly Glynn, Berryman, & Glynn (2000), highlight the importance of the nature of the relationship between home and school in determining effective student learning and achievement. Studies by Christensen (2003, 2004) note the issue of student language fluency on achievement. Christensen (2004) argues that there is a strong link between students' proficiency levels in te reo Māori and their progress through the stages in the Number Framework. He notes that there is significant correlation between language proficiency and performance in the diagnostic interview (Christensen, 2003, p. 27). What is not known at this point in time is what impact the teacher's own language proficiency has on student learning and achievement in Māori-medium education.

Māori-medium education, particularly the factors that contribute to student achievement, is a developing area of study. Hopefully, this small study will confirm the results of the earlier studies and provide additional data to support Māori-medium schools and associated policies and initiatives.

Methodology

The major component of this research is a qualitative case study of two schools. The two schools were chosen on the basis of the results of their students' performance in the diagnostic interviews and feedback from the Te Poutama Tau facilitators.

Case Study

In this research, each case study is a study of a social unit (a kura) that attempts to determine the factors that led to its success in the Te Poutama Tau project. The case studies provide a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply presenting them with abstract theories or principles. Case studies can establish cause and effect. Indeed, one of the strengths of case studies is that they observe effects in real contexts, recognising that context is a powerful means of determining both cause and effects (Nisbet and Watt, 1984). Sturman (1999) argues that a distinguishing feature of case studies is that human systems have a wholeness or integrity to them rather than being a loose connection of traits. Furthermore, contexts are unique and dynamic, investigating and reporting the complex, dynamic, and unfolding interaction of events, human relationships, and other factors in a unique instance.

There are several types of case studies (Yin, 1984) and they can be identified in terms of their outcomes, including exploratory (as a pilot to other studies or research questions), descriptive (providing narrative accounts), and explanatory (testing theories). This particular case study is both descriptive and explanatory.

Data Collection Strategies

In the Te Poutama Tau project, students were assessed individually at the beginning of the project, using a diagnostic interview, and again at the end of the year. The diagnostic interview was designed to provide teachers with quality information about the knowledge and mental strategies of their students and to assist in the location of each student's position on the Number Framework. The results for each student, classroom, and school were entered on the national database. The database shows the progress that students made on the Number Framework from the initial to the final diagnostic interviews. Participating schools were ranked in terms of their mean stage gain overall.

There was a cluster of schools and/or classrooms that achieved similar results. Schools who participated in Te Poutama Tau did so either as whole schools (in general, these were the kura kaupapa Māori and special character schools) or Māori-medium units (classes in English-medium schools). Initially, it was decided to concentrate on only two schools. This decision was based primarily on manageability of the data and the process, assuming that if there were more schools, there would be less time available to investigate the results of each one. Secondly, two schools provided the opportunity to identify the common factors that may have contributed to their positive outcomes.

The two schools selected for the case studies, kura A and kura E, were chosen on the basis of their very positive mean stage gains, and they also were able to provide bigger data samples to minimise the chance of "one-off" spikes and dips. The advice of the Te Poutama Tau numeracy facilitators for the two schools was also sought in order to gain their perspectives on the implementation of Te Poutama Tau in these particular schools. In both cases, the numeracy facilitators confirmed the case study schools' positive attitudes and commitment to the project.

As a key component of the research methodology, consideration was given to relevant and appropriate approaches to working in Māori immersion and kura kaupapa Māori schools. Once the two schools were selected, the principals were sent a letter informing them of the rationale and aims of the project. This was followed by a phone call to organise a visit to the school and to establish the identity of the researcher, in order to establish positive relationships and to confirm the schools' willingness to participate. Māori-medium schools have tended to be resistant to research projects that do not benefit the school directly or Māori-medium education in general. In this instance, the principals could clearly see the benefits of the project and were very positive about being involved.

The two principals and those teachers from the case study schools who were involved in the 2004 Te Poutama Tau project were also sent questionnaires (see Appendix A), essentially to promote thinking and discussion on the following questions:

- What role did the socio/cultural and demographic features of the school and its community have on the positive outcomes of the project?
- Did the relationships between the school and its local community, including links to the local iwi and hapū, also influence and or impact on outcomes?
- Did the experience and qualifications of management and teachers, particularly in relation to pāngarau (Māori-medium mathematics), also influence outcomes?
- What were the attitudes and involvement of school management and teachers in Te Poutama Tau?
- What was the effect of the Te Poutama Tau project on classroom practice?
- What are the teachers' reflections on the implementation of Te Poutama Tau?

This was followed up by a personal visit (kanohi ki te kanohi) to discuss issues relating to mutual benefits of the project, to outline the research process, to establish cultural legitimisation, and, of course, to discuss the research questions. Most of the interaction with the interviewees was carried out in the medium of Māori to validate and to establish the commitment of the researcher to the importance of te reo Māori to the kura.

Each teacher was interviewed for 15 to 20 minutes. This was followed by an interview with the principal. The interview responses and the reflections of the staff involved in Te Poutama Tau are discussed in the following section.

Results

Case Study 1: Kura A

Based on feedback from the participants, this initial section examines the brief history of the school and its relationship with its whānau community. This kura is located on the outskirts of one of the bigger cities of Aotearoa in a semi-rural locality and currently has a decile rating of 3. Some of the students attending this kura are drawn from all parts of the city, but in general, the majority live close by. Initially, the whānau established a kōhanga reo, then, subsequently, a kura (primary school) to cater for the needs of the kōhanga reo graduates. Essentially, parents desired the continued development of te reo and tikanga Māori for their children. The kura opened in the early 1990s in a garage, which illustrates the whānau commitment to the language revitalisation cause. The kura was subsequently relocated to a high school and eventually to its present site in 1994. Subsequently, the whānau developed educational activities that branched out into a range of community services operating from a Māori perspective.

The kura is dedicated to whole-brain, holistic, and accelerated learning that connects students to learning for life. The overall vision of the kura and the whānau is to provide quality education from early childhood through to adult education. The school aims to provide an environment that nurtures and develops the students' self esteem whilst holding fast to the taonga of their tūpuna, te reo me ngā tikanga Māori. The kura caters for children and whānau who have an active commitment to te reo Māori and its continued development as a national language of Aotearoa.

Students and parents are required to participate in an initial interview to clarify the school's expectations, particularly in the area of students' te reo Māori language competency. Parents are required to sign an initial parental agreement, which clearly makes explicit the expectations of the kura and its community.

The principal and teachers highlight the close relationships the kura has with the community, the local hapū, and the marae. The whānau is expected to work closely with the school and support students and staff. It is also expected that whānau develop their own knowledge and understanding of reo and tikanga independently and by attending wānanga organised by the school.

School and project leadership

The Te Poutama Tau facilitators and the teachers noted on a number of occasions the key role of the principal in curriculum leadership in general and her influence on the success of this professional development project. The principal had been teaching for a number of years, including 11 years in the role of principal, and was highly instrumental in the development of this particular kura from its conception. She closely monitored the students' progress in the project and provided a range of support strategies to staff, including release time to organise support resources. In her view, the school made significant progress in the project. She intrinsically believed all students could achieve success in mathematics. The teachers did comment, however, that not all students had positive attitudes to pāngarau prior to the Te Poutama Tau project.

The principal argued that successful practice not only required positive attitudes and knowledge of the subject but also an environment that was conducive to effective learning. In her view, this required "great classroom management" and the development of "good relationships" with and between students.

The role of the teacher

The teachers who participated in Te Poutama Tau had spent most, if not all, their teaching career in this particular school. In all cases, their qualifications were general teaching qualifications with no specific emphasis on teaching pāngarau. They had participated in previous mathematics workshops based on Montessori methodology.

Class sizes were between 15 and 20, with multiple age groups in each class. In general, the teachers judged the majority of students to be fluent in te reo Māori.

Teachers identified the Te Poutama Tau professional development model as critical to the successful student outcomes. A key feature of the model was the development of their own content and pedagogical knowledge. An additional feature of the model was the collaborative nature of the project, in particular, the development of a community of teacher practitioners who could share ideas and resources and support one another in implementing the project. The teachers also identified the importance of associated support mechanisms of the project, including the Te Poutama Tau facilitators and the teaching resources. The critical role of support resources has also been identified in studies by Christensen (2003, 2004) and Trinick (2005b). This is easy to understand, considering the paucity of Māori-medium resources across other curriculum areas.

The Number Framework

Teachers consistently referred to the support and guidance provided by the Number Framework: not only did it provide clear progressions of learning for students, but also, with the major shift to emphasising the teaching and learning of strategies, teachers felt their own content knowledge had improved.

The teachers themselves had high expectations of their students and believed the school and its management provided a clear vision and set high, but realistic, targets of achievement for students.

Case Study 2: Kura E

This kura is also relatively new and mirrors the experiences of a number of other schools involved in the revitalisation of te reo Māori in the late 1980s and early 1990s. The catalyst for its development was the concerns parents had for their children when graduating from kōhanga reo. Essentially, these concerns were based on a number of factors, including language loss and the feeling of disempowerment of the parent group.

Consequently, parents associated with the kōhanga reo set up the kura, with minimal state support, in a city council building. The kura was subsequently relocated in 1995, with the majority of its student population made up from two contributing kōhanga reo. The school has a close relationship with the contributing kōhanga reo and a large number of whānau had children in both institutions.

The school has a decile rating of 5 and is located on the fringe of one of the bigger cities. The school has close links to local hapū but also identified the fact that many students were from other tribal areas. The interviewees highlighted high parental expectations for their tamariki. All the parents/caregivers are interviewed to ensure that parents/caregivers commit to the philosophy of the school and its expectations, and vice versa.

School leadership

The facilitators and the staff identified the Principal as playing a key role in the positive outcomes in pāngarau for the kura. The Principal had an excellent grasp of Te Poutama Tau and its aims and was therefore able to support the staff in the development and implementation of the project. She provided valuable criticism of the programme to ensure positive student outcomes.

The Number Framework and the professional development model

As noted in the previous case studies (Trinick, 2005b), the interviews noted the Number Framework as a key factor. The Framework provided much more explicit information on the key content, teaching strategies, and language models. In the absence of the range of resources that exist for English-medium schools, this is critical for Māori-medium schools.

In English-medium education, there are large communities of mathematics practitioners, for example, teaching colleagues, advisors, and resources developers. All these various individuals and groups provide support to the class teacher directly and indirectly. In comparison, this community of practice is very small in Māori-medium education.

Teachers also appreciated the "in-class modelling" and the subsequent feedback from the facilitator and/or their colleagues. They believed they were open to changing aspects of their teaching after observing and trialling the strategies (Ministry of Education, 2006). The teaching and learning model provided by the project also provided significant guidance (Hughes, 2002).

Teachers involved in the project noted its positive impact on their classroom management. The consistency of learning routines provided more stable learning environments, with students taking more control over their own learning.

The interviewees all noted the positive crossover from Te Poutama Tau into other areas and strands of the pāngarau curriculum. For example, concepts learned in numeracy, such as multiplying and dividing by 10, made converting between measuring scales easier for a number of students.

Teachers identified a range of factors that supported the achievement of successful outcomes for students in pāngarau. One of the critical factors was the linking of Te Poutama Tau to the school's strategic planning to develop high standards of literacy and numeracy for all students. The plan consisted of a range of key interrelated components, such as the setting of clear targets for improving student achievement. The school set the target that all year 4–6 students will make gains in the grouping and place value aspect of the Framework by the second diagnostic assessment. A range of review mechanisms were put in place to answer questions such as "Have we met the target?", taking note of trends, strengths, and weaknesses and their contributing factors. These were followed up with a list of recommendations on "Where to next?"

Reporting to parents involved showing students' current strategy and knowledge stage and gains during 2004. Reporting to the community involved showing the mean student stage gains in 2003. Finally, reports to the Board of Trustees included comparison of school performance to national norms.

Additional components of the plan to support the targets included comments on the teaching and learning programme development and/or focus, staff and personal professional development, key agencies to provide support, community involvement, and finally a resourcing plan.

The teachers

The staff's experience in a teaching role ranged between 3.5 and 11 years. Some had spent the majority of their teaching years in this particular school, while others had only recently joined the teaching staff. Their teaching qualifications were general teaching qualifications with no specific specialisation in mathematics. However, all the staff had achieved in mathematics to a high level in secondary school, which partly explains their positive attitudes to teaching pāngarau.

Classes ranged in size from 20 to 30, with a number of age groups in each class. Teachers rated the majority of students as fluent in te reo Māori. There was mixed student attitudes to pāngarau, but the teachers noted a large number did not like it prior to Te Poutama Tau. Subsequently, the success that students achieved in the project assisted in fostering more positive attitudes to pāngarau for the majority of students.

The teachers highlighted the changes that occurred in their own teaching practice, particularly in the teaching of strategies. They utilised the various activities provided in the resources to get students to think both critically and creatively.

The teachers believed they had been very well supported by school management, but there was a strong feeling that there was a need for ongoing professional development in mathematics beyond their involvement in the Te Poutama Tau project.

Results and Discussion

While it is difficult to isolate individual items, the outcomes of this study suggest that the following key points that the two kura have in common contributed to the positive progress of the students in the Number Framework. It would seem also the following points cannot be seen in isolation from each other.

The Sociocultural and Demographic Features

The schools were middle- to low-decile, with the parents and caregivers coming from a variety of professions and occupations. With such a small sample, it is difficult to identify the impact of the parents/caregivers' own professions and socio-economic status on student achievement in the case study schools. However, the unifying bond throughout both school communities is based on the desire and aspirations of these school communities to revitalise Māori language, knowledge, and culture. These aspirations are manifested in the philosophies and principles of Te Aho Matua that underpin the various sociocultural practices of both schools. A key component of Te Aho Matua is the need for the school and its community to work closely together on aspects such as developing the curriculum. A number of studies have noted the key role of parental involvement in the education of their children to achieve positive educational outcomes (Hohepa, 1993; Bishop, et al., 2001).

Key Relationships

The discussions with the interviewees suggest that the positive relationships between the facilitators, the schools, and the school management that were established as part of the project were critical to student outcomes. In both instances, the facilitators had established strong links with the schools and the local communities. They closely identified with teaching colleagues in the project schools, and in many cases had built up professional development profiles in their communities over a period of time. In general, the facilitators worked alone in their regions as Māori-medium facilitators.

The case study schools shared practices that worked for each of them and drew strength from each other. As well as collaboration between schools, there has been frequent collaboration among the teachers, who provided peer support and mentoring for each other. A key factor that has been identified in the development of successful programmes is the establishment of an effective learning environment in which participants have opportunities to share ideas and to develop supportive long-term relationships with their colleagues (Timperley et al., 2003).

School and Curriculum Leadership

Both principals participated in the professional development programmes with the teachers, working alongside staff to develop a shared sense of purpose and direction. By modelling desired dispositions and actions, the principals enhanced the rest of the school's belief in the project and in their own capacities and enthusiasm for change. The lead teachers also played a significant role in the implementation of the project and were well supported by the Te Poutama Tau facilitators.

Development of New Knowledge and Skills

There is little argument that what a teacher knows is one of the most important influences on what happens in a classroom and ultimately on what students learn. Recent research confirms that teachers' knowledge of subject matter, student learning, and development and teaching methods is critical to teacher effectiveness (Higgins, 1999; Bobis, 2000; Bishop, Berryman, & Richardson, 2001).

The Te Poutama Tau professional development model consisted of a series of national professional development workshops for facilitators who, in turn, developed a series of workshops for participating schools and teachers. These workshops focused on the mathematical content of the Number Framework and the interconnection between concepts (for example, the relationship between multiplicative thinking and solving ratio and proportion problems). For teachers, the Framework provided a much more explicit picture of the required content and how students progress through the content. This point is closely associated with the setting of goals and the monitoring of performance.

Teachers of Māori-medium mathematics have an additional challenge with regard to support materials in the medium of Māori. Many of the facilitators and participating teachers are second-language learners of Māori language, and the support provided by these materials is critical to success. Māori-medium mathematics language development is relatively recent and, for some areas of the Framework (for example, ratio and proportion), there are no established patterns of discourse. Interpreting content is challenging for those who are not very proficient in Māori language and/or not familiar with the discourse. Therefore, Te Poutama Tau also supports facilitators and teachers to develop appropriate Māori language models. In some cases, this requires the development of new terminology. Māori-medium educators are also concerned with language revitalisation and development and, consequently, the linguistic aspects of the programme have been an inevitable subject of discussion and debate, centred on syntax, semantics, and issues of tribal dialect (Christensen, 2003).

The Role of Student and Teacher Beliefs

Teachers' beliefs and prior experiences affect what and how they learn (White, 2002; Smith & Lowrie, 2001; Beswick & Dole, 2001). This is particularly so in mathematics where there have been substantial changes in philosophies over the last 10 years or so. In the case study schools, teachers were open to changes in teaching practice if it improved student outcomes. The pedagogical approaches associated with Te Poutama Tau required teachers to develop learning experiences and to communicate in ways that guided learners to construct their own mathematical knowledge and understanding.

Prior to Te Poutama Tau, a number of teachers and students had negative attitudes towards pāngarau. Despite their professional training, many teachers still lack confidence, based on memories of their own mathematical learning experiences. In the case study schools, teachers and principals felt there had been significant change over the duration of the project in teacher and pupil attitude to pāngarau. Much of the change on the teachers' part was that they could see the positive outcomes and thus felt more inclined to change their practice.

For students, the ways in which numeracy was taught in the project eased many of their anxieties and increased knowledge and confidence. For teachers, the focus on students' development of mathematical thinking also provided an opportunity for teachers to develop their own understanding. In general, these factors have all contributed to a positive change in attitude to mathematics exhibited by many of the teachers and facilitators in the Te Poutama Tau project (Christensen, 2003, 2004; Trinick, 2005b).

Understanding of Outcomes of Practice for Students

It has been argued that the professional knowledge and skills of the teacher have a direct link to student achievement, behaviour, and attitude (Parsons, 2001). The Te Poutama Tau project provides teachers with diagnostic tools such as the diagnostic interview that give them quality information aligned to the Number Framework about the knowledge and thinking strategies of their students.

The case study schools used the data they sent to the national database to establish targets for planning and reporting. The data was used to group students according to ability and to set achievement targets for the year. The principals and lead teachers closely monitored the school performance during the year, setting clear goals for teaching staff. The goals were evaluated throughout the year.

As a result of the project, the principal and staff focused on student learning, including the development of positive attitudes as well as the knowledge and strategies of the Number Framework.

Reflective Practitioners

There is widespread agreement among teacher educators that encouraging teachers to reflect on the success of their teaching is a necessary first step towards change (Cobb, 1986; Artzt & Armour Thomas, 2002; Higgins, 2003). Teachers need to be encouraged to be reflective and to analyse their own practice. They also need opportunities to reconstruct and develop further knowledge and pedagogy around teaching and learning in order to be effective “agents of change” (Stokes et al., 1997). Change takes time and needs to be related to classroom practice. Teachers also need to be committed to the kaupapa (event) to perceive a need for change (Fullen, 1993).

Feedback from the case study interviewees suggests that the participants constantly reflected on their practices in relation to the pāngarau teaching programme. Teachers, with facilitator support, videoed and reviewed some of their teaching practices. The principals were constantly critiquing and reviewing the programme to ensure benefits for the learners.

Needs of Participants

Teachers working in Māori-medium contexts are faced with a number of challenges, including lack of resources, workload, content, and language. As noted previously, many of the teachers are second-language learners of Māori and trained in English-medium pre-service teacher education programmes, although this is changing as Māori-medium teacher education has grown. Many Māori-medium teachers teach all curriculum areas across multiple levels and ages and also perform multiple roles in their schools and communities. These roles include participation in community events, for example, sporting and cultural events, and administrative roles within the family and tribe (iwi).

Teachers in the case study kura considered the time spent with other teachers in planning and learning was most valuable and felt that this professional development module had eased their workloads in the long term. For teachers, the Number Framework has provided a much more explicit picture of the required content and related student progress. The Framework also provided clear links to the Māori-medium pāngarau curriculum statement. Significant ranges of print and electronic resources have been developed to support the project, often modified as a result of teacher feedback (www.nzmaths.co.nz). The project has also provided opportunities for parents and whānau to be involved in their children's learning in positive ways (Trinick, 2005b), thus developing stronger relationships within communities.

Summary

The outcomes of this study suggest that the following key points have contributed to the positive progress of the students in the Te Poutama Tau project. It would seem also that the following points cannot be seen in isolation from each other, but in combination. It is also important to note that these factors are also consistent with the 2004 case study schools (Trinick, 2005b).

- The leadership provided by the principal and the lead teacher was one of the key elements in the success of the Te Poutama Tau project in these two case study schools.
- The principals, lead teachers, and staff closely monitored the school performance during the year, setting clear goals for teaching staff. The goals were evaluated throughout the year. As a result of the project, the principal and staff focused on student learning – not only the knowledge and strategies of the Number Framework but also the development of positive attitudes.
- For teachers, the Number Framework provided a much more explicit picture of the required content and how students progress through the content. This point is closely associated with the setting of goals and the monitoring of performance.
- Both kura had a commitment to teaching and learning in the medium of Māori, but it is not clear in this research of the impact of the level of proficiency of the students on progress through the Number Framework.

There may well be features of the two case study schools unique to each of the schools that contributed to the positive results. Although the two schools follow the Te Aho Matua philosophies, they are different in their histories, their staff, and their relationships with their local communities.

Future Research

There has been minimal research done in New Zealand on Māori-medium schooling as noted in the introductory section. Therefore this report recommends:

- Developing a set of criteria to identify successful schools in Māori medium and profiling a range of successful schools. This report is limited in that it focuses only on the mean stage gains in the Numeracy Development Project as a success indicator, when in fact there are a considerable range of success indicators.
- Examining effective strategies that teachers use in the various curriculum areas and the quality of the relationship between student and teacher. This study is limited in that it does not explore the effective teaching and learning strategies used by teachers to improve achievement.

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Sustaining the Numeracy Project: The Lead Teacher Initiative 2005

Gill Thomas
Maths Technology Ltd.
<gill@nzmaths.co.nz>

Jenny Ward
Maths Technology Ltd.
<jenny@nzmaths.co.nz>

As the Numeracy Development Projects move into a phase of supporting schools to sustain and build on the initial professional development programme, it is important to identify the factors that support or hinder sustainability. This paper reports on the perspectives of principals, teachers, lead teachers, and facilitators who participated in the 2005 lead teacher initiative, aimed at developing the numeracy capabilities of lead teachers within schools. Participants believed the initiative had been effective, with workshops and facilitator visits to schools seen as the most valuable components of the programme. Schools and teachers identified ongoing facilitator support, lead teacher leadership within schools, and principal support as central to sustaining and developing effective numeracy teaching and learning. Some facilitators stressed that schools ultimately need to take responsibility for their ongoing numeracy professional learning if they are to successfully sustain numeracy practices.

Background

The New Zealand Numeracy Development Project (NDP) was implemented in 2001 following the 2000 pilot of the New South Wales Count Me In Too programme. The NDP was initiated as a result of the relatively poor results of New Zealand students in the 1995 Third International Mathematics and Science Study (TIMSS). Initial phases of the project involved the development of a comprehensive numeracy policy and strategy and several pilot projects focusing on the professional development of teachers (Higgins, Parsons, & Hyland, 2003; Ministry of Education, 2001). The NDP is part of the New Zealand Ministry of Education's Literacy and Numeracy Strategy and so reflects the key themes of that strategy: clarifying expectations, improving professional capability, and involving the community (Ministry of Education, 2002).

There are three essential elements of the NDP. They are the Number Framework, which describes the mental processes (strategies) that students use to solve number problems as well as the knowledge required to do so, the diagnostic interview, and the professional development programme (Bobis, et al., 2005). As the NDP has progressed, comprehensive evaluations have informed and shaped the development.

> The NDP is moving into a phase in which the emphasis is not only on improving the teaching and learning of mathematics in New Zealand schools but also on enhancing the capacity of schools to sustain and build on that learning. (Ministry of Education, 2005, p. 4)

In 2004, one of the evaluations commissioned by the Ministry of Education examined the sustainability of NDP practices within schools across the regions of New Zealand (Thomas, Ward, & Tagg, 2005). The evaluation report focused on the roles of facilitators and lead teachers within schools because these roles had been found to be important to the internalisation of NDP practices into school structures and classroom practices (Higgins, 2003). Evidence of the development of numeracy professional learning communities within schools was also examined, with the use of student achievement information seen as key to sustaining developments (Timperley, 2003).

In 2005, the Ministry of Education commissioned a further study into the sustainability of the NDP. The evaluation focused on the effects of the 2005 lead teacher initiative, an initiative aimed at sustaining NDP practices by developing the numeracy capabilities of lead teachers within schools. This paper reports on the findings of this study.

Method

Participants

The sample comprised all schools and facilitators involved in the lead teacher initiative in 2005. Regional numeracy co-ordinators were asked to provide lists of schools involved, along with the number of lead teachers working in each school. All lead teachers received two questionnaires, one for themselves and one for one teacher in their school, whom they were asked to select on the basis of the alphabetical order of surnames. Accordingly, 937 lead teacher and teacher questionnaires and 643 principal questionnaires were distributed to the 643 schools involved in the lead teacher initiative.

Regional numeracy co-ordinators were asked how many facilitators were working in the lead teacher initiative in 2005. Accordingly, 53 facilitator questionnaires were sent to regional co-ordinators for distribution in the seven regions involved. Table 1 shows the distribution of questionnaires across the seven regions with corresponding return rates.

Table 1
Regional Distribution of Questionnaires and Return Rates

	Number of questionnaires distributed						
	Facilitator		Principal		Lead teacher and teacher		
	Number distributed	% return	Number distributed	% return	Number distributed	Lead teacher % return	Teacher % return
Otago/Southland	2	100	67	40	100	37	41
Canterbury	8	25	65	35	135	41	41
Wellington	8	100	134	43	196	32	36
Massey	6	67	48	29	56	29	30
Waikato	14	43	161	42	212	38	40
Auckland	12	75	108	42	178	36	35
Northland	3	100	60	37	60	25	35
Total	53	62	643	40	937	35	38

Procedure

Questionnaires were developed to gather information from key participants: principals, lead teachers, teachers, and facilitators. Questions focused on the continued use of NDP practices in classrooms, school-wide developments in numeracy, the development of numeracy communities of practice, and the factors necessary to sustain effective numeracy teaching and learning. The majority of the questions involved closed responses, with participants being asked to rank factors or use Likert scales.

Questionnaires were distributed to all participants in early November, with returns requested approximately two weeks after distribution. An email was sent to regional co-ordinators at the end of November asking them to remind the participants in their regions to return the questionnaires. All questionnaires received prior to December 14 were included in the evaluation.

Findings

The findings are presented as responses to six key questions. Participant comments have been used to illustrate themes and are taken directly from questionnaires. Even though all but one of the items in the questionnaires were closed, many of the participants added comments. Some participants did not respond to all questions, so some percentages do not add up to 100.

To What Extent Are Teachers Continuing to Use Numeracy Project Practices in Their Mathematics Programmes?

One of the items in the questionnaires asked the lead teachers and teachers to make a general judgment about the extent to which they incorporate NDP practices into their classroom mathematics programmes. Eighty-eight percent of lead teachers and 83% of teachers report using Numeracy Project practices considerably or fully.

> As a staff we have put much energy and effort into the NDP this year. Staff have been reflective in their practice and are enthusiastic to maintain the impetus the project has created. (Principal)

Lead teachers and teachers were also questioned more specifically about the numeracy practices used in their classrooms. The most widely utilised numeracy practices were the grouping of students based on strategy stage, the use of numeracy activities from resource books or the website, and the use of resources such as tens frames and number lines.

> Resources are great, material masters are easy to use. (Teacher)

In general, a high degree of utilisation of numeracy practices were reported, with all seven components surveyed being used by the majority of lead teachers. Table 2 summarises these findings. Overall, lead teachers reported higher usage of Numeracy Project practices than teachers and this could be attributed to their continued training.

Table 2
Utilisation of NDP Components

	Lead teachers	Teachers
Numeracy activities (from books or website)	97%	90%
Student groupings based on strategy stage	97%	89%
Resources, e.g., tens frames, number lines, other material masters	96%	91%
NumPA assessment	89%	82%
Planning templates	66%	64%
GloSS Assessment	56%	42%
Numeracy Planning Assistant (online)	53%	25%

One common theme among participants was the way numeracy practices have become a regular part of school and classroom programmes. Lead teachers and teachers made it clear that their future practice would continue to incorporate numeracy, with over 95% of teachers and lead teachers reporting that they intended to use NDP ideas and materials in their classroom mathematics programmes to the same or a greater extent in 2006.

> We are a numeracy school: numeracy practices are fully integrated in our school. (Lead teacher)

> I always begin with a numeracy starter, even when covering another topic unit, for example, measurement. (Teacher)

What Numeracy Developments Have Occurred in Schools as a Result of the Lead Teacher Initiative 2005?

Some of the developments that occurred within schools were focused on the lead teachers themselves, reflecting the priority that is placed on lead teachers within the sustainability component of the NDP. Most of the lead teachers (83%) reported an increased confidence in their own ability to lead numeracy within their school as a result of the lead teacher initiative in 2005 and were generally positive about their role.

> I love teaching numeracy. (Lead teacher)
>
> I feel there are resources in place and a clear direction for numeracy. (Lead teacher)

Lead teachers (87%) also reported development of their professional knowledge of mathematics and attributed this to aspects of the professional development they were involved in.

> I have found the lead teacher workshops invaluable to guide me as to where I should be leading the school and learning about new developments and initiatives. (Lead teacher)

In addition to the professional development of the lead teacher, school-wide developments in numeracy were reported. Seventy-nine percent of lead teachers identified that the teaching and learning of numeracy had changed as a result of the 2005 sustaining numeracy initiatives. Principals concurred with this, with 90% rating the lead teacher as moderately to very successful in developing the teaching and learning of numeracy within the school. Those principals that stated numeracy developments had not occurred attributed this to a variety of factors, including a lack of time for the lead teacher to implement change or an inappropriate selection of lead teacher.

> We need to give her [the lead teacher] more time to work in classes with teachers. Wasn't in 2005 budget. (Principal)
>
> We chose the wrong person [to be the lead teacher]. (Principal)

Lead teachers were asked to identify the nature of the numeracy developments occurring in their schools in 2005. Table 3 summarises these findings.

Table 3
Numeracy Development in Schools

	Total	Development initiated by:		
		School	Facilitator	School and facilitator
Numeracy or maths assessment practices	79%	33%	22%	24%
Numeracy teaching practices within classrooms	77%	23%	26%	28%
Expectations for student achievement	70%	39%	17%	14%
School-wide plans for numeracy	66%	38%	10%	18%
Numeracy reporting practices (to parents, Board of Trustees)	62%	46%	6%	10%

The most widely reported numeracy developments in schools were changes in the assessment and teaching practices of teachers.

> We have sustained the numeracy programme successfully for the last three years. However, consistency of teaching methods for strategies and assessment is a factor which needed to be investigated to help student achievement. (Teacher)

School-wide numeracy developments occurred as a result of schools' initiative and support from facilitators. Nearly half of the schools initiated changes in numeracy reporting practices, and over a third of schools independently undertook developments in numeracy planning and changing expectations for student achievement. Approximately a quarter of schools made changes to both teaching and assessment practices as a direct result of facilitator support.

Lead teachers identified the types of actions they had undertaken to develop numeracy within their schools. Table 4 summarises these results.

Table 4
Numeracy Developments Implemented by the Lead Teacher

	Have implemented	Would like to implement
Regular (at least 2 per term) staff meetings focused on numeracy teaching and learning	25%	27%
Occasional (1 per term) staff meetings focused on numeracy teaching and learning	42%	21%
In-class support/mentoring of new teachers to the school (including beginning teachers)	46%	27%
In-class support/mentoring of numeracy-trained teachers	41%	31%
A school-wide focus on numeracy achievement	66%	14%
Developing school-wide plans for numeracy	55%	23%
External support (e.g., private consultants)	9%	8%
Collaborating with other schools	20%	21%

The most widely reported developments implemented by lead teachers were staff meetings focused on numeracy teaching and learning and the establishment of a school-wide focus on numeracy achievement. Two-thirds of lead teachers made developments in these areas. Just over half of the lead teachers developed school-wide plans for numeracy, and in-class lead teacher support for new teachers and trained teachers was widespread.

Principals reported supporting lead teachers in their role of leading numeracy development in a variety of ways. Over 80% of principals gave support by providing opportunities for staff meetings focused on numeracy and by informal discussions with lead teachers.

> We are starting to develop capabilities and now need a consistent approach and consolidated understandings. We meet to develop this further. (Principal)

Principals also supported lead teachers by meeting with them formally and by providing release time to enable lead teachers to work with staff.

> It's difficult to provide support for new teachers without release time. Release time is essential. (Principal)

Table 5 summarises the types of support that principals provided to lead teachers.

Table 5
Principal Support of Lead Teachers

Support provided	% provided with support
Informal discussion with the lead teacher	86
Opportunity for staff meetings focused on numeracy	84
Formal meetings with the lead teacher	48
Release time for lead teacher to support new staff	47
Release time for lead teacher to mentor numeracy trained staff within class	35
Management units (from one to four reported)	32

To What Extent Do Schools Track and Use Numeracy Achievement Data to Inform Their Decisions about the Teaching and Learning of Number?

Nearly all schools collect information on student achievement in numeracy, with 97% of teachers and lead teachers identifying that they do so. The most widely reported use of achievement information was for reporting to parents. Ninety-two percent of lead teachers reported this, with the majority of these teachers using achievement information to report to parents two to four times a year. Use of achievement information to report to Boards of Trustees was also high (86%).

Student achievement information was used by over 80% of schools to identify student learning needs and to develop teaching programmes, with a third of schools using the information for these purposes more than four times a year. Table 6 summarises the reported uses of achievement information.

Table 6
Reported Uses of Achievement Information by Lead Teachers

		Frequency of Use		
	Information used	Once a year	2–4 times a year	More than 4 times a year
Reporting to parents	92%	14%	76%	2%
Identification of individual learning needs	90%	8%	47%	35%
Reporting to Board of Trustees	86%	45%	40%	1%
Development of teaching programmes	81%	15%	38%	28%
Development of school targets or benchmarks	80%	52%	28%	
Comparison with national achievement data	73%	51%	22%	
Staff appraisal process	55%	35%	20%	

Lead teacher and teacher views on the development of numeracy targets to track student achievement varied, with the majority of lead teachers (79%) stating numeracy targets had been developed but less than half (40%) of teachers reporting this.

Lead teachers and teachers who responded that their school had targets were asked to state what those numeracy targets were. Lead teachers were asked to provide targets that had been developed for the school. Copies of school documents were supplied by 17% of lead teachers and were generally comprehensive. Teachers were asked to supply the targets for the year levels they teach and 35% of teachers did so. The nature and detail of the targets provided by teachers varied greatly, with some teachers stating the strategy stages students were expected to achieve, others describing the skills to be focused on, and some using levels from the New Zealand Mathematics Curriculum to describe achievement targets. Some teachers described achievement goals in terms of progress made by students. Examples given by teachers include:

> By the end of year 6, 80% will be working with stage 5+. (Teacher)
>
> Complete stage 7 by end of year 6. (Teacher)
>
> Target for year 2: Move up at least one level from the beginning of the year. (Teacher)
>
> Target for years 4 and 5: To raise student achievement in all the add/sub and mult/div domains and give them vocabulary to work with. (Teacher)
>
> 80% pass rate in basic facts to 20 and tables to 10. (Teacher)
>
> Children's abilities in problem solving to be raised. 80% of year 4 children working at or beyond level 2 [of the curriculum]. (Teacher)

What Have Been the Most Effective Elements of the Lead Teacher Initiative (2005)?

All participants were asked to identify the most effective elements of the lead teacher initiative, and participants' views provided results from a variety of different perspectives. In general, principals thought the initiative was effective, with the majority of principals (92%) viewing the lead teacher programme as moderately or very effective in developing the numeracy capability of the lead teacher in their school:

> Found the whole professional development increased her knowledge. (Principal)

Participants were asked to rank components of the lead teacher initiative to provide a measure of the relative effectiveness of each component. While some participants ranked elements from one to five, as asked, others appeared to misunderstand the question and rated each element individually on a one-to-five scale. This means that the relative effectiveness of each component is less clear than if the ranking had been consistently used by participants. Both types of response have been combined and used as an indication of the effectiveness of each of the initiative's elements.

The majority of lead teachers received support by attending workshops (87%) and having a facilitator visit their school (62%). Lead teachers found this support valuable:

> Workshops are crucial for staff development opportunities and to further own understanding of where the project is headed, children's targets, and MOE expectations. (Lead teacher).
>
> Facilitator visits to school maintain programme and keep it vital. (Lead teacher)

Lead teachers also found facilitator visits to classrooms valuable and facilitator support to mentor teachers and lead numeracy within the school helpful. Table 7 summarises the support received by lead teachers and the perceived effectiveness of this support.

Table 7
Support Received by Lead Teachers

	Percentage received lead teachers	Ranking/Rating[1] (as percentage)		
		1–2	3–5	6–7
Workshops	87	56	29	4
Facilitator visits to school	62	54	28	9
Facilitator visits to classes	48	53	31	10
Facilitator support with mentoring teachers	43	42	39	11
Facilitator support with providing numeracy leadership within school	53	46	36	8
Lead teacher material on NZ Maths website	74	39	40	9

Facilitators' views are in accordance with the views of lead teachers because they both identified workshops and facilitator visits to schools as the two most utilised components of numeracy support, with approximately 90% of facilitators involved in their delivery. Facilitators viewed both these elements as helpful and identified the use of email as another important support mechanism:

> Workshops provide professional learning time for numeracy. (Facilitator)
>
> When visiting a school, I withdraw each teacher from the classroom and work with them related to their needs. I demonstrate with children where necessary. (Facilitator)

Table 8 summarises the numeracy support elements delivered by facilitators.

Table 8
Support Delivered by Facilitators

	Percentage delivered facilitators	Ranking/Rating (as percentage)				
		1	2	3	4	5
Workshops	95	68	18	15		
Facilitator visits per school	89	53	24	6	3	9
Facilitator visits to class	56	26	12	15		6
Other (state)	60	29	6	15	9	3

Teachers received support from lead teachers in a variety of ways. The most common and most valuable forms of support were the introduction of new resources and the use of staff meetings to focus on numeracy. Over 70% of teachers received support in these ways. Approximately half of the teachers were involved in in-class mentoring or professional readings in numeracy.

> I find support from the lead teacher useful. Helpful resources and advice about particular students. We have an extremely able syndicate leader. (Teacher)

[1] Ranking/Rating is given as a percentage of the lead teachers who received the elements listed. Where the percentages do not add up to 100, it is due to some participants not rating or ranking the element.

Table 9 describes the support that teachers received from lead teachers.

Table 9
Support Received by Teachers

	Percentage received teachers	Ranking/Rating (as percentage)				
		1	2	3	4	5
Introduction of new resources	78	31	28	18	10	6
Staff meeting focused on numeracy	70	33	23	22	10	7
In-class mentoring or support from lead teacher	53	35	18	20	8	13
Professional numeracy readings	50	15	16	30	23	12
Other (state)	11	39	17	12	5	20

One interesting aspect of the response to questions about elements of the lead teacher initiative was the misunderstanding that some participants appeared to be under as to the scope of the programme. Some participants viewed the development as the workshop component only and answered questions accordingly.

> Question: Has your confidence in your own ability to lead numeracy within your school been further developed through the lead teacher initiatives this year?
>
> Response: Those that I have been able to attend. (Lead teacher)

Such comments indicate a narrow view of the professional development programme provided by the lead teacher initiative. Also of interest were the participants who regarded ongoing workshops and facilitator support for all staff as vital:

> Need support of facilitators for all teachers to maintain programme and keep it vital. (Lead teacher)
>
> Not lead teacher's role to develop numeracy teaching, facilitator has done that ... Lead teacher duties have mainly been to ensure teachers are prepared for facilitator's visits. (Principal)

What Elements of Numeracy Support Do Schools and Teachers Believe They Need in Order to Sustain or Further Develop Effective Numeracy Teaching and Learning?

All participants were asked to rank a number of factors in order to determine their relative helpfulness for sustaining and developing numeracy within schools. Some participants answered this question as asked, ranking all factors.[2] Others appeared to misunderstand the question and rated each component individually as on a Likert scale. The average rankings presented in Table 10 are based on the responses of those who ranked the factors as requested. This represents 61% of principals, 46% of teachers, 45% of lead teachers, and 50% of facilitators. Factors reported are those in common across the questions asked to all groups of participants.

2 Elements were ranked from 1 (highest) to 5 for principals and lead teachers, from 1 to 7 for teachers, and from 1 to 8 for facilitators.

Table 10
Factors for Sustaining and Developing Numeracy

	Average ranking			
	Principal	Teacher	Lead teacher	Facilitator
Ongoing facilitator support	2.6	3.7	2.4	3.4
Release time for lead teachers to mentor and support teachers	3.3	4.4	3.4	3.8
Collaboration between teachers about numeracy practices	3.0	2.8	2.8	3.8
Lead teacher leadership within school	2.6	4.0	not asked	2.5
Principal support and leadership of numeracy within school	not asked	5.2	3.2	2.5

Participants' views on the relative importance of the five factors listed in Table 10 varied. Principals and lead teachers believed ongoing facilitator support was most helpful for sustaining and developing numeracy into the future, with principals believing lead teacher leadership within the school was equally helpful. Facilitators also identified lead teacher leadership as important and believed principal support and leadership within the school to be equally important. Teachers rated collaboration between teachers about numeracy practices as the most helpful factor for sustaining numeracy.

Each group of participants was also asked to rate other factors that they would have a unique perspective on. When all factors were considered, principals identified ongoing facilitator support as most helpful for sustaining and developing numeracy in the future and the monitoring of school-wide numeracy achievement as least helpful. Teachers believed collaboration between teachers about numeracy practices was most helpful and principal support of numeracy was least helpful. Lead teachers concurred with principals and ranked ongoing facilitator support as most helpful and believed opportunities to collaborate with other numeracy lead teachers to be least helpful. Facilitators believed that effective lead teachers within schools were most beneficial for sustaining and developing numeracy and release time for all teachers was least important.

Participants were asked what they believed the barriers to sustaining and developing numeracy were in the schools they worked in. This was an open-ended question, with respondents required to identify barriers. Results for this question are summarised in Table 11. Items listed are those that at least one group of participants identified as a barrier, with a reporting threshold of 10%.

Table 11
Barriers to Sustaining and Developing Numeracy

Barrier	Percentage of respondents identified			
	Principal	Teacher	Lead teacher	Facilitator
New staff lack training	48	17	26	58
Lack of teacher time to plan, teach, and assess numeracy	11	25	38	17
Teacher resistance to new ideas or lack of motivation	13	10	25	17
Lack of focus on numeracy due to participation in other PD projects	13	4	11	25
Lack of funding for release time and resources	18	11	18	17
Lack of ongoing PD in numeracy	12	13	10	
Resource issues: availability, ease of use, lack of updates	8	21	16	
Lack of principal support for numeracy		1	3	33
Ineffective lead teachers	3	1		31

Participants were generally in agreement about the barriers that exist to sustaining numeracy, with five barriers identified by all four groups of participants. The barrier identified by the highest number of participants was the challenge presented to a school when teachers who lack numeracy training are added to the staff. This was identified by approximately two-thirds of facilitators, half the principals, a quarter of the lead teachers, and 17% of teachers.

> Change of staff, already lost one lead teacher during contract. Not always possible to replace with numeracy-trained teachers. (Principal)
>
> Change of key staff, e.g., those who have been trained leave the school and are replaced by untrained people so school is back to square one. (Facilitator)
>
> Some new teachers (even beginning teachers) have had very little Numeracy Project experience and they can only go to catch-ups which aren't as effective as the full programme with intensive facilitator input. (Lead teacher)

Lack of teacher time to plan, teach, and assess numeracy was also widely reported as a barrier, with over a third of lead teachers and a quarter of teachers identifying this, as well as over 10% of facilitators and principals.

> Lack of time, materials, and equipment having to be made by classroom teachers. (Lead teacher)
>
> I believe planning for the non-teaching group is the biggest hurdle. Teachers need to spend up to 45 minutes a week to sort out meaningful follow-up tasks. This is not unreasonable at all, but many are not used to doing this. (Lead teacher)
>
> Teachers need ideas about how to build assessment into their programmes without it becoming too cumbersome. (Teacher)

Other barriers to sustaining numeracy that all four groups of participants also identified were lack of motivation, teacher resistance to new ideas, lack of focus on numeracy due to participation in other professional development projects, and lack of funding for release time and resources.

> Not all teachers on board, some are reluctant and not willing to change. (Teacher)
>
> Commitments to other contracts make it harder to maintain a watching brief in numeracy. (Principal)
>
> All these [professional development] programmes are excellent, but there is too much. If we had been able to concentrate solely on numeracy, I think greater progress would have been made. (Principal)

Principals, lead teachers, and teachers believed a lack of ongoing professional development in numeracy and issues surrounding resources, including resource availability and ease of use, were also barriers to sustaining and developing numeracy.

> We have found that resources have been lacking in the senior area, which has been frustrating for the classroom teachers. (Principal)

Approximately a third of facilitators identified lack of principal support and ineffective lead teachers within schools as barriers. Small percentages of teachers, lead teachers, and principals were in agreement.

In addition to those factors identified in common with other respondents, facilitators described two other barriers to sustaining numeracy. These were the lack of a numeracy professional development culture within schools (22%) and the lack of support for numeracy from schools' senior management teams (14%).

To What Extent Are Schools Developing Numeracy Communities of Practice?

Teachers were asked about the numeracy practices they participated in, to get a picture of the types of professional numeracy activities happening in schools. Results are presented in Table 12.

Table 12
Professional Numeracy Activities

	Percentage of teachers participated	Frequency (as percentage)		
		Once a year	2–4 times a year	More than 4 times a year
Reflection on own teaching practice	86	6	24	56
Collaboration with other teachers	83	3	24	56
Use of numeracy achievement information	81	13	48	20
Numeracy meetings (staff, syndicate)	80	8	43	29
Use of numeracy targets	54	15	28	11

Over 80% of teachers were involved in reflecting on their own teaching practice, collaborating with other teachers, and using student achievement information in numeracy. Just over half of the teachers made use of numeracy targets.

The frequency of these activities varied, with just over half of the teachers reflecting on their own teaching practice and collaborating with others more than four times a year. Nearly half of the teachers were involved in using achievement information and staff meetings based on numeracy between two and four times a year. Twenty-eight percent of teachers report using numeracy targets between two and four times a year.

One of the facilitators provided a comprehensive summary of two schools that she believed were successfully sustaining numeracy developments. The following excerpts are taken from her comments and illustrate the belief that successfully sustaining schools are those that are empowered to take responsibility for ongoing numeracy professional learning within their school and have strong leadership through effective and committed lead teachers and principals.

> Having worked with two successfully sustaining schools, it is apparent these schools face all the same barriers as other schools – teacher mobility, resistance by individual teachers, etc. However these schools have high quality lead teachers ... The principals have numeracy sustainability in the strategic plan for the school and both schools have a strong professional learning community ... Expectations are set and barriers are not viewed as barriers but as problems to be solved. There is the expectation that the schools themselves are empowered to solve the problems rather than wait for someone else to do it for them ... The lead teachers felt it was imperative for the workshop element of the NDP to continue to be offered for catch-up teachers but felt in-school support should come from themselves.

Concluding Comment

Lead teachers and teachers continue to incorporate NDP ideas and materials into their classroom mathematics programmes, with the most widely utilised numeracy components being the grouping of students based on strategy stage and the use of numeracy resources and activities. School-wide numeracy developments were reported, with changes in the teaching and assessment practices of teachers being widespread. Lead teachers also reported an increased confidence about leading numeracy developments and an enhanced professional knowledge of mathematics.

Participants believed the lead teacher initiative had been effective in developing the numeracy capability of lead teachers in their schools, with workshops and facilitator visits to schools seen as the most valuable components of the programme. Schools and teachers identified ongoing facilitator support, lead teacher leadership within schools, and principal support as key to sustaining and developing effective numeracy teaching and learning. Barriers to sustaining and developing numeracy that were reported included the challenge presented to schools when teachers who lack numeracy training are added to the staff and a lack of time for teachers to plan, teach, and assess numeracy.

Schools appear to be developing numeracy communities of practice, with teachers involved in reflecting on their own teaching practice, collaborating with other teachers, and using student achievement information in numeracy. The majority of schools reported using student achievement information to identify student learning needs, to develop teaching programmes, and to measure progress against targets.

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Sustained Numeracy Project Practices in Two Schools

Fiona Ell
University of Auckland
<f.ell@auckland.ac.nz>

Kay Irwin
University of Auckland
<k.irwin@auckland.ac.nz>

The numeracy practices of two schools were considered in depth in order to investigate the factors contributing to whether or not schools and teachers sustain Numeracy Development Project (NDP) approaches after they have completed their initial participation. Lead teachers and four other teachers from a large, urban, contributing primary school and from a small, rural, full primary school gave interviews. Videos of mathematics lessons were taken in three classrooms. The two schools took contrasting approaches to establishing NDP practices. In both schools, lead teachers played a key role in motivating and resourcing staff. Analysis of interviews alongside videos showed that these teachers described their practice accurately. Teachers on the videos were organising their lessons in accordance with NDP suggestions and were asking for, and responding, to students' strategies.

Background

As the introduction of the Numeracy Development Project (NDP) to primary schools nears the end of its initial phase, the question of sustainability becomes of increasing importance. Schools involved in the NDP have received facilitation programmes, intended to result in long-term changes in teachers' numeracy teaching practices. An in-depth review of sustainability as a concept is beyond the scope of this paper; rather a brief summary of work related to the New Zealand NDP is provided.

The Ministry of Education has let contracts to provide ongoing support for schools through the six Colleges of Education, who employ facilitators to work on sustainability issues. Thomas, Ward, and Tagg (2004) provided an evaluation of these initiatives and found that, although teachers who had participated in the sustainability programmes reported using NDP materials and intended continuing to use NDP approaches in their classrooms, this was not necessarily driven by raising student achievement in numeracy. The teachers appeared to be at the stage of wanting resources and ideas rather than engaging with students' thinking. This engagement with students' thinking is a key element of internalising NDP approaches (Bobis, et al., 2005) and therefore underpins sustained practice (Higgins, 2004).

Higgins (2004) addresses sustainability in her evaluation of the Advanced Numeracy Project. She suggests three factors that contribute to sustained practice, which she defines as "... continuing enactment of the structural elements of the project" (p. 53). These factors are:

> innovation which leads to internalisation, which leads to sustainability. Innovation in teaching practices amongst a small group of staff ... (and) ... a collective, internalised effort creates a dynamic from which changes can be sustained at the school level. (Higgins, 2004, p. 59)

In Higgins' analysis, individual teachers are seen as the starting point for sustained NDP practice, with these individuals supporting each other in teams and generating a school-wide commitment to continuing NDP practices.

New Zealand's NDP is one of a number of initiatives internationally that aim to address the learning and teaching of numeracy in primary schools. Bobis et al. (2005) describe the implementation and effectiveness of three such programmes – the New Zealand NDP, the New South Wales' Count Me In Too project, and the Victorian Early Numeracy Research Project. A common factor in all three programmes was the desire to overcome the drawbacks of a one-off professional development experience by encouraging teachers to:

> take research information from external sources and from their own children, reflect on it with colleagues, and make adjustments to planning for individuals and groups, with this iterative process continuing over an extended period of time. (Bobis et al., 2005, p. 50)

This gives a picture of sustainability as an ongoing process of engagement with the ideas and principles of the projects. Higgins (2004) discusses "structure" as a significant concept in understanding sustainability. This notion of structure embodies the iterative process described above and is further defined by Higgins (2004) for the New Zealand NDP as "the Number Framework and the Teaching Model" (Higgins, 2004, p. 53).

The NDP seeks to change teachers' current practice to teaching that is based on accurate observation of students' strategies and needs, that utilises interaction and discussion, and that emphasises the development of robust and flexible concepts about number. These classroom practices are framed by broader system practices at syndicate, school, and Ministry of Education level that enable, constrain, and shape what occurs in the classroom. The classroom practice builds a school-wide commitment (Higgins, 2004); the school-wide approach at the same time shapes the classroom innovations. This study therefore considered three different types of data: school-wide documentation and strategies, interviewing teachers about their practice, and video observation of classes.

Method

Participants

Teachers from two schools participated in this study. The schools were nominated by NDP area co-ordinators and were in different areas. Co-ordinators were asked to nominate schools that had what they felt was a typical response to the facilitation, rather than the most enthusiastic or most confident schools. "City School" was a high-decile, 20-teacher, urban, contributing school, and "Country School" was a mid-decile, seven-teacher, rural, full primary school. Five teachers from each school volunteered to participate in the study. In each school, this was the lead teacher and four other teachers from differing areas of the school. The teachers' experience, class level, and facilitation history are summarised in Table 1 below.

Table 1
Summary of Participants

City School				Country School			
Participant	Years of teaching experience	Length of time since facilitation	Class level	Participant	Years of teaching experience	Length of time since facilitation	Class level
F	25	One year	Yr 5–6	A#	25	One year	Yr 6–7–8
G*	7	Two years	On leave	B#	23	Two years	Yr 5–6
H	3	Two years	Yr 1–2	C*	12	One year	Yr 6–7–8
I	3	Two years	Yr 3–4	D	1	Current	NE
J#	4	Two years	Yr 5–6	E	5	One year	Yr 3–4

*Numeracy lead teacher # Video participant

Method

Data was collected on school-wide planning and organisation, teachers' responses to the programme, and classroom practice. The school-wide data was obtained by interview with the lead teacher and collecting documentation. The teacher data was collected in individual interviews of approximately 15–20 minutes. These were audio-recorded and transcribed. One 45-minute lesson was videoed in each of three classrooms, two at Country School and one at City School. (The video from a fourth classroom was withdrawn by the participant.) The teachers were asked to follow their normal mathematics lesson format, and the responses of the students suggested that this was the case. Well-established routines appeared to be in place, and artefacts such as recording in modelling books and group boxes and task boards were present. The videos were analysed using a protocol derived from the Manurewa Enhancement Initiative's (MEI) indicators of NDP practices (Hughes, 2006, pers. comm.).

Findings

School-wide Implementation

City School and Country School took contrasting approaches to school-wide implementation of the NDP post-facilitation. Both schools had enthusiastic lead teachers. These teachers were cited as a key influence on practice by colleagues in the interviews and had taken responsibility for carrying on the NDP in their schools.

Table 2 summarises the schools' approaches to continuing with the NDP.

Table 2
City School's and Country School's Approaches to Continuing with the NDP

	City School	Country School
Policy	New policy written to specify NDP practices and ideals.	Policy unchanged – waiting until practice is "embedded" before they write new policy.
Planning	Standard planning formats developed with facilitator; planning standardised across classrooms.	Planning individually and on a range of formats. Discussion to take place after teachers had worked through issues in their classrooms.
Assessment	Separate assessment sheet system developed for tracking. Benchmarking system used alongside this, with comparison to national norms.	Using Numeracy Project Assessment diagnostic tool (NumPA) data from previous year, observing and re-assessing if necessary. Individual records kept – range of formats.
Equipment and resource books	Resource books in classrooms. Equipment provided for each classroom, stored in the room. Teachers making own games.	Resource books in classrooms. Equipment provided for each classroom, stored in the room. Teacher aide hired to make games. (Applied for a community grant for extra equipment and teacher-aide time.)
Outside support	Contract facilitator hired to help finish planning and assessment schemes.	Teacher aide hired to make resources, using community grant.

Country School had deliberately left school-wide policy changes until practice changes were firmly in place. They felt that the key for continuing to use the NDP practices effectively was having the resources readily available. They spent money and time resourcing each classroom and making sure that everyone had the necessary equipment to teach the lessons in the NDP booklets.

City School had hired a contract facilitator to help them complete school-wide planning and assessment systems. They had also invested in equipment and asked each student to buy a small whiteboard as part of their stationery order, but the teachers were making their own games and activity materials.

Teachers Talk about Their Practice

Teachers were asked a range of questions about their practice in teaching numeracy concepts. Responses to questions about what made the biggest difference for students and what was the most useful aspect of the programme focused on the hands-on nature of the lessons, students talking about their ideas, and how teaching was based on how students think.

Table 3 summarises the responses of the teachers to the question "What is the most difficult aspect of the Numeracy Project for you in your classroom?" The responses are indicative of the teachers' understanding of the NDP approaches. Some teachers focused on practical aspects of implementation, while others were concerned with the mathematics and students' thinking. This did not differ systematically by school or teaching experience, pointing to the construction of individual understandings about the NDP approach.

Table 3
Responses of Teachers to the Question "What is the most difficult aspect of the Numeracy Project for you in your classroom?"

Teacher	Response
D	Is each child in the group getting it, or are the group getting it?
C	Acting out the lessons yourself to make sure you get the maths.
B	Getting the children who are not with you to do something useful.
A	Planning – needs to be responsive and you need to understand the lesson.
E	Sharing with them all the strategies so they can choose what's best – they tend to stick with what they know.
I	Assessment is time-consuming, resourcing lessons, fitting in two groups.
J	Knowing when they have really grasped it.
F	Retraining myself after 25 years of doing it my way.
H	Resourcing – making games and independent activities.

Nine of the 10 interviews followed a similar pattern of response. The teachers were enthusiastic and felt that the NDP had made a positive, and permanent, difference to their teaching. Their talk about the NDP suggested that they saw it as "their practice" – it had been internalised. One teacher, "F", from City School, expressed the same enthusiasm but used language that revealed that, for her, the NDP approaches were not internalised. This can be seen in Table 4, which contrasts "F" with "A" from Country School. These two teachers had similar lengths of teaching experience and had undergone the facilitation one year ago.

Table 4
Summary of A and F's Responses to Interview Questions

Question	A	F
Previous practice	Maths Plus in three groups.	Maintenance, taught in groups, more bookwork.
Facilitation	Probably the biggest shift in my teaching since I started. Probably the whole insight of how children think – it was just amazing for me.	Never very comfortable teaching maths, worried about doing it the right way/ coverage, now have a sequence to follow.
Most useful	Real analysing of the way kids think and the stages, in-depth teaching every day with your group. Just so exciting.	Facilitator being practical, coming from a classroom base, not too much theory to sift through. Junior-based, so already had task board and equipment, see how it fits into a senior class more clearly now.
Most difficult	Planning – quite easy before – go from this page to this page, this activity, that activity, but now you are looking at what you are teaching the children. You have to get your head around the session before you start. I always used to plan my maths for a week, and now I only plan for tomorrow. I will have an idea about the next day, but I won't plan it because I have to see how they respond.	Just putting it all into practice. I got a bit overwhelmed with doing it their way after the 25 years I had done it my way, so that was sort of the most difficult part, I guess, just retraining myself. The classroom organisation was reasonably easy to take on, but you think, "oh my goodness, you are going to let children play games for 15 minutes".
How do you teach now?	Cross-grouped by strategy stage across three classes, two-group rotation, use teaching model and resources.	Pretty much how they taught me because for my own self I just think I should stick to the model and make a few variations within the classroom but, you know, stick to it so that I have got that on board, and then next year I will launch out and do the things that I really feel either philosophically or just management-wise would work better. I think all learners need revision and reminding and practice, and I don't think quick warm-up games is enough to actually maintain that knowledge. I also have a few issues with the assessment sheets … (alignment) is not as clear for me, and I fling these sheets in and out of the numeracy file all the time, and it's crazy.

Table 4
Summary of A and F's Responses to Interview Questions – continued

Question	A	F
What do you do that is not from the project?	Do a bit of formal geometry, to prepare them for secondary school.	This term I am only teaching one group. I am just trying to teach one group all the way through to number properties … the second group is really just checking up on what we have done today. I find it's too frustrating. I would rather for my peace of mind and the kids' learning take them all the way through … it's usually practice with number properties where they hit the wall. I have built in more revision and maintenance, and that's basically out of a book or worksheet … I am just going along with the assessment the way they like it.
Biggest difference to students	There is lot more focus teaching that goes on – group strategy teaching, like 20 minutes a day just focusing on strategy and being able to verbalise their thinking.	More engaging and more fun, if you are motivated you will learn it.

The contrast between these two teachers is apparent in the language they use and the substance of what they say. For A, the NDP has caused a revolution in practice.

> It was for me personally probably the biggest shift in my teaching since I started. It was huge – the whole insight into how children think. (A, Country School)

For F, the NDP approaches remained external to her core practice. She has concerns about maintenance and bookwork that have not been overcome, but she feels obliged to "stick to it" as "they" have told her to. The NDP has the feel of an imposition for this teacher. This is an important finding because it may be representative of a large number of teachers whose views may not be captured by research that asks for volunteers to contribute their ideas. The teachers who were videoed demonstrated practice that was consistent with what they had described in the interviews. In this small sample, it appears that teachers describe their practice accurately in an interview situation. The language they used to describe their practice was indicative of what was observed in the videoed lessons.

Evidence from Classroom Lessons

Three classroom lessons were videoed, two at Country School and one at City School. All three teachers used superficial features of NDP practice. Their lessons drew from the resource books, they used project games and equipment, the classes were grouped by strategy, and a group rotation was in operation. In each lesson, the teachers shared the learning intention with the groups they were teaching and used a modelling book to record the discussion. The lessons all followed the format suggested by the NDP, with an introductory whole-class phase, group teaching, and a wind-up session to close the lesson. Beyond this organisational framework, however, the teachers' interaction with their students was of key interest. To attempt to systematically consider the deeper lesson features, observation protocols from the MEI were used to derive a list of key teacher actions that could be observed on the videos.

The elicitation of students' strategies and the types of responses given by teachers to these strategies were analysed. All three teachers sought strategy explanations in their group teaching, and one teacher sought strategy explanations during the warm-up game and plenary session. The frequency of strategy elicitation depended on the lesson content. The teachers had their preferred responses to strategy contributions, with one teacher offering evaluative responses (yes, good, no) and revoicing contributions, one inviting students to record, and one questioning and recording herself. This suggests that the teachers had internalised the weaving of strategy contributions into discussion, resulting in responses that suited their teaching. In all the lessons, the teachers frequently called on students to make public contributions. However, the sharing of ideas zigzagged between the teacher and the students rather than generating dialogue between the students. Sharing methods between students so that they understood each other's strategies was not evident, and there was no discussion of the most effective strategy. There was only one instance of a teacher providing a strategy.

Public recording of strategies was done in modelling books in the Country School lessons. Most of the recording in the City School lesson was on individual whiteboards. The methods of public recording contributed to the sharing pattern in the groups, with more sharing occurring when there was group rather than individual recording (this issue is addressed more fully by Higgins in this compendium, p. 65).

The teachers in these three lessons were emphasising strategies with the students. They appeared to understand the students' contributions, as evidenced by their responses.

Discussion

The findings of this study are necessarily limited by the small number of participants. The narratives about practice told by these teachers are highly individual but at the same time help to "flesh out" the patterns found in interview and survey data (Thomas, Ward, & Tagg, 2004; Higgins, 2004). As the issue of sustainability of NDP practices becomes more central, we need to develop a clearer idea of what sustained practice might look like. Thomas, Ward, & Tagg (2004) list the five criteria that might be used to evaluate the sustainability projects. Further clarification of what the key elements of classroom practice are would help in evaluating the project's effectiveness. Using the MEI criteria to analyse the videoed lessons produced the suggestion that evidence for teachers noticing students' strategies and understanding them could be found in observing teachers' elicitation of and response to students' contributions. This is an area for further consideration.

The comparison of A and F confirms Higgin's (2004) notion that internalisation forms the basis for innovation and change to NDP approaches. A's enthusiasm for the NDP was revealed in his interview responses and suggested that the approaches had become an integral part of his practice. F, however, spoke about the NDP as a separate entity, using words such as "they" and "as I have been told to". F found the NDP approach much harder to use and sustain without support than A did. Further evidence for the idea that individual-teacher change culminates in sustained school-wide change comes from Country School, where a deliberate decision was taken to wait until practice was embedded before making policy-level changes. City School's contrasting approach may have contributed to F's feeling of disconnection from the NDP, as she refers to the assessment system in particular as not aligning with her practice and being frustrating for her.

The readily observable elements of the videoed lessons, such as the sharing of learning intentions and the use of groups based on strategy, need to be considered in combination with analysis of the interaction that occurs within the lesson. The lesson organisation creates a "space" for meaningful interaction to occur, but it is not sufficient in itself to count as "sustained practice" if we are looking for change in understanding of students' thinking in numeracy. Teachers may be able to sustain the system aspects of running an NDP-based numeracy programme, but they may not be able to sustain the depth of insight and interaction as the students progress. Further study of both teachers' narratives about their practice and observation of their teaching will enhance our understanding of what sustainability means for the NDP.

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Appendix A (Patterns of Performance and Progress)

Composition of the year 0–8 cohorts 2002–2005

		2002	2003	2004	2005
Number of students		*61 208*	*138 829*	*70 027*	*51 771*
Gender					
Boys		51.4%	50.0%	51.0%	51.7%
Girls		48.6%	49.0%	49.0%	48.3%
Ethnicity					
European		58.0%	57.8%	60.4%	63.3%
Māori		23.4%	23.6%	19.7%	19.5%
Pasifika		9.6%	9.7%	10.2%	7.3%
Asian		4.9%	4.7%	5.4%	5.4%
Other		4.2%	4.1%	4.3%	4.4%
Project					
ENP (yrs 0–3)		34.2%	35.2%	35.0%	29.9%
ANP (yrs 4–6)		55.6%	52.8%	42.6%	46.4%
INP (yrs 7–8)		10.2%	12.0%	22.4%	23.7%
Decile Ranking	1	14.1%	14.1%	10.8%	5.9%
	2	9.8%	11.3%	8.4%	4.8%
	3	12.8%	12.2%	9.7%	7.4%
	4	13.5%	10.5%	10.6%	11.7%
	5	12.3%	12.2%	9.4%	9.5%
	6	6.2%	7.3%	8.2%	10.6%
	7	6.9%	7.2%	10.7%	12.9%
	8	8.4%	7.9%	8.7%	10.9%
	9	7.2%	7.2%	9.1%	13.1%
	10	8.8%	10.0%	14.5%	13.2%
Decile Band					
Low (1–3)		36.7%	37.6%	28.9%	18.1%
Medium (4–7)		38.9%	37.2%	38.9%	44.7%
High (8–10)		24.4%	25.1%	32.3%	37.2%
Average Decile Ranking					
Low (1–3)		1.96	1.95	1.96	2.08
Medium (4–7)		5.16	5.30	5.48	5.55
High (8–10)		9.02	9.08	9.18	9.06

Appendix B (Patterns of Performance and Progress)

Percentages of students at each stage on the Number Framework as a function of year group 2005

Year		0–1	2	3	4	5	6	7	8
Number of students		*4737*	*5048*	*5719*	*6966*	*8353*	*8689*	*6348*	*5911*
Addition and Subtraction									
Initial									
0	Emergent	16.1	2.8	0.6	0.3	0.2	0.2	0.1	0.1
1	1:1 Counting	33.4	13.8	5.4	1.6	0.6	0.4	0.2	0.1
2	Count from one w. materials	40.9	37.7	14.8	4.8	2.4	1.0	0.6	0.3
3	Count from one w. imaging	7.5	23.7	15.0	7.0	3.3	1.6	1.1	0.7
4	Advanced Counting	1.9	18.7	45.1	47.0	38.3	29.7	25.7	21.1
5	Early Additive P–W	0.2	3.2	18.2	35.3	45.4	50.1	49.0	44.8
6	Advanced Additive P–W	0.0	0.1	0.9	4.0	9.3	15.4	20.1	27.5
7	Adv. Multiplicative P–W		0.0	0.0	0.1	0.5	1.5	3.1	5.4
	Total Stages 6–7	***0.0***	***0.1***	***0.9***	***4.1***	***9.8***	***16.9***	***23.2***	***32.9***
Final									
0	Emergent	2.2	0.4	0.2	0.1	0.1	0.2	0.1	0.1
1	1:1 Counting	10.9	2.8	0.8	0.3	0.2	0.1	0.1	0.1
2	Count from one w. materials	44.2	16.3	4.4	1.4	0.5	0.4	0.3	0.1
3	Count from one w. imaging	24.2	20.0	7.4	2.5	1.1	0.6	0.4	0.1
4	Advanced Counting	16.4	45.1	41.9	29.9	20.5	12.9	10.9	8.4
5	Early Additive P–W	2.0	14.8	40.0	48.7	51.0	45.1	41.4	32.4
6	Advanced Additive P–W	0.0	0.6	5.2	16.1	24.0	33.3	36.3	39.8
7	Adv. Multiplicative P–W	0.0	0.1	0.2	0.9	2.7	7.4	10.6	19.0
	Total Stages 6–7	***0.0***	***0.7***	***5.4***	***17.0***	***26.7***	***40.7***	***46.9***	***58.8***
Multiplication and Division									
Initial									
	Not entered	49.6	40.8	20.5	7.0	2.9	2.0	1.2	0.8
n/a	Not Applicable	49.7	44.2	23.4	9.8	5.1	2.5	1.1	0.8
2–3	Count from one	0.5	8.6	22.8	18.3	11.9	7.0	5.0	3.3
4	Advanced Counting	0.2	5.3	26.5	42.7	38.6	30.7	24.9	18.9
5	Early Additive P–W	0.0	0.9	5.4	15.8	25.1	29.3	30.0	27.3
6	Advanced Additive P–W	0.0	0.1	1.2	6.0	14.2	22.5	28.2	33.8
7	Adv. Multiplicative P–W			0.1	0.6	2.1	5.2	8.1	12.9
8	Adv. Proportional P–W				0.0	0.1	0.7	1.5	2.2
	Total Stages 7–8	***0.0***	***0.0***	***0.1***	***0.6***	***2.2***	***5.9***	***9.6***	***15.1***
Final									
	Not entered	47.0	28.2	10.7	3.9	1.7	1.1	0.7	1.0
n/a	Not applicable	43.7	23.6	6.9	3.0	1.2	0.8	0.6	0.3
2–3	Count from one	4.5	12.9	11.8	6.1	3.7	2.0	1.3	0.8
4	Advanced Counting	4.3	28.9	44.0	36.6	24.8	14.9	11.5	7.6
5	Early Additive P–W	0.4	5.5	18.6	28.5	30.1	25.0	22.9	18.8
6	Advanced Additive P–W	0.2	0.9	7.3	17.9	29.3	35.7	37.6	35.4
7	Adv. Multiplicative P–W		0.0	0.8	3.8	8.2	16.5	19.6	25.3
8	Adv. Proportional P–W		0.0	0.0	0.2	1.0	3.9	5.9	10.8
	Total Stages 7–8	***0.0***	***0.0***	***0.8***	***4.0***	***9.2***	***20.4***	***25.5***	***36.1***

Percentages of students at each stage on the Number Framework as a function of year group 2005

Year		**0–1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Number of students		*4737*	*5048*	*5719*	*6966*	*8353*	*8689*	*6348*	*5911*
Proportion and Ratio									
Initial									
	Not entered	49.7	40.6	20.9	7.1	3.3	2.4	1.5	1.6
n/a	Not applicable	49.5	44.2	23.4	9.8	5.1	2.6	1.1	0.8
1	Unequal sharing	0.3	6.4	16.9	15.4	10.8	7.4	4.8	2.8
2–4	Equal sharing	0.4	8.4	34.2	50.7	47.7	40.4	32.3	24.6
5	Early Additive P–W	0.0	0.4	4.1	13.1	23.2	27.5	30.3	28.7
6	Advanced Additive P–W		0.0	0.4	3.3	8.0	14.3	19.5	24.5
7	Adv. Multiplicative P–W			0.0	0.4	1.9	4.9	9.3	14.5
8	Adv. Proportional P–W			0.0	0.0	0.1	0.5	1.1	2.4
	Total Stages 7–8	***0.0***	***0.0***	***0.0***	***0.4***	***2.0***	***5.4***	***10.4***	***16.9***
Final									
	Not entered	46.8	27.9	11.5	4.3	1.9	1.4	1.1	1.5
n/a	Not applicable	44.0	23.8	6.8	3.0	1.2	0.8	0.6	0.3
1	Unequal sharing	2.8	5.6	5.9	3.3	2.2	1.2	0.8	0.5
2–4	Equal sharing	6.1	38.5	52.9	44.9	31.2	21.6	15.5	11.5
5	Early Additive P–W	0.4	3.9	18.4	30.0	34.6	30.4	29.0	23.1
6	Advanced Additive P–W	0.0	0.4	3.9	11.5	20.2	26.5	27.8	27.5
7	Adv. Multiplicative P–W		0.1	0.6	3.0	8.1	15.6	21.2	27.2
8	Adv. Proportional P–W		0.0		0.1	0.6	2.4	3.9	8.4
	Total Stages 7–8	***0.0***	***0.1***	***0.6***	***3.1***	***8.7***	***18.0***	***25.1***	***35.6***
Fractions									
Initial									
	Not entered	49.6	40.5	19.3	6.2	2.5	1.4	1.4	0.6
n/a	Not applicable	49.7	43.6	23.5	9.7	4.0	2.1	1.0	0.8
2–3	Unit fractions not recognised	0.7	14.6	46.1	47.4	34.2	24.4	13.5	9.0
4	Unit fractions recognised	0.0	1.0	8.3	21.9	29.9	30.2	26.0	23.0
5	Ordered unit fractions	0.0	0.2	2.7	13.9	25.2	32.9	41.2	39.0
6	Co-ord. num'rs & denom'rs			0.2	0.7	3.3	5.8	10.9	15.0
7	Equivalent fractions		0.0		0.1	0.7	2.2	4.1	8.2
8	Ordered fractions				0.1	0.2	1.0	1.9	4.3
Final									
	Not entered	46.3	27.4	9.6	3.0	1.4	0.6	0.6	0.4
n/a	Not applicable	44.1	24.4	7.0	3.0	1.3	0.8	0.6	0.2
2–3	Unit fractions not recognised	6.8	22.4	20.7	13.6	9.3	5.1	3.2	2.4
4	Unit fractions recognised	2.2	17.5	30.2	27.6	21.7	16.2	14.3	10.6
5	Ordered unit fractions	0.6	8.2	30.5	44.0	46.7	45.0	42.0	33.8
6	Co-ord. num'rs & denom'rs	0.0	0.1	1.7	7.4	14.3	19.1	20.1	22.0
7	Equivalent fractions		0.0	0.2	1.1	3.6	8.3	11.8	16.8
8	Ordered fractions		0.0	0.0	0.4	1.8	4.8	7.4	13.8

Percentages of students at each stage on the Number Framework as a function of year group 2005

	Year	**0–1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	Number of students	*4737*	*5048*	*5719*	*6966*	*8353*	*8689*	*6348*	*5911*
Place Value									
Initial									
	Not entered	2.0	0.7	0.3	0.5	0.8	0.3	0.2	0.3
0–1	Emergent	53.8	21.2	6.9	2.8	1.1	0.6	0.3	0.2
2–3	One as a unit	42.6	63.1	50.3	32.4	19.7	13.9	8.3	5.7
4	Ten as a counting unit	1.6	14.8	40.1	56.1	59.5	54.8	43.6	30.1
5	Tens in nos. to 1000		0.2	2.2	7.0	14.7	21.6	32.3	36.4
6	Ts, Hs, Ths in whole nos.		0.0	0.3	1.1	3.7	6.8	10.6	16.9
7	10ths in decimals/orders decs		0.0		0.1	0.5	1.7	3.3	7.1
8	Decimal conversions				0.0	0.0	0.4	1.4	3.3
Final									
	Not entered	0.8	0.2	0.1	0.0	0.0	0.0	0.0	
0–1	Emergent	17.0	5.3	2.2	0.6	0.4	0.4	0.2	0.2
2–3	One as a unit	63.3	45.8	25.6	14.6	7.6	4.6	2.7	1.8
4	Ten as a counting unit	18.6	46.4	60.2	59.7	48.3	34.5	22.8	14.5
5	Tens in nos. to 1000	0.2	2.1	10.5	20.1	29.3	33.5	37.3	31.9
6	Ts, Hs, Ths in whole nos.	0.0	0.1	1.3	4.2	11.0	17.0	21.6	24.7
7	10ths in decimals/orders decs			0.1	0.6	2.9	7.4	10.4	15.8
8	Decimal conversions		0.0	0.0	0.2	0.5	2.7	5.0	11.1
Basic Facts									
Initial									
	Not entered	2.5	0.2	0.3	0.2	0.4	0.1	0.2	0.2
0–1	Non-grouping with 5	92.0	74.6	38.5	18.9	8.8	5.2	2.8	1.4
2–3	Within & with 5, within 10	4.9	18.6	28.3	23.5	14.6	9.6	5.8	4.2
4	Add'n with 10s & doubles	0.6	6.2	27.6	38.0	34.3	23.7	17.0	12.9
5	Addition facts	0.0	0.4	4.4	15.4	27.6	30.8	30.3	25.9
6	Subtr'n & mult'n facts		0.1	0.7	3.5	12.2	24.4	34.0	37.9
7	Division facts			0.1	0.4	1.9	5.5	8.7	14.7
8	Common factors & multiples	0.0			0.1	0.2	0.7	1.4	2.8
Final									
	Not entered	1.4	0.1	0.1	0.0		0.0		
0–1	Non-grouping with 5	58.2	28.5	9.8	4.6	2.0	1.3	0.9	0.4
2–3	Within & with 5, within 10	29.6	32.8	18.2	9.4	5.5	2.9	1.9	1.3
4	Add'n with 10s and doubles	10.2	32.8	43.2	34.5	20.7	11.4	8.3	5.2
5	Addition facts	0.5	5.0	23.1	34.6	36.4	28.1	22.8	16.6
6	Subtr'n & mult'n facts	0.0	0.7	4.9	13.5	25.5	33.3	38.0	35.7
7	Division facts		0.0	0.6	3.0	8.6	18.7	22.7	29.2
8	Common factors & multiples	0.0	0.0	0.1	0.5	1.3	4.4	5.4	11.6

Appendix C (Patterns of Performance and Progress)

Percentages of year 0–8 students at each framework stage as a function of gender, ethnicity, and school decile band in 2005

	Gender		Ethnicity				Decile Band		
	Boys	Girls	European	Māori	Pasifika	Asian	Low	Middle	High
Number of students	*26760*	*25011*	*32787*	*10090*	*3785*	*2809*	*9199*	*22774*	*18926*
				Addition and Subtraction					
Initial									
0: EM	2.2	1.6	1.5	3.1	2.7	1.4	3.2	2.1	1.2
1: OT	5.3	5.5	5.0	6.7	6.3	4.3	6.1	5.8	4.9
2: CM	10.5	10.2	10.1	11.3	10.3	8.6	11.0	10.7	10.1
2: CM	10.1	10.1	11.6	11.0	11.1	9.5	8.4	8.8	
3: CI	6.4	6.8	6.2	7.5	8.2	5.7	7.9	6.6	6.2
4: AC	27.0	33.2	28.2	34.1	38.6	25.6	37.4	29.7	27.0
5: EA	33.8	34.1	35.5	29.7	28.9	38.6	27.8	33.2	37.3
6: AA	12.8	7.7	12.0	6.8	4.4	13.2	6.0	10.2	12.0
7: AM	1.9	0.7	1.5	0.8	0.6	2.6	0.5	1.7	1.3
Stages 6–7	***14.7***	***8.4***	***13.5***	***7.6***	***5.0***	***15.8***	***6.5***	***11.9***	***13.3***
Final									
0: EM	0.4	0.3	0.3	0.6	0.4	0.2	0.8	0.3	0.1
1: OT	1.5	1.4	1.2	2.4	1.7	0.7	1.9	1.6	1.1
2: CM	6.6	6.3	6.0	7.9	7.8	5.2	7.5	7.1	5.5
3: CI	5.5	5.8	5.4	6.4	6.1	4.5	5.6	6.2	5.3
4: AC	20.2	24.6	20.4	26.4	30.3	18.2	29.5	21.9	19.8
5: EA	35.3	39.2	37.3	36.5	39.1	35.2	37.8	36.7	37.2
6: AA	23.8	18.6	23.4	16.7	12.3	26.2	13.9	20.9	24.8
7: AM	6.7	3.8	5.9	3.2	2.3	9.8	3.1	5.3	6.1
Stages 6–7	***30.5***	***22.4***	***29.3***	***19.9***	***14.6***	***36.0***	***17.0***	***26.2***	***30.9***
				Multiplication and Division					
Initial									
Not given	16.6	16.5	15.8	19.9	16.1	13.4	18.6	16.4	15.8
2–3	10.8	12.3	10.4	13.1	17.0	11.3	15.6	11.0	10.2
4: AC	27.8	31.8	28.3	32.9	37.6	25.1	35.6	29.6	27.1
5: EA	20.3	22.1	22.0	19.6	17.8	22.5	18.0	21.3	22.5
6: AA	17.8	14.4	17.9	12.0	9.7	20.1	10.1	16.6	18.5
7: AM	5.6	2.7	4.9	2.4	1.7	6.3	1.9	4.5	5.0
8: AP	1.1	0.2	0.8	0.2	0.1	1.3	0.2	0.7	0.8
Stages 7–8	***6.7***	***2.9***	***5.7***	***2.6***	***1.8***	***7.6***	***2.1***	***5.2***	***5.8***
Final									
Not given	9.0	8.7	8.3	11.5	8.4	5.9	9.5	9.3	8.1
2–3	5.1	6.0	4.7	7.1	8.0	6.3	7.8	5.4	4.7
4: AC	22.5	26.0	22.5	27.9	32.7	19.8	31.6	23.4	21.7
5: EA	21.1	24.0	22.5	22.9	24.5	19.5	23.2	22.5	22.2
6: AA	25.2	24.4	26.0	21.7	20.6	27.4	20.2	25.4	26.3
7: AM	12.7	9.1	12.4	7.2	5.2	15.8	6.3	10.8	13.4
8: AP	4.2	1.8	3.5	1.6	0.7	5.3	1.4	3.2	3.7
Stages 7–8	***16.9***	***10.9***	***15.9***	***8.8***	***5.9***	***21.1***	***7.7***	***14.0***	***17.1***

Percentages of year 0–8 students at each framework stage as a function of gender, ethnicity, and school decile band in 2005

	Gender		Ethnicity				Decile Band		
	Boys	Girls	European	Māori	Pasifika	Asian	Low	Middle	High
Number of students	*26760*	*25011*	*32787*	*10090*	*3785*	*2809*	*9199*	*22774*	*18926*
				Proportion and Ratio					
Initial									
Not given	16.7	16.5	15.9	19.8	16.0	13.2	18.6	16.5	15.9
1	10.2	9.5	8.3	11.8	15.8	11.1	13.5	9.8	8.2
2–4	35.4	40.3	36.7	40.1	45.7	34.7	44.0	37.2	35.6
5: EA	19.5	20.6	20.9	18.8	16.1	20.4	16.4	20.4	21.3
6: AA	11.6	9.6	12.0	7.1	4.8	13.6	5.6	11.0	12.6
7: AM	5.7	3.2	5.4	2.2	1.4	6.2	1.8	4.5	5.7
8: AP	0.8	0.3	0.7	0.2	0.3	0.8	0.1	0.7	0.6
Stages 7–8	***6.5***	***3.5***	***6.1***	***2.4***	***1.7***	***7.0***	***1.9***	***5.2***	***6.3***
Final									
Not given	9.1	8.7	8.4	11.5	8.2	5.9	9.7	9.3	8.2
1	3.1	2.8	2.3	4.3	4.1	2.9	4.8	2.5	2.5
2–4	30.0	32.5	29.2	35.4	40.9	27.7	38.2	31.2	28.0
5: EA	24.1	27.4	25.3	27.1	27.8	22.8	27.1	25.7	25.0
6: AA	18.0	17.8	19.1	14.5	13.8	22.0	13.8	18.1	19.6
7: AM	12.9	9.3	13.1	6.4	4.7	14.8	5.6	10.9	14.1
8: AP	2.9	1.3	2.6	0.8	0.5	3.8	0.8	2.2	2.6
Stages 7–8	***15.8***	***10.6***	***15.7***	***7.2***	***5.2***	***18.6***	***6.4***	***13.1***	***16.7***

Appendix D (Patterns of Performance and Progress)

Percentages of year 0–8 students at each framework stage as a function of ethnicity and gender in 2005

2005	European		Māori		Pasifika		Asian	
	Boys	Girls	Boys	Girls	Boys	Girls	Boys	Girls
Number of students	*16868*	*15919*	*5279*	*4811*	*1927*	*1858*	*1462*	*1347*
Addition and Subtraction								
Initial								
0: EM	1.8	1.3	3.5	2.7	3.2	2.2	1.4	1.4
1: OT	4.9	5.1	6.7	6.8	6.6	5.9	4.2	4.4
2: CM	10.1	10.1	11.6	11.0	11.1	9.5	8.4	8.8
3: CI	6.0	6.3	7.6	7.4	7.2	9.4	5.3	6.2
4: AC	24.9	31.7	31.4	37.1	36.8	40.4	23.6	27.8
5: EA	35.1	36.0	30.5	28.7	28.3	29.5	37.6	39.6
6: AA	15.0	8.7	7.7	5.8	5.9	3.0	15.9	10.4
7: AM	2.2	0.8	1.0	0.5	0.9	0.2	3.7	1.4
Stages 6–7	***17.2***	***9.5***	***8.7***	***6.3***	***6.8***	***3.2***	***19.6***	***11.8***
Final								
0: EM	0.3	0.2	0.7	0.5	0.6	0.3	0.2	0.2
1: OT	1.2	1.2	2.6	2.2	2.0	1.5	1.0	0.4
2: CM	6.2	5.9	8.2	7.5	8.3	7.3	4.9	5.6
3: CI	5.3	5.6	6.5	6.2	5.8	6.4	4.4	4.6
4: AC	17.9	22.9	24.6	28.3	28.4	32.2	16.8	19.7
5: EA	35.2	39.6	35.4	37.7	37.4	40.9	32.8	37.7
6: AA	26.3	20.4	18.3	14.9	14.5	10.1	27.8	24.6
7: AM	7.6	4.2	3.7	2.7	3.1	1.5	12.1	7.2
Stages 6–7	***33.9***	***24.6***	***22.0***	***17.6***	***17.6***	***11.6***	***39.9***	***31.8***
Multiplication and Division								
Initial								
Not given	15.8	15.8	20.5	19.2	16.1	16.0	13.3	13.4
2–3	9.6	11.2	12.3	13.9	17.0	17.1	11.0	11.6
4: AC	26.3	30.4	31.3	34.6	34.4	40.8	22.8	27.7
5: EA	20.9	23.2	19.6	19.6	18.1	17.5	20.8	24.4
6: AA	19.6	16.0	12.9	11.0	11.4	7.9	21.8	18.2
7: AM	6.6	3.1	3.0	1.7	2.7	0.7	8.2	4.2
8: AP	1.2	0.3	0.3	0.0	0.2	0.0	2.1	0.4
Stages 7–8	***7.8***	***3.4***	***3.3***	***1.7***	***2.9***	***0.7***	***10.3***	***4.6***
Final								
Not given	8.4	8.3	11.9	11.0	8.7	8.0	6.1	5.7
2–3	4.1	5.2	6.6	7.7	8.1	7.9	6.5	6.0
4: AC	20.7	24.4	26.6	29.3	30.7	34.8	17.9	22.0
5: EA	21.0	24.1	21.6	24.3	24.3	24.7	18.3	20.8
6: AA	26.3	25.7	23.0	20.4	20.7	20.5	26.5	28.5
7: AM	14.5	10.2	8.3	6.1	6.3	4.0	17.8	13.6
8: AP	4.9	2.0	2.0	1.2	1.2	0.2	6.9	3.4
Stages 7–8	***19.4***	***12.2***	***10.3***	***7.3***	***7.5***	***4.2***	***24.7***	***17.0***

Percentages of year 0–8 students at each framework stage as a function of ethnicity and gender in 2005

2005	**European**		**Māori**		**Pasifika**		**Asian**	
	Boys	**Girls**	**Boys**	**Girls**	**Boys**	**Girls**	**Boys**	**Girls**
Number of students	*16868*	*15919*	*5279*	*4811*	*1927*	*1858*	*1462*	*1347*
				Proportion and Ratio				
Initial								
Not given	15.9	15.9	20.6	19.1	15.9	16.1	13.3	13.0
1	8.6	8.0	11.9	11.7	16.3	15.3	11.9	10.2
2–4	34.1	39.5	38.5	41.8	44.4	47.0	30.8	39.0
5: EA	20.3	21.6	18.6	19.1	15.7	16.5	20.4	20.6
6: AA	13.1	10.9	7.8	6.3	5.4	4.2	14.3	12.8
7: AM	7.0	3.8	2.5	1.9	1.9	0.8	8.1	4.1
8: AP	1.0	0.3	0.3	0.1	0.4	0.1	1.3	0.3
Stages 7–8	***8.0***	***4.1***	***2.8***	***2.0***	***2.3***	***0.9***	***9.4***	***4.4***
Final								
Not given	8.5	8.4	12.0	11.1	8.8	7.6	6.2	5.7
1	2.5	2.2	4.3	4.3	4.1	4.0	3.3	2.4
2–4	27.9	30.6	35.1	35.8	40.0	41.9	25.4	30.3
5: EA	23.4	27.3	25.6	28.7	26.6	29.0	22.1	23.6
6: AA	19.2	19.0	14.7	14.4	13.8	13.7	21.6	22.5
7: AM	15.1	10.9	7.5	5.2	5.7	3.6	16.1	13.5
8: AP	3.5	1.6	0.9	0.6	0.9	0.1	5.4	2.0
Stages 7–8	***18.6***	***12.5***	***8.4***	***5.8***	***6.6***	***3.7***	***21.5***	***15.5***

Appendix E (Patterns of Performance and Progress)

Table E1

Percentage of students who progressed to a higher stage on addition and subtraction for each initial stage 2002–2005

Addition and Subtraction	2002			2003		
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stage 0 Emergent	*1080*	*623*	*346*	*2416*	*1629*	*874*
Up 1	25.6	27.3	28.6	21.9	26.0	22.8
Up 2	40.6	40.1	40.2	39.2	39.5	44.1
Up 3	6.8	7.1	9.8	5.8	6.7	10.1
Up 4	7.8	5.0	4.6	8.2	7.3	8.1
Up 5+	0.6	0.2		9.6	5.2	3.1
Total	***81.4***	***79.7***	***83.2***	***84.7***	***84.7***	***88.2***
Stage 1 One-to-one Counting	*1803*	*791*	*411*	*4933*	*2278*	*1153*
Up 1	52.3	50.1	55.0	50.6	50.8	53.3
Up 2	17.2	15.4	15.3	17.7	18.1	17.3
Up 3	13.9	12.6	12.9	16.5	13.9	12.7
Up 4+	0.6	0.5	0.7	1.2	1.1	1.2
Total	***84.0***	***78.6***	***83.9***	***86.0***	***83.9***	***84.5***
Stage 2 Counting All w. Materials	*3779*	*1581*	*700*	*12749*	*5618*	*2728*
Up 1	24.4	27.1	27.0	25.0	24.8	27.4
Up 2	38.0	30.4	29.0	37.6	34.3	32.2
Up 3+	4.1	1.9	1.9	5.7	3.9	3.5
Total	***66.5***	***59.4***	***57.9***	***68.3***	***63.0***	***63.1***
Stage 3 Counting All w. Imaging	*2714*	*1539*	*844*	*4474*	*1946*	*1030*
Up 1	48.7	43.5	44.4	59.8	57.8	55.7
Up 2	14.6	13.9	8.1	16.0	13.5	11.2
Up 3	2.7	0.8	0.9	0.6	0.3	0.2
Total	***66.0***	***58.2***	***53.4***	***76.4***	***71.6***	***67.1***
Stage 4 Advanced Counting	*12743*	*6156*	*2668*	*24685*	*11679*	*4876*
Up 1	52.7	48.6	41.1	51.5	46.5	39.1
Up 2	4.7	3.7	2.5	4.6	4.2	3.2
Total	***57.4***	***52.3***	***43.6***	***56.1***	***50.7***	***42.3***
Stage 5 Early Additive P–W	*11779*	*4081*	*1123*	*7583*	*2221*	*565*
Up 1	34.7	27.8	21.3	33.7	28.7	24.1

Table E1 – *continued*

Percentage of students who progressed to a higher stage on addition and subtraction for each initial stage 2002–2005

Addition and Subtraction		**2004**			**2005**	
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stage 0 Emergent	*1119*	*495*	*292*	*528*	*323*	*106*
Up 1	18.3	23.4	25.0	25.9	32.5	29.2
Up 2	31.0	39.4	42.1	40.2	37.5	49.1
Up 3	7.2	8.9	8.9	10.4	5.6	0.9
Up 4	13.6	6.1	7.9	5.3	4.3	3.8
Up 5+	21.2	11.7	5.8	1.2		
Total	***91.3***	***89.5***	***89.7***	***83.0***	***79.9***	***83.0***
Stage 1 One-to-one Counting	*2544*	*885*	*478*	*1639*	*685*	*240*
Up 1	44.2	47.1	42.5	45.4	43.5	40.8
Up 2	22.8	18.5	22.0	21.7	20.7	20.0
Up 3	16.9	15.8	18.6	16.8	15.0	22.1
Up 4+	1.3	1.1	1.3	2.7	2.5	2.9
Total	***85.2***	***82.5***	***84.4***	***86.6***	***81.7***	***85.8***
Stage 2 Counting All w. Materials	*5478*	*1794*	*963*	*3328*	*1152*	*398*
Up 1	29.0	25.4	30.7	27.7	26.8	27.9
Up 2	35.5	37.9	33.0	35.9	36.5	34.2
Up 3+	5.1	3.6	4.2	6.7	5.1	4.3
Total	***69.6***	***66.9***	***67.9***	***70.3***	***68.4***	***66.4***
Stage 3 Counting All w. Imaging	*2728*	*920*	*567*	*2044*	*771*	*315*
Up 1	60.9	61.7	63.8	57.4	57.7	61.3
Up 2	15.7	12.3	11.8	19.7	19.3	12.4
Up 3	0.6	0.2		1.0	1.4	0.6
Total	***77.2***	***74.2***	***75.6***	***78.1***	***78.4***	***74.3***
Stage 4 Advanced Counting	*12397*	*4813*	*2806*	*9558*	*3630*	*1521*
Up 1	49.3	44.3	40.5	50.7	46.3	45.4
Up 2	5.0	4.6	3.3	8.1	7.0	5.9
Total	***54.3***	***48.9***	***43.8***	***58.8***	***53.3***	***51.3***
Stage 5 Early Additive P–W	*13198*	*3815*	*1682*	*12689*	*3376*	*1187*
Up 1	34.8	29.4	27.3	41.1	36.2	29.7

Table E2

Percentage of students who progressed to a higher stage on multiplication and division for each initial stage 2002–2005

Multiplication and Division		**2002**			**2003**	
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stages 2–3 Counting All	*3271*	*1684*	*785*	*7258*	*3827*	*1588*
Up 1	55.4	52.5	57.8	56.0	54.6	58.9
Up 2	17.9	17.2	12.0	19.0	16.1	12.7
Up 3	5.5	4.9	3.2	5.9	5.0	3.8
Up 4	0.4	0.2		0.4	0.2	0.1
Total	***79.2***	***74.8***	***73.0***	***81.3***	***75.9***	***75.5***
Stage 4 Skip Counting	*10111*	*4640*	*1818*	*18643*	*8513*	*3203*
Up 1	41.7	39.8	36.7	40.6	38.6	35.8
Up 2	16.4	12.0	10.0	18.1	14.0	12.3
Up 3	1.8	1.3	0.5	1.7	1.3	0.8
Total	***59.9***	***53.1***	***47.2***	***60.4***	***53.9***	***48.9***
Stage 5 Repeated Addition	*7302*	*2617*	*766*	*13164*	*4847*	*1406*
Up 1	47.2	43.8	41.5	48.7	44.3	42.0
Up 2	11.3	7.3	5.7	10.3	7.7	7.0
Total	***58.5***	***51.1***	***47.2***	***59.0***	***52.0***	***49.0***
Stage 6 Early Multiplicative P–W	*5045*	*1189*	*269*	*9843*	*2504*	*618*
Up 1	40.5	36.3	40.5	39.5	34.0	34.0

Multiplication and Division		**2004**			**2005**	
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stages 2–3 Counting All	*3780*	*1559*	*1008*	*2936*	*1108*	*542*
Up 1	56.8	57.3	56.0	56.4	55.7	58.7
Up 2	17.9	14.2	15.3	19.9	17.1	14.4
Up 3	6.8	5.0	4.7	6.2	5.5	5.2
Up 4	0.3	0.2	0.1	0.4	0.5	
Total	***81.8***	***76.7***	***76.1***	***82.9***	***78.8***	***78.3***
Stage 4 Skip Counting	*9908*	*3798*	*1965*	*8042*	*2852*	*1201*
Up 1	39.2	39.2	38.0	40.1	36.3	36.2
Up 2	19.1	14.4	11.7	19.9	16.0	16.6
Up 3	1.5	1.2	1.0	2.4	2.1	1.1
Total	***59.8***	***54.8***	***50.7***	***62.4***	***54.4***	***53.9***
Stage 5 Repeated Addition	*7669*	*2321*	*1034*	*6312*	*1738*	*570*
Up 1	50.4	47.0	43.0	48.7	46.8	45.3
Up 2	10.1	8.9	5.3	11.7	8.1	6.5
Total	***60.5***	***55.9***	***48.3***	***60.4***	***54.9***	***51.8***
Stage 6 Early Multiplicative P–W	*5941*	*1411*	*527*	*5210*	*1073*	*311*
Up 1	40.7	34.2	27.3	36.7	29.4	29.6

Table E3

Percentage of students who progressed to a higher stage on proportion and ratio on each initial stage 2002–2005

Proportion and Ratio	**2002**			**2003**		
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stage 1 Unequal Sharing	*5194*	*2797*	*1137*	*8723*	*4528*	*1871*
Up 1	54.8	55.4	59.7	57.6	58.0	62.4
Up 2	20.2	16.9	13.5	20.0	16.8	14.8
Up 3	7.7	5.5	2.9	6.9	5.2	2.9
Up 4+	1.4	0.8	0.6	1.1	0.7	0.5
Total	***84.1***	***78.6***	***76.7***	***85.6***	***80.7***	***80.6***
Stages 2–4 Equal Sharing	*10883*	*4561*	*1685*	*22068*	*9287*	*3347*
Up 1	35.4	35.1	34.7	35.1	33.4	32.1
Up 2	14.7	11.7	8.0	14.3	11.0	9.3
Up 3	3.8	1.8	1.7	3.1	1.6	1.8
Up 4	0.2	0.0	0.1	0.1	0.1	0.1
Total	***54.1***	***48.6***	***44.5***	***52.6***	***46.1***	***43.3***
Stage 5 Early Additive P–W	*5930*	*1925*	*577*	*11120*	*3718*	*1060*
Up 1	37.4	35.2	31.5	38.1	34.8	30.9
Up 2	16.7	12.1	12.0	14.7	10.2	10.5
Up 3	1.4	0.8	1.4	1.1	0.7	0.8
Total	***55.5***	***48.1***	***44.9***	***53.9***	***45.7***	***42.2***
Stage 6 Advanced Additive P–W	*3298*	*729*	*213*	*6315*	*1408*	*397*
Up 1	39.4	34.3	30.5	39.4	32.8	30.2
Up 2	6.3	3.6	4.7	5.6	4.4	4.0
Total	***45.7***	***37.9***	***35.2***	***45.0***	***37.2***	***34.2***
Stage 7 Adv. Multiplicative P–W	*1867*	*288*	*69*	*2953*	*486*	*115*
Up 1	31.1	28.1	17.4	30.9	25.3	20.9

Table E3 – *continued*

Percentage of students who progressed to a higher stage on proportion and ratio on each initial stage 2002–2005

Proportion and Ratio	**2004**			**2005**		
Initial Stage	**European**	**Māori**	**Pasifika**	**European**	**Māori**	**Pasifika**
Stage 1 Unequal Sharing	*3302*	*1358*	*797*	*2321*	*989*	*495*
Up 1	61.6	62.2	69.5	60.1	63.2	62.4
Up 2	21.6	18.3	16.4	22.4	18.4	19.2
Up 3	5.7	3.7	3.0	6.8	3.3	4.8
Up 4+	0.7	0.1	0.3	0.8	0.4	0.2
Total	***89.6***	***84.3***	***89.2***	***90.1***	***85.3***	***86.6***
Stages 2–4 Equal Sharing	*12900*	*4687*	*2597*	*10354*	*3433*	*1454*
Up 1	36.9	35.4	34.8	36.8	37.1	35.4
Up 2	13.1	9.9	9.4	14.4	10.2	11.5
Up 3	2.8	1.2	0.8	3.8	1.8	1.6
Up 4	0.1	0.1		0.1	0.0	
Total	***52.9***	***46.6***	***45.0***	***55.1***	***49.1***	***48.5***
Stage 5 Early Additive P–W	*6935*	*2174*	*861*	*6018*	*1679*	*513*
Up 1	37.7	33.2	31.5	36.5	32.2	36.1
Up 2	13.3	8.5	6.4	14.5	9.5	7.6
Up 3	0.9	0.7	0.6	0.7	0.3	
Total	***51.9***	***42.4***	***38.5***	***51.7***	***42.0***	***43.7***
Stage 6 Advanced Additive P–W	*3819.0*	*833.0*	*271.0*	*3492*	*623*	*155*
Up 1	41.6	34.2	30.3	42.2	31.8	41.9
Up 2	5.3	4.4	2.6	4.8	2.1	1.9
Total	***46.9***	***38.6***	***32.9***	***47.0***	***33.9***	***43.8***
Stage 7 Adv. Multiplicative P–W	*1729*	*233*	*52*	*1601*	*189*	*44*
Up 1	27.6	21.5	11.5	24.7	19.6	20.5

Appendix F (Patterns of Performance and Progress)

Means & standard deviations (in brackets) for final framework stages on addition and subtraction as a function of ethnicity and initial stage 2002–2005

	2002		2003		2004		2005	
Initial stage	**Final stage European**		**Final stage European**		**Final stage European**		**Final stage European**	
0	1.61	(1.13)	2.03	(1.56)	2.68	(1.74)	1.67	(1.08)
1	2.30	(0.93)	2.40	(0.97)	2.46	(0.99)	2.50	(1.02)
2	3.11	(0.96)	3.16	(0.97)	3.14	(0.94)	3.19	(0.97)
3	3.84	(0.79)	3.90	(0.73)	3.91	(0.71)	3.96	(0.75)
4	4.61	(0.60)	4.59	(0.63)	4.58	(0.64)	4.65	(0.66)
5	5.32	(0.54)	5.30	(0.58)	5.33	(0.54)	5.40	(0.59)
Initial stage	**Final stage Māori**		**Final stage Māori**		**Final stage Māori**		**Final stage Māori**	
0	1.50	(1.06)	1.81	(1.33)	2.16	(1.51)	1.44	(0.99)
1	2.19	(0.98)	2.31	(0.97)	2.36	(0.99)	2.41	(1.05)
2	2.92	(0.92)	3.03	(0.97)	3.11	(0.95)	3.13	(0.95)
3	3.72	(0.76)	3.80	(0.79)	3.85	(0.67)	3.97	(0.76)
4	4.54	(0.62)	4.51	(0.72)	4.52	(0.65)	4.58	(0.65)
5	5.23	(0.54)	5.21	(0.72)	5.25	(0.60)	5.34	(0.60)
Initial stage	**Final stage Pasifika**		**Final stage Pasifika**		**Final stage Pasifika**		**Final stage Pasifika**	
0	1.57	(1.03)	1.90	(1.21)	1.98	(1.29)	1.47	(0.91)
1	2.27	(0.91)	2.29	(0.95)	2.47	(1.02)	2.59	(1.07)
2	2.88	(0.92)	2.99	(0.98)	3.07	(0.95)	3.10	(0.92)
3	3.61	(0.72)	3.71	(0.81)	3.82	(0.73)	3.83	(0.71)
4	4.44	(0.60)	4.32	(0.96)	4.46	(0.62)	4.54	(0.61)
5	5.16	(0.50)	4.97	(1.20)	5.23	(0.54)	5.26	(0.58)
Initial stage	**Final stage High Decile**		**Final stage High Decile**		**Final stage High Decile**		**Final stage High Decile**	
0	1.69	(1.12)	2.29	(1.70)	2.03	(1.27)	1.73	(1.00)
1	2.40	(0.94)	2.51	(0.99)	2.55	(0.97)	2.59	(1.04)
2	3.21	(0.93)	3.21	(0.97)	3.23	(0.93)	3.23	(0.95)
3	4.00	(0.84)	3.96	(0.71)	3.94	(0.67)	4.02	(0.74)
4	4.65	(0.59)	4.62	(0.62)	4.61	(0.61)	4.67	(0.69)
5	5.34	(0.53)	5.31	(0.55)	5.34	(0.51)	5.42	(0.60)
Initial stage	**Final stage Low Decile**		**Final stage Low Decile**		**Final stage Low Decile**		**Final stage Low Decile**	
0	1.57	(1.06)	1.90	(1.33)	2.77	(1.81)	1.41	(1.03)
1	2.24	(0.96)	2.35	(0.99)	2.45	(1.01)	2.54	(1.08)
2	2.92	(0.94)	3.03	(0.97)	3.13	(0.93)	3.20	(0.99)
3	3.68	(0.72)	3.76	(0.81)	3.85	(0.68)	3.96	(0.76)
4	4.51	(0.62)	4.45	(0.81)	4.49	(0.65)	4.56	(0.63)
5	5.24	(0.54)	5.16	(0.90)	5.26	(0.58)	5.31	(0.61)

Appendix G (Patterns of Performance and Progress)

Mean differences, pooled variance, and effect sizes for differences between subgroups in progress on addition and subtraction as function of initial stage 2002–2005

	European–Māori			European–Pasifika			High–Low Decile		
Initial Stage	**Difference bet. Gps**	**Pooled SD**	**Effect Size**	**Difference bet. Gps**	**Pooled SD**	**Effect Size**	**Difference bet. Gps**	**Pooled SD**	**Effect Size**
2002									
0	0.120	1.110	0.11	0.040	1.109	0.04	0.120	1.080	0.11
1	0.110	0.948	0.12	0.030	0.929	0.03	0.160	0.952	0.17
2	0.190	0.954	0.20	0.230	0.961	0.24	0.290	0.943	0.31
3	0.120	0.783	0.15	0.230	0.784	0.29	0.320	0.771	0.42
4	0.070	0.610	0.11	0.170	0.605	0.28	0.140	0.616	0.23
5	0.090	0.537	0.17	0.160	0.535	0.30	0.100	0.539	0.19
Median			***0.14***			***0.26***			***0.21***
2003									
0	0.220	1.473	0.15	0.130	1.476	0.09	0.400	1.463	0.27
1	0.080	0.972	0.08	0.100	0.969	0.10	0.170	0.992	0.17
2	0.130	0.973	0.13	0.170	0.976	0.17	0.180	0.978	0.18
3	0.100	0.748	0.13	0.190	0.748	0.25	0.200	0.773	0.26
4	0.080	0.663	0.12	0.270	0.705	0.38	0.170	0.747	0.23
5	0.090	0.622	0.14	0.330	0.674	0.49	0.160	0.755	0.21
Median			***0.13***			***0.21***			***0.22***
2004									
0	0.530	1.688	0.31	0.710	1.678	0.42	–0.740	1.671	–0.44
1	0.100	0.988	0.10	–0.010	0.991	–0.01	0.110	0.989	0.11
2	0.030	0.942	0.03	0.070	0.943	0.07	0.110	0.930	0.12
3	0.070	0.702	0.10	0.090	0.716	0.13	0.090	0.678	0.13
4	0.060	0.639	0.09	0.120	0.633	0.19	0.110	0.634	0.17
5	0.080	0.553	0.14	0.100	0.540	0.19	0.080	0.542	0.15
Median			***0.10***			***0.16***			***0.14***
2005									
0	0.229	1.051	0.22	0.201	1.056	0.19	0.324	1.029	0.31
1	0.086	1.029	0.08	–0.094	1.025	–0.09	0.054	1.052	0.05
2	0.060	0.966	0.06	0.097	0.965	0.10	0.024	0.966	0.02
3	–0.014	0.752	–0.02	0.124	0.744	0.17	0.059	0.750	0.08
4	0.068	0.658	0.10	0.108	0.654	0.17	0.113	0.667	0.17
5	0.061	0.592	0.10	0.141	0.589	0.24	0.115	0.605	0.19
Median			***0.09***			***0.17***			***0.13***

Appendix H (Patterns of Performance and Progress)

Performance of students at stages 4–7 on multiplication and division on other domains 2002–2005

Multiplication and Division 2002–2005	**Stage 7 Adv. Mult've**	**Stage 6 Early Mult've**	**Stage 5 Repeated Add'n**	**Stage 4 Skip Counting**
Number of students	*36238*	*68604*	*68689*	*76663*
Proportion and Ratio				
5: Early Additive	5.8	31.7	48.3	17.5
6: Adv. Additive	22.0	43.8	12.8	1.9
7: Adv. Multiplicative	51.6	13.2	1.6	0.1
8: Adv. Proportional	19.2	0.8	0.0	0.0
Total Stages 7–8	***70.8***	***14.0***	***1.6***	***0.1***
Fractions				
5: Orders units fractions	16.9	44.1	50.0	30.2
6: Co-ordinates numerators & denoms	25.3	30.6	11.4	2.5
7: Equivalent fractions	26.9	7.0	0.9	0.1
8: Orders fractions w. unlike n/d	26.2	2.4	0.3	0.0
Total Stages 7–8	***53.1***	***9.4***	***1.1***	***0.1***
Decimals and Percentages (02–03)				
5: ID to 3 places	22.0	15.6	5.2	1.0
6: Orders to 3 places	19.5	8.7	1.8	0.3
7: Rounds to nearest whole, Tth, Hth	20.1	4.1	0.5	0.1
8: Converts dec. to %	19.8	1.4	0.2	0.0
Total Stages 7–8	***39.9***	***5.5***	***0.7***	***0.1***
Grouping and Place Value (02–03)				
5: Tens in 100	13.9	42.9	44.6	21.9
6: Tens and Hs in whole nos	19.1	26.1	9.3	1.6
7: Tens, Hs, Ths in whole nos	30.6	9.4	1.2	0.1
8: Tth, Hths, Thths in decimals	32.3	3.7	0.3	0.1
Total Stages 7–8	***62.8***	***13.0***	***1.5***	***0.2***
Place Value (04–05)				
5: Tens in nos to 1000	17.3	40.0	41.7	20.8
6: Tens, Hs, Ths in whole nos	31.7	32.5	13.4	2.8
7: Tths in dec/orders dec. to 3 places	28.6	8.0	1.2	0.1
8: Converts dec. to %	18.1	1.2	0.3	0.1
Total Stages 7–8	***46.6***	***9.2***	***1.4***	***0.2***
Basic Facts (04–05)				
5: Add'n & mult'n facts 2, 5, 10	5.4	25.7	43.9	26.2
6: Sub'n & mult'n facts	23.0	45.8	24.6	5.8
7: Division facts	45.7	17.9	3.8	0.4
8: Common factors & multiples	22.2	2.1	0.4	0.1
Total Stages 7–8	***67.9***	***20.0***	***4.2***	***0.5***

Appendix I (Patterns of Performance and Progress)

Performance of students at stages 4–8 on proportion and ratio on other domains 2002–2005

Proportion and Ratio 2002–2005	Stage 8 Advanced Proport'al	Stage 7 Advanced Mult've	Stage 6 Advanced Additive	Stage 5 Early Additive	Stage 4 Advanced Counting
Number of students	*7638*	*28675*	*48110*	*70235*	*98510*
Multiplication and Division					
5: Early Additive	0.3	3.6	18.1	46.6	24.6
6: Adv. Additive	6.5	30.9	62.1	30.9	7.0
7: Adv. Multiplicative	***93.0***	***65.2***	***16.8***	***3.0***	***0.4***
Fractions					
5: Orders unit fractions	2.9	17.8	41.1	53.4	31.6
6: Co-ordinates nums & denoms	8.7	31.5	36.0	13.6	3.2
7: Equivalent fractions	22.8	29.5	8.4	1.2	0.2
8: Orders fractions with unlike n/d	64.0	17.4	2.6	0.3	0.1
Total stages 7–8	***86.7***	***46.9***	***10.9***	***1.5***	***0.2***
Decimals and Percentages (02–03)					
5: ID to 3 places	8.0	27.0	16.2	7.6	1.5
6: Orders to 3 places	11.3	22.9	9.4	2.6	0.4
7: Rounds to nearest whole, Tth, Hth	22.0	19.8	4.6	0.7	0.1
8: Converts decimals to %	54.5	11.5	1.4	0.2	0.1
Total stages 7–8	***76.5***	***31.2***	***6.0***	***0.8***	***0.1***
Grouping and Place Value (02–03)					
5: Tens in 100	3.2	17.5	40.9	46.5	25.3
6: Tens & Hs in whole nos	6.7	22.3	29.5	12.1	2.9
7: Tens, Hs, Ths in whole nos	23.7	33.4	10.0	2.3	0.2
8: Tths, Hths, Thths in decimals	64.3	22.5	3.9	0.5	0.1
Total stages 7–8	***88.0***	***55.9***	***13.8***	***2.8***	***0.3***
Place Value (04–05)					
5: Tens in nos to 1000	3.8	19.5	36.6	43.0	22.6
6: Tens, Hs, Ths in whole nos	10.0	35.1	37.0	16.7	4.4
7: Tth in dec/orders dec. to 3 places	29.9	29.9	10.3	1.6	0.2
8: Converts decimals to %	54.7	11.3	1.4	0.3	0.1
Total stages 7–8	***84.5***	***41.2***	***11.7***	***1.9***	***0.3***
Basic Facts (04–05)					
5: Add'n & mult'n facts 2, 5, 10	1.1	6.8	20.3	40.5	27.8
6: Sub'n & mult'n facts	6.9	26.2	46.7	29.6	8.7
7: Division facts	32.4	48.6	22.5	5.7	1.0
8: Common factors & multiples	56.3	14.9	3.2	0.5	0.2
Total stages 7–8	***88.6***	***63.5***	***25.7***	***6.2***	***1.2***

Appendix J (Patterns of Performance and Progress)

Performance of students at stage 7 at the end of 2005 on other domains assessed at the same time

Stage 7 M/D on Addition and Subtraction 2005	A/S St 7 Final Adv. Mult've	M/D St 7+ Final Adv. Mult've	P/R St 7+ Final Adv. Mult've
Number of students	*2742*	*6612*	*6250*
Addition and Subtraction			
5 Early Additive		5.7	7.5
6 Adv. Additive		58.0	57.3
7 Adv. Multiplicative	***100.0***	***36.2***	***35.1***
Multiplication and Division			
5 Early Additive	1.6		2.6
6 Adv. Additive	10.5		25.3
7 Adv. Multiplicative	47.4	78.2	50.8
8 Adv. Proportional	40.2	21.8	21.0
Total stages 7–8	***87.6***	***100.0***	***71.8***
Proportion and Ratio			
5 Early Additive	2.9	6.0	
6 Adv. Additive	16.0	24.9	
7 Adv. Multiplicative	52.3	53.6	83.9
8 Adv. Proportional	27.9	14.3	16.1
Total stages 7–8	***80.2***	***67.9***	***100.0***
Fractions			
5 Orders unit fractions	8.5	20.5	18.0
6 Co-ordinates numerators & denoms	16.1	24.8	25.2
7 Equivalent fractions	32.2	27.5	29.0
8 Orders fractions with unlike n/d	41.7	24.5	26.3
Total stages 7–8	***73.9***	***52.0***	***55.3***
Place Value (04–05)			
5 Tens in nos to 1000	10.9	22.9	22.3
6 Tens, Hs, Ths in whole nos	20.7	29.5	28.8
7 Tth in dec/orders dec. to 3 places	33.7	25.7	26.8
8 Converts dec. to %	33.2	17.3	18.3
Total stages 7–8	***66.9***	***43.0***	***45.1***
Basic Facts (04–05)			
5 Add'n & mult'n facts 2, 5, 10	2.7	6.2	6.5
6 Sub'n & mult'n facts	16.1	26.5	26.2
7 Division facts	46.1	46.5	46.8
8 Common factors and multiples	34.6	19.9	19.7
Total stages 7–8	***80.7***	***66.4***	***66.5***

Appendix K (Patterns of Performance and Progress)

Performance of students at stage 8 at the end of 2005 assessed on other domains at the same time

Stage 8 P/R on Multiplication and Division 2005	M/D St 8 Final Adv. Prop'al	P/R St 8 Final Adv. Prop'al
Number of students	*1444*	*1008*
Addition and Subtraction		
5 Early Additive	0.8	1.2
6 Adv. Additive	23.1	23.1
7 Adv. Multiplicative	***76.1***	***75.7***
Multiplication and Division		
5 Early Additive		0.3
6 Adv. Additive		5.7
7 Adv. Multiplicative		29.3
8 Adv. Proportional		64.6
Total stages 7–8	***100.0***	***93.9***
Proportion and Ratio		
5 Early Additive	0.3	
6 Adv. Additive	8.7	
7 Adv. Multiplicative	45.8	
8 Adv. Proportional	45.1	
Total stages 7–8	***90.9***	***100.0***
Fractions		
5 Orders unit fractions	4.4	3.0
6 Co-ordinates numerators & denoms	10.0	6.6
7 Equivalent fractions	27.8	22.1
8 Orders fractions with unlike n/d	57.3	67.9
Total stages 7–8	***85.1***	***90.0***
Place Value (04–05)		
5 Tens in nos to 1000	6.4	4.7
6 Tens, Hs, Ths in whole nos	13.6	9.2
7 Tth in dec/orders dec to 3 plces	32.0	30.4
8 Converts dec. to %	47.2	55.0
Total stages 7–8	***79.2***	***85.4***
Basic Facts (04–05)		
5 Add'n & mult'n facts 2, 5, 10	1.0	0.9
6 Sub'n & mult'n facts	10.4	8.9
7 Division facts	41.2	35.1
8 Common factors and multiples	47.4	54.8
Total stages 7–8	***88.6***	***89.9***

Appendix L (Algebraic Thinking)

Table L1

Percentage of Year 7 Students from Intermediate School 1 Correct on Each Item

Operation	Item 1	Item 2	Item 3	Item 4	Item 5
Addition	70	35	3	1	1
Multiplication	35	16	3	0	0
Subtraction	23	13	4	0	0
Division	21	10	3	0	0

Table L2

Percentage of Year 8 Students from Intermediate School 1 Correct on Each Item

Operation	Item 1	Item 2	Item 3	Item 4	Item 5
Addition	77	57	6	3	1
Multiplication	49	28	6	1	0
Subtraction	38	25	9	2	2
Division	38	22	3	2	2

Table L3

Percentage of Year 9 Students from Four Secondary Schools Correct on Each Item

Operation	Item 1	Item 2	Item 3	Item 4	Item 5
Addition	68	57	16	15	7
Multiplication	43	32	13	12	6
Subtraction	26	21	12	10	7
Division	34	29	14	13	7

Table L4

Percentage of Year 10 Students from Four Secondary Schools Correct on Each Item

Operation	Item 1	Item 2	Item 3	Item 4	Item 5
Addition	79	64	32	27	17
Multiplication	54	44	22	19	13
Subtraction	30	23	18	15	11
Division	45	36	22	21	15

Appendix M (Te Poutama Tau – A Case Study of Two Schools)

THE UNIVERSITY OF AUCKLAND
NEW ZEALAND

TE PUNA WANANGA
FACULTY OF EDUCATION

11 Epsom Campus
Gate 3, 74 Epsom Ave
Auckland, New Zealand
Telephone 64 9 623 8899
Facsimile 64 9 623 8898
www.education.Auckland.ac.nz
The University of Auckland College of Education
Private Bag 92601, Symonds Street
Auckland 1035, New Zealand

Te Rārangi Pātai mō te Hunga Tumuaki

Te Tumuaki

1. E hia ō tau e whakaako ana?
2. E hia ō tau e mahi ana hei tumuaki?
3. He aha ō tohu whakaako?
4. Kua uru atu koe ki tētahi/ētahi akoranga, whakawhanaketanga ngaio rānei mō te pāngarau? Whakamārama mai.
5. He aha ō kaingākautanga ki te pāngarau?

Ngā āhuatanga ā-iwi, ā-whānau

6. He pēhea te āhua o te whanaungatanga i waenganui i te kura me te iwi kāinga, ngā iwi/ hapū?
7. He aha ngā kapapori pāpori, kapapori ohaoha *(socio-economic background)* o ngā tamariki?
   He teitei — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
   Kei waenganui — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
   He hakahaka — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
8. Te āhua o te whānau. He pēhea te āhua o ngā whānau?
   Kotahi anake te matua — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
   Tokorua ngā mātua — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
   He whānau whānui — te katoa/te nuinga/ētahi/ruarua noa iho/ kāre kau
9. Kōrero ai te whānau i te reo Māori i te kāinga?
   I ngā wā katoa — te katoa o ngā whānau/ ētahi/ruarua noa iho/ kāre kau
   I te nuinga o te wā — te katoa o ngā whānau/ ētahi/ruarua noa iho/ kāre kau
   I ētahi wā — te katoa o ngā whānau/ ētahi/ruarua noa iho/ kāre kau
10. Te takiwā o te kura. He aha te āhua o te takiwā o te kura?
   He taiwhenua *(rural)* He tāone iti *(small town)* He tāone nui *(big town/city)*

Te tataunga o te kura

11. He aha te tataunga o te kura? *(What is the school decile?)*
    He teitei (8–10)   Kei waenganui (4–7)   He hakahaka (1–3)

12. E hia ngā tamariki kei roto i te kura?

Ngā whakaritenga o te pāngarau

13. Ka pēhea tō whakarite i te kura mō te whakaako pāngarau? Whakamārama mai.

Te Poutama Tau

14. He pēhea tō tautoko i ngā pouako ki te whakatinana i Te Poutama Tau?

15. Ki ō whakaaro, ka pēhea te haere whakamua o Te Poutama Tau i roto i tō kura?
    He tino neke   He āhua neke   He iti noa   Kāore i neke

16. Ki ō whakaaro e tautoko ana Te Poutama Tau i te piki whakarunga o ngā tamariki kei roto i te pāngarau?
    He tino tautoko   He āhua tautoko   He iti noa   Kāore i tautoko

17. Ki ō whakaaro he aha ngā āhuatanga tērā i whakarite kia angitū tō kura i roto i Te Poutama Tau?
    Hei tauira:
    Te tautoko mai o ngā pouako.
    Te tautoko mai o te whānau.
    Te matatau o ngā pouako ki te pāngarau.
    Te kaingākaunui o ngā tamariki ki te pāngarau.

18. He kōrero anō āu mō Te Poutama Tau, mō te whakaako rānei i te pāngarau?

Te Rārangi Patapatai mō te Hunga Tumuaki
(Principals' Questionnaire)

(These are indicative questions only.)

Principal

1. How many years have you been teaching?
2. How many years have you been principal?
3. What academic qualifications do you have?
4. Have you done any courses or professional development?
5. What are your own interests in pāngarau?

Demographic Characteristics

6. Iwi identification. Is the school closely connected to iwi/hapū?
   One iwi/hapū mixture of iwi/hapū
7. What is the socio-economic background of the tamariki?

High	All/Most/Some/Few/None
Middle	All/Most/Some/Few/None
Low	All/Most/Some/Few/None

8. Family Type. What are the characteristics of the whānau?

Single Parent	All/Most/Some/Few/None
Nuclear family	All/Most/Some/Few/None
Extended family	All/Most/Some/Few/None

9. Do the whānau speak te reo Māori?

All the time	All/Most/Some/Few/None
Most of the time	All/Most/Some/Few/None
Sometimes	All/Most/Some/Few/None

10. School locality.
    What are the characteristics of the local area?
    Rural Minor urban *(small town)* Major urban *(big town/city)*

School Characteristics

11. What is the school decile?
    High (8–10) Medium (4–7) Low (1–3)
12. What is the school roll?
13. How do you organise the school for pāngarau?

Te Poutama Tau

14. What kinds of support do you provide to teachers for the implementation of Te Poutama Tau?
15. How well do you rate your school's progress in Te Poutama Tau?
16. Do you think Te Poutama Tau has raised general pāngarau achievement?
17. What do you think are the factors that have lead to your school's success in Te Poutama Tau? For example, teacher support/attitudes, whānau involvement/support, peer (teachers' and students') support, resource quality, facilitator support.
18. Do you have anything else to add about Te Poutama Tau or mathematics?

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Telephone 64 9 623 8899
Facsimile 64 9 623 8898
www.education.Auckland.ac.nz
The University of Auckland College of Education
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Auckland 1035, New Zealand

Te Rārangi Pātai mō te Hunga Pouako

Ngā mahi whakaako

1. E hia ō tau e whakaako ana?
2. E hia tō roa e whakaako ana i tēnei kura?
3. He aha ō tohu whakaako?
4. Kua uru atu koe ki ētahi atu akoranga, whakawhanake ngaio rānei i kō atu i Te Poutama Tau?
5. He aha ngā marautanga e tino kaingākautia ana e koe?

Te āhua o te akomanga

6. Ko te āhua o tō kura/akomanga:
   - He kura kaupapa Māori?
   - He kura rumaki?
   - He kura ā-iwi?
   - He akomanga rumaki i te kura auraki?
   - He akomanga reo rua?
   - He momo kura kē atu?
7. Tokohia ngā tamariki i tō akomanga?
   5–10 11–15 15–20 20–30 30+
8. Nō ēhea reanga ā-tau ngā tamariki i tōu akomanga?
   T1, 2, 3, 4, 5, 6, 7, 8, 9, 10
9. E hia ō tau e whakaako ana i taua/aua reanga ā-tau/akomanga?
10. He pēhea te matatau o tō akomanga ki te reo Māori?
   - He tino matatau te katoa.
   - He matatau te nuinga.
   - He āhua matatau te nuinga.
   - Kāore i te matatau te nuinga.

11. He pēhea ngā waiaro o ngā tamariki ki te pāngarau i mua i Te Poutama Tau?
    - He tino ngākaunui te katoa.
    - He ngākaunui te nuinga.
    - He āhua ngākaunui te nuinga.
    - He iti nei ō rātau ngākaunui.

12. He pēhea te whakaaro o ngā tamariki ki te pāngarau i nāianei?
    He ōrite tonu He āhua rerekē He tino rerekē

Te whakaako i Te Poutama Tau

13. E hia ō tau e whai wāhi mai ana ki te kaupapa o Te Poutama Tau?

14. I whakahaeretia Te Poutama Tau i te whānuitanga o te kura?
    Āe Kao

15. Mehemea ko koe te kaiwhakahaere o Te Poutama Tau ki tō kura, he aha ētahi o ngā whakaritenga matua mō tēnei kaupapa?

16. He pēhea tō whakaako i te pāngarau i nāianei? He rite tonu, he rerekē? Whakamārama mai.

17. He aha ngā rautaki whakaako o Te Poutama Tau e tino pai ana ki a koe? Whakamārama mai.

18. He aha ngā wāhanga tino pai o Te Poutama Tau ki a koe? Whakamārama mai.

19. He aha ngā wāhanga tino pai o Te Poutama Tau ki ō tamariki?

20. I pēhea koe i whakamahi ai ngā rauemi o Te Poutama Tau? Whakamārama mai.

21. Ki ō whakaaro, he aha ngā rauemi matua? Whakamārama mai.

Te tautoko a te kura

22. He pēhea nei te tautoko mai o tō kura i a koe e whai atu ana i Te Poutama Tau?
    - Ka tino tautoko.
    - Ka āhua tautoko mai.
    - Kāore e tino tautoko mai i ētahi wā.
    - Kāore i te tino tautoko.

23. He kōrero anō āu mō Te Poutama Tau, mō te whakaako rānei i te pāngarau?

Te Rārangi Pātai mō te Hunga Pouako
(Teachers' Questionnaire)

(These are indicative questions only.)

Teaching Experience

1. How many years have you been teaching?
2. How many years have you been teaching in this school?
3. What academic qualifications do you have?
4. Have you done any courses or professional development in pāngarau outside of Te Poutama Tau?
5. What are your main curriculum areas?

Characteristics of Class

6. Is/was your class:
   - kura kaupapa Māori?
   - total immersion school?
   - total immersion class in an English-medium school?
   - bilingual class?
   - another type of class?
7. How many children did you have in your class?
   5–10 11–15 15–20 20–30 30+
8. What year group were they?
   Y1, 2, 3, 4, 5, 6, 7, 8, 9, 10
9. How many years have you been teaching this age group?
10. How would you rate te reo Māori fluency of your class?
11. What is/was the attitude of the children to pāngarau?
12. Has their attitude to pāngarau changed?

Teaching Te Poutama Tau

13. How many years have you been involved in the Te Poutama Tau project?
14. Do you have school-wide responsibilities for Te Poutama Tau?
15. If you are the lead teacher, what are some of the main factors to consider?
16. Has your own teaching style been affected by Te Poutama Tau?
17. What are some of the effective strategies of Te Poutama Tau?
18. What do you find most effective about Te Poutama Tau? Explain.
19. What aspects of Te Poutama Tau do your children enjoy most?
20. How have you used the equipment?
21. What has been the key equipment?

School support

22. How has the school supported you in the Te Poutama Tau project?
23. Do you have anything else to add about Te Poutama Tau or mathematics?